from 0 to 8 N can be found (Y.M. Tseytlin, 1972) on the basis of the following linear relationship $s_1 = 0.83\dfrac{Q_1 L^3}{12\,H}$. (8-9)

Let us verify the condition (2-13), which proves the rightful choice of the approximating polynomial by inequality $\dfrac{|\gamma(Q_{1r})| - |\gamma(Q_{1c})|}{2} = \dfrac{0.1-0.06}{2} = 0.02$ mm≤{Δs}.

Therefore, the largest error {Δs} of calculation with the proposed method can be equal 0.02 mm. If this error is permissible, then the condition (2-13) is satisfied and the choice of the polynomial is right. The relative error of the approximate s_1 calculation is not more than 1.73% when the largest driving force Q_{1m} =8 N.

Calculation of displacement s_1 on the basis of common linear bending strain theory in the corresponding region yields an error of 20% (*see* Y.M. Tseytlin, 1972). We see the same deflection from the common linear theory in the stiffness expression, which follows from (9)

$$Q_1/s_1 = 1.2\frac{12H}{L^3} \tag{8-10}$$

One can derive a similar formula for the stiffness of cantilever at large deflections from (9) if to assume that the elastica in the figure (8-10a) consists of two cantilevers with length l=1/2L. In this case, we have

$Q/s = 1.2\dfrac{3H}{l^3}$, instead of $Q/s = 3H/l^3$ in accordance with common linear mechanics for small cantilever's deflections. A similar 20%increase in stiffness (spring constant) with increase of bending forces and flexural displacement was found in the experiments on fabricated ultrashort nanocantilevers with length of 2μm, width of 150 nm, and thickness of 50 nm.

The more complicated approach is to yield the approximate relationship for longitudinal displacement s_2. We will use the equation (2-9) for this purpose because the absolute values of s_2 are small. Let us consider Q_{10} =0.4 N (displacements s_2 under smaller forces are negligible indeed), Q_{1m} = 4N, and Q_2 again is equal to zero. For the purpose of convenience, we can use the dimensionless force coefficient $(\lambda_0'')^2 = \dfrac{1}{4}\dfrac{Q_1 L^2}{H}$, instead of force Q_1, and then apply values $\dfrac{s_2}{(\lambda_0'')^2 L}$.

As a result, we have the sought expression as follows

$$0.0456 - \frac{s_2}{(\lambda_0'')^2 L.}(Q_{1s}) = 0, \text{ which yields } \frac{s_2}{(\lambda_0'')^2 L.}(Q_{1s}) = 0.0456. \tag{8-11}$$

The smallest force value Q_{10} yields the following equation $\dfrac{s_2^{(0)}}{(\lambda_0'')^2 L.}(Q_{10}) = 0.0736$.

The approximate relationship for displacements s_2 can then be established (Table 8-10) on the basis of the similar triangles

$$s_2 = \left[0.027\frac{0.46-(\lambda_0'')^2}{0.4} + 0.0456\right](\lambda_0'')^2 L\,. \tag{8-12}$$

We assume that condition (2-13) for displacements s_2 is satisfied if an error in s_2 calculation is less than 10%. This error slightly increases if force Q_{1m} values are http://www.springer.com/978-0-387-25156-1

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