

Notes to Table 5-3: \*\* This formula is based on the frequency equation  $\cos \alpha L \cosh \alpha L = -1$  with the first root  $\alpha L = 1.875$ . \*\*\*The formula is based on the frequency equation  $\cos \alpha L \cosh \alpha L = 1$  with the first root  $\alpha L = 4.730$  (see Y.G.Panovko, 1968); if the beam has one fixed and one simply supported end as in the contact mode of AFM (see Chap. 8), its frequency equation is  $\tan \alpha L = \tanh \alpha L$  with the first root  $\alpha L_n = 3.927$ , and the natural frequency of the simply supported beam increases with addition of axial tensile force  $Q_{ax}$  and decreases with the axial compression force  $-Q_{ax}$  in accordance with (S.P. Timoshenko *et al.*, 1964) the following

expression :  $f_0 = \frac{h n_f^2 L^{-2}}{0.702 \pi} \sqrt{E / \rho_\gamma} \sqrt{1 + \frac{Q_{ax} L^2}{n_f^2 E J \pi^2}}$ , where we accept  $J = b h^3 / 12$  and  $n_f$  is

the mode number. \*\*\*\*Influence of surrounding air or liquid (Weaver *et al.*, 1990)

with density  $\rho_\gamma$  reflects in the factor  $\gamma_f = 1 / \sqrt{1 + \beta_f}$ , where  $\beta_f = 0.6689 \frac{\rho_\gamma l}{\rho_\gamma} R / h$ .

Stress  $\sigma_{\max}$  is the largest acceptable stress for the material under working conditions:  $\sigma_e$ ,  $\sigma_f$ , or  $\sigma_{en}$  (see Table 1-14).

The last column in Table 5-3 shows the relationship between natural frequency  $f_0$ , minimal sag  $\delta_{\min}$ , and maximum stress  $\sigma_{\max}$  without any elastic element's geometrical dimensions figuring in these formulae. It is important to note that in the above reasoning a complete utilization of the elastic element's strength characteristics was assumed. The similar procedure was used for the round plane diaphragms subjected to a distributed load (pressure) whose formulae are also shown in Table 5-3's row 4. Analysis of the formulae in Table 5-3 shows that the product of the natural frequency  $f_0$  of the elastic element on its minimal sag  $\delta_{\min}$  does not depend on the element's size, but is completely determined by the selected material features. Moreover, this product is structurally the same for all studied elastic systems. The computation's results of  $f_0$  for elastic elements made of the different materials (steel, beryllium bronze, titanium, and aluminum alloys), in the case when minimal sag  $\delta_{\min} = 10 \mu\text{m}$ , show that the natural frequencies are in the range from 50 to 113 kHz. However, we should recall that the value of the minimal sag was selected in this case as a typical for the pressure electromechanical transducers of different types known in 1970s. It should be noted that this value is a conditional one and is changed in new structures, which have much higher fundamental frequencies. The largest natural frequency for the flat strings can be defined from the expression  $f_n = 0.5 n_f L^{-1} \sqrt{(\sigma_e) / \rho_\gamma}$ , (5-86)

where  $n_f$  is the harmonic's number and  $\{\sigma_e\}$  is the permissible yield stress for the string's material. Then at the minimum value of the string's length  $L = 1 \text{ mm}$  (this number is conditional with respect to the mesoscopic structure's options), the largest yield stress value  $\{\sigma_e\} \approx 1800 \text{ MPa}$  of the elastic materials in the bulk state, and the density of the material  $\rho_\gamma \approx 8 \cdot 10^{-6} \text{ kg/mm}^3$ , we obtain  $f_{n=1} \leq 230 \text{ kHz}$ .

*Revisited estimations.* Today these options have large progress in the micro and nanotechnology field. The nanowire-based very-high-frequency electromechanical resonator is already fabricated with natural (fundamental) frequency of 105.3 MHz (A. Hussain *et al.*, 2003). The resonator is built of suspended platinum nanowire and



<http://www.springer.com/978-0-387-25156-1>

Structural Synthesis in Precision Elasticity

Tseytlin, Y.M.

2006, XVI, 400 p. 96 illus., Hardcover

ISBN: 978-0-387-25156-1