Chapter 1

AN OVERVIEW

1. Introduction

1. This book deals with the two topics cited in the title: biased technical change and economic conservation laws. At first glance, the two might appear to have no connection with one another, yet when viewed from the perspective of the optimal control behavior of the primary agents in an economic analysis, both topics have much in common and are profoundly related as phenomena that are the result of optimized behavior.

The term "biased technical change," needless to say, refers to a situation in which the factors of production (in this case, capital and labor) each achieve technical change at different rates. As a result, the efficiency of capital and the efficiency of labor exhibit different growth rates. Consequently, even though the ratio of capital to labor remains constant, the shares of each are affected by technical change. This is the essence of biased technical change.

2. The accepted postulate among economists has been that, as a special case of biased technical change, only Harrod neutral technical change is valid as a condition for balanced growth in a mature economy. In other words, for long-run macroeconomic stability, technical change must of necessity be Harrod neutral technical change. To be more precise, this means that the only hypothesis conducive to long-run balanced growth is one in which capital-augmenting technical change does not exist.

What economists have learned from serious empirical analysis over the past few decades, however, is that capital-augmenting technical change is not zero. On the contrary, it varies significantly in an economic upswing or downturn, increasing greatly during the former and decreasing during the latter. The trend is not zero, however. In short, technical change during an upswing is not completely canceled out by technical change during a downturn. Consequently, Harrod neutral technical change, which assumes a zero rate of capital-augmenting technical change not just over the short run but even in the long run, is an unrealistic hypothesis. It is merely the simplest mathematical hypothesis to ensure long-run macroeconomic stability.

3. It makes no sense to regard advances in information technology, computer functions, for example, which have been growing by leaps and bounds, as all the result of a rise in Harrod neutral labor efficiency. In the ten years that the author has used a computer, the technical change of his labor has remained virtually the same. Nevertheless, it is a fact that every time I buy a new computer, the efficiency of my work rises rapidly.

This fact is readily analyzable if one simply accepts that the computer's capital efficiency has risen. In order to explain why the author is able to make more expeditious use of the computer while his labor efficiency remains unchanged, an indirect explanation-the assumption that new computers are improving because the productivity of those workers involved in manufacturing them has gone up-is far-fetched. This would mean believing indirectly in a Ricardo-Marxian labor theory of value in which all values derive from labor.

4. If labor and capital are regarded as independent production factors, a company/agent might focus its energy on increasing the efficiency of labor over that of capital, depending on the economic situation at the time. Or, faced with too little capital, it might invest more resources into increasing the efficiency of capital rather than labor.

At the time of the oil crises of the 1970s, companies confronted by soaring oil prices focused on technical change as a way to economize on capital, i.e. oil. Indeed, technological development was precisely what saved Japan's resource-poor economy. In Japan, incidentally, this is called "sho-ene" or energy-saving technology development.

5. The differences between this book and other works are (1) its main theme is an analysis of precisely this sort of technical change in which the efficiencies of capital and labor rise at different rates, and (2) it shows that long-run economic stability is possible under a biased technical change other than Harrod neutral. The basic concept here is that, of the different types of technical change, capital-augmenting technical change occurs endogenously. To be more precise, the theory that an economy achieves stable balanced growth only under the existing Harrod neutral technical change is limited to a situation in which technical change of labor and of capital is exogenous; in short, the theory applies only to those situations in which technical change is bestowed upon a nation or one of its companies by another nation or another company or by Heaven without that nation or its company using any resources of its own. But technical change ought to be regarded as attainable only when investment is made in resources. This is the theory of endogenous technical change put forward in this book.

6. The essence of the theory of endogenous technical change is the question of optimization: i.e. how much of the resources that a nation or a company possesses should it invest in technical change? Specifically, this book attempts to analyze this problem using the Pontryagin-Hesteness-Bellman calculus of variation or an optimal control method.

7. The theory of economic conservation laws that makes up the second theme may be less familiar to many economists than the first concept of biased technical change. But in the fields of physics, applied mathematics and other modern sciences, it is becoming a tool for studying the question of optimization in greater depth.

The above mentioned calculus of variation or optimal control theory is a suitable tool to explain natural or economic phenomena; behind variables and systems that are mathematically observable are phenomena that cannot at first glance be observed. To take a well-known example in physics, there is a rule (conservation law) that, while the movement of a billiard ball is observable, the sum of the kinetic energy and the potential energy behind it, which are impossible to observe, is constant; this rule is constancy of the Hamiltonian function.

8. In endogenous occurrences of biased technical change or in the maximization of a company's long-run profits, we are able to observe fluctuations over time in technical change rates or in profit rates. Likewise, in models dealing with a country's optimum capital savings or its optimum technical change rate, we can observe variables in savings and investment, consumption and GDP, just as we can observe the movement of a billiard ball, but, of course, the more essential but invisible and hidden law behind these optimized phenomena cannot be observed. Yet, it is this law, which the eye cannot see, that embodies the essence of the endogeneity question.

