typical multivibrator $L/S_o = 9$. The difference between (14) and the known analogous relationship for the flat string is in the substitution of the radius of gyration for the parameter $\sqrt{(1/2)J_p/A}$. At $J_p \to 0$ expression (14) corresponds to the formula for the vibration of the regular flat string under tension. By substituting the expression $v = Y(x)e^{-2\pi ift}$ into (12), we obtain the ordinary quartic differential equation of the fourth order with periodic coefficients:

$$-QY'' + [f(x)Y'']'' + \chi g_\chi Y = 0, \tag{5-15}$$

where $f(x) = \kappa_j^2 [(2\pi/S_o)x]$, $g(x) = 4\pi^2\rho_\gamma A$; $\chi = f^2$. (5-16)

This conjugate equation with the conjugate boundary-value conditions $Y(0) = Y(L) = Y'(0) = Y'(L) = 0$ has eigenvalues with multiplicity not more than two. Equation (15) fully corresponds to the problem for vibration of the rigid string with the section radius of gyration equal to the mean value of the radius of gyration for the initially pretwisted helicoidal string $k_j = \bar{k}_j = \sqrt{J_p/2A}$. (5-17)

Thus, we again come to the same expression (14) for the natural frequency of the helicoidal multivibrator, which was established earlier by the physical approach to the problem.

Uncertainty of the derived equation. The approximate expression similar to (14) for the rigid uniform flat or round string is an effective one at $EA\kappa_j^2 < L^2Q_0$. But the differential equation for vibration of the rigid string in general can be solved only by numerical methods. In Table 5-1 the uncertainty of the solution for (14) is shown in comparison with the numerical solution to a similar flat string.

Table 5-1. Estimation of an uncertainty in the application of (14)

$\frac{Q_0L^2}{AE\bar{k}_j^2}$	$\sqrt{\frac{\rho_\gamma L^4}{Ek_j^2}}\,\omega_0$ for (12)	$\omega\sqrt{\frac{\rho_\gamma L^4}{E\bar{k}_j^2}}$ from (14)	$\frac{\omega-\omega_0}{\omega_0}$ %	(i)		(ii)		(iii)	
				Q_0 cN	$\theta°$	Q_0 cN	$\theta°$	Q_0 cN	$\theta°$
106.5	41.9	41.42	-1.10	2.17	48.4	-	-	-	-
243.2	57.3	57.08	-0.39	4.97	110.5	2.15	86.3	0.74	73.7
564.3	82.2	82.07	-0.16	11.54	256.4	5.00	200	1.71	170.9
1358.0	122.9	122.8	-0.08	27.76	617.0	12.05	500	4.11	411.3

Note: $\omega_0 = 2\pi f_0$; $\omega = 2\pi f$. Helicoidal strip of zinc bronze with dimensions in mm :
(i) 0.008x 0.12x π x 18; (ii) 0.006x 0.10x π x 18; (iii) 0.004x 0.08x π x 18.

The uncertainty of natural frequency calculation in the case of the helicoidal multivibrator with preliminary untwist $\theta = 90°$ and $Q_0L^2/(AE\bar{k}_j^2) = 243.2$ is less than 0.4%, and an uncertainty is less than 0.16% for the more typical adjustment with $\theta \geq 200°$ and $QL^2/(AE\bar{k}_j^2) \geq 564.3$. However, the influence of the section rigidity on the frequency value ω for the helicoidal multivibrator is significant in accordance with the formula

$$\delta(\bar{k}_j^2, Q_0) = 2(EA\bar{k}_j^2/Q_0L^2)^{1/2} + (4+\pi^2/2)\,EA\bar{k}_j^2/(Q_0L^2), \tag{5-18}$$

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