where $d_{ii} = \frac{(x^{**}_{in})^3}{q^3}$; $d_{ij}=d_{ji}$; $d_{ij} = \frac{(x^{**}_{jn})^2}{2q^3}(3x^{**}_{in} - x^{**}_{jn})$ at $i<j$; $d_{iw} = \frac{(x^{**}_{in})^2}{2q}(3 - \frac{x^{**}_{in}}{q^2})$.

Note that all coordinates marked with two asterisks are equal to corresponding nodal points' coordinates [derived from (4a)] divided by the length of cantilever L that makes them dimensionless. Now one can estimate the deflection at the cantilever's quasi-end offset section caused by superposition of forces W, W_1 , $W_{2,}$ W_j action which is equal to: $s = s_{qend}(W) - \sum_j^{(n-1)} s_{qend}(W_j) = A_{qn}\frac{L^3}{3EJ}$,

where $s_{Wqend} = \frac{WL^3q^3}{3EJ}$; $s_{W_jqend} = \frac{W_j\,(x^{**}_{jn})^2L^3}{6EJq}(3 - x^{**}_{jn}/q^2)$. Substituting these expressions, we obtain the total deflection in the offset moving end section and the effective statical spring constant for the cantilever in the n-th mode of resonant oscillations as $K_{neff} = \frac{3EJ}{A_{qn}L^3}$. This allows us to estimate the ratio between the effective spring constant K_{neff} at n resonant mode and the same parameter K_{1eff} $=3EJ/(L^3)$ for the first resonant mode of prismatic cantilever that equals $K_{neff}/K_{1eff}=A_q^{-1}$. Therefore the ratio between effective mass factor for the n-th mode and the first mode of oscillations is $n^*_n/n^*_1 = \frac{f_1^2}{f_n^2}K_{neff}/K_{1eff} = \frac{(\alpha L)_1^4}{(\alpha L)_n^4}A_{qn}^{-1}$.

Let us recall that $n^*_1 \approx 0.24$ for prismatic rectangular cantilever. Therefore, the effective mass factor for n-th mode of oscillations of the same cantilever is equal to:

$n^*_n = 0.24\frac{(\alpha L)_1^4}{(\alpha L)_n^4}A_{qn}^{-1}$. Hence the ratio between sensitivity to additional mass Δm at the n-th mode of resonant oscillations and the first resonant oscillation is as follows: $\frac{\Delta f_n(\Delta m)}{\Delta f_1(\Delta m)} = \frac{f_1(K_{neff}/K_{1eff})n^*_1(n^*_1m_c+\Delta m)}{f_n n^*_n(n^*_n m_c+\Delta m)} \approx \frac{f_n n^*_1}{f_1 n^*_n}$, where m_c is the mass of the cantilever. As a result, the sensitivity to additional mass at the higher mode equals

$\Delta f_n/\Delta m = \frac{f_n n^*_1}{f_1 n^*_n}\Delta f_1/\Delta m = \frac{(\alpha L)_n^2 n^*_1}{(\alpha L)_1^2 n^*_n}(\Delta f_1/\Delta m)$. It is clear that at the nodal points

$K_{neff}=\infty$, $n^*_n=\infty$, and $\Delta f_n/\Delta m=0$. *See* Y.M. Tseytlin (2005), idem. ibid., *Rev. Sci.Instrum.* **79**, 025102 (2008) for more detail. *See* also M.B. Bauza *et al*. (2005), *Rev. Sci. Instrum.*,**76**, 095112 with application of vibrating shank virtual probe. Analysis indicates that nanomechanical resonators offer immense potential for mass sensing ultimately with the resolution at the level of individual molecules. A mass responsivity of approximately 5 fg/ Hz has been observed for micrometer sized cantilever when operating the cantilever in the fourth mode. The largest attainable natural frequency of the triangular cantilever is higher than that of the rectangular one with the same dimensions (h, L) and of the same material (*see* section 5-4).

Internal and external damping.

It was found in the experimental study that Q-factor of silicon triangular cantilevers and their arrays with length from 500 nm to 100μm and thickness from 30 to 100 nm

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