Several hidden conservation laws may exist in a single optimized system. The methodology for making a close and accurate analysis of this sort of problem is the Emy Noether theory of invariance using Lie groups. To sum up, the two main themes are bound together by the common terms endogeneity and conservation laws.

9. A Short History of Biased Technical Change. The work that formally introduced the concept of technical change to modern economics was The Theory of Wages (1932) by Sir John Hicks. Hicks analyzed the impact of inventions and technical change on the shares of labor and capital in Britain's post-industrial-revolution economy and society. Even in situations where the ratio of capital to labor was constant and unchanging, if technical change occurred, it might have an effect on the shares of capital and labor or their distributive shares.

Even though the ratio of capital to labor remained unchanged, a new invention might make the share of capital increase and the share of labor decline. Or the opposite might occur. As the simplest case, Hicks conceived of a situation in which a new invention or technical change had absolutely no impact whatsoever on the shares of labor and capital. To be more precise, one in which, under a constant capital-to-labor ratio, a new invention or technical change would have no effect on the income distributions of labor or capital, or the effect would be neutral. This subsequently came to be called Hicks neutral technical change.

10. The 1930s when Hicks advanced his theory of technical change was the era of the Great Depression, which had started in the United States and spread throughout the globe. The world's economists paid no attention to Hicks' contribution to the analysis of questions related to growth and technical change. Keynes' General Theory came out in 1936, a few years after the publication of Hicks' book, at the very time when an academic theory of how to get through the depression was in high demand. Needless to say, Keynes' work attracted the attention not only of economists but of politicians and policy-makers.

After World War II at the beginning of the 1950s, economists primarily in the United States began to think that, by skillfully combining Keynes' prescription and market principles, it would be possible for the world economy to escape recession and achieve stable growth. This was the theory of a mixed economy set forth by Paul Samuelson.

11. By the mid 1950s the belief had become prevalent that the world economy, and especially the American economy, would never experience another Great Depression like the one in the 1930s. On the contrary, given the prosperity and growth of the US economy resulting from postwar technological advances, there was a growing view that, in addition to the growth of labor and capital, reconsideration needed to be given to the significance of the contribution of technical change. For the first time, the theory of technical change that Hicks had analyzed in the 1930s began to attract economists' interest. Representative of this trend was Professor Robert Solow of MIT.

12. Solow analyzed growth trends in the non-agricultural private sector using Kendrick data for the US economy between 1909 and 1949 (Review of Economics & Statistics, 1957). Using a neoclassical production function of constant returns, he hypothesized that the growth rate for income in the non-farm private sector would also be affected by the growth rate for technical change over and above the growth rates for capital and labor. The technical change that Solow used here was the very same neutral type that Hicks had analyzed in the 1930s.

Solow ascertained that part of the income growth rate which is not dependent on the growth rates of labor and capital occurs as a result of technical change. Moreover, he succeeded in estimating the growth rate of technical change simply by subtracting the sum of the growth rates of labor and capital weighted by their distributive shares from the growth rate of income. According to Solow's estimates, approximately one-third of the income growth rate is dependent on technical change. Thus, Hicks' technical change of the 1930s was revived by Solow in the 1950s.

13. Solow's achievement had an enormous impact on young economists at the time. In particular, Solow's method opened the way to estimate from the same data both the production function, which is hard to estimate, and the rate of technical change. The paper by Professor John Kendrick and the author (American Economic Review, 1963) brought this work to completion.

Using the same Kendrick data as Solow had, it was possible to estimate the technical change rate and the production function simultaneously by estimating the elasticity of factor substitution, a concept that Professor Hicks had developed in the 1930s. It was estimated that the technical change rate in the period 1909–1920 was an average of 2.1 percent, and the elasticity of factor substitution was 0.6. Thus, it was proposed that the most suitable theory for explaining the growth of the American economy in that period is a production function with an elasticity of factor substitution of 0.6 (a CES production function).

Another important point made in this article was that the estimated value of the elasticity was smaller than the Cobb-Douglas production function (=1) emphasized in the Solow estimate, i.e. a production function with an elasticity of factor subsitution of 0.6, more approximately explained the growth of the American economy.

14. There were doubts at the time whether the Hicks neutral concept might be too unrealistic to explain technical change. Economists began to perceive a need to reconsider once again the concept of neutral technical change and to analyze how technical change is related to a neoclassical production function.

In addition to the Hicks' concept of neutral technical change, in the late 1940s Harrod had studied technical change which would enable a mature economy to achieve a long-run, stable, balanced growth path, one in which a constant income-capital ratio is maintained. In other words, technical change in which the shares of labor and capital are unchanged when the ratio of income to capital is constant. This came to be called Harrod neutral technical change.

Analysis was also made of a type of technical growth which is the antithesis or mirror image of this and which applied to the economies of developing countries, i.e. one in which there is absolutely no impact on the shares of capital and labor both before and after technical change when the ratio of income to labor is constant. This is Solow-Ranis-Fei neutral technical change.

15. In the empirical field of technical change, on the other hand, a multiