Mechanical oscillators. The typical differential equations for the oscillators 0I, III, IV, and V at the small natural vibrations can be represented as follows:

for oscillator 0I $\quad m_1\ddot{x}+j_1x=0,\quad m_1\ddot{x}+j_1x=Q\delta(x-a)$, (5-11a)

for oscillator III $\quad M_{L1}\ddot{y}+j_3\ y=0,$ (5-11b)

for oscillator IV $\quad M_{L2}\ddot{z}+C_x\dot{z}=2L^{-1}Q(z-z_1)+2j_\theta i_\theta L^{-2}(z-z_1)^3,$ (5-11c)

$$J_Q\ddot{\theta}+C_\theta\dot{\theta}=-2(j_{\tau\tau}\,\theta/L)+j_{k\tau}z^2/L^2, \tag{5-11d}$$

and for the oscillator V with cantilever pointer whose two segments supported in one intermediate point $\quad (EJ_{1,2}m^{-1}_{1,2}\ \partial^4v/\partial x^4)+\partial^2v/\partial t^2=0.$ (5-11e)

Solution to the last equation is shown in Chap. 7. In this section we will study oscillators I, II, and IV, which have not been previously studied in all options and are essential for the helicoidal multivibrators. Let us discuss this problem in more detail.

Pretwisted string. The helicoidal (initially pretwisted) beam-strip with fixed ends has flexural (in two planes), longitudinal, and torsional vibrations that are mutually related (*see* B.F. Shorr, 1961). In the linear problem for quasi-constant relations at the vibration of the pretwisted beam-strip with double-symmetrical profile under longitudinal tension, this system splits into two independent systems: the flexural and the longitudinal-torsional vibrations. Let us note that the inductance voltage in the helicoidal multivibrator is generated only by its transverse motion which is perpendicular to the magnetic lines, i.e., by the flexural vibration in one plane. Hence, in the linear problem with the elimination of non-used expressions for torsional vibrations, the transverse vibration of the helicoidal strip-string can be described by the partial differential equation with the periodic coefficients such as follows

$$Q_0(\partial^2v/\partial x^2)-EA\ \partial^2/\partial x^2\,[\kappa_j^2(x)\partial^2v/\partial x^2]=\rho_\gamma A(\partial^2v/\partial t^2), \tag{5-12}$$

where $\kappa_j^2(x)=(1/A)\,[J_\eta\sin^2\varphi(x)+J_\xi\cos^2\varphi(x)]$ is the equivalent of radius of gyration for the helicoidal strip's cross section, Q_0 is the longitudinal pretension of the strip, ρ_γ is the density of the material, A is an area of the strip's cross section, J_η, J_ξ are the section's moments of inertia, $\varphi=(2\pi/S_0)x$, and S_0 is the pitch of the pretwisted helix. Note that function $\kappa_j^2(x)$ is a periodical one. Let us evaluate the mean value $\bar{k}_j^2(x)$ in the limits of one helix pitch S_0 on the pretwisted strip

$$\bar{k}_j^2(x)=[1/(S_0\mathrm{A})]\int_0^{S_0}dx\ [J_\eta\sin^2(2\pi/S_0)x+J_\xi\cos^2(2\pi/S_0)x]=$$

$$\tfrac{1}{2}(J_\xi/A)+\tfrac{1}{2}(J_\eta/A)=\tfrac{1}{2}J_p/A. \tag{5-13}$$

Furthermore, if the radius of gyration is constant and equal to the found $\overline{\kappa_j(x)}$, we have from (12) a solution similar to that for the rigid flat string with $\kappa_j^2(x)=\bar{k}_j^2(x)$ and at the fixed ends $Y(0)=Y(L)=Y'(0)=Y'(L)=0$, which yields

$$f_n\approx(n_f/2L)\sqrt{Q/\rho_\gamma A}\,[1+(2/L)\sqrt{EJ_p/2Q}+(4+\frac{n_f^2\pi^2}{2})EJ_p/(2QL^2)], \tag{5-14}$$

where f_n is the natural frequency of vibration for the elastic helicoidal multivibrator at $n_f \ll L/S_0$. So the number of harmonics n_f should be not more than 3 because in the http://www.springer.com/978-0-387-25156-1

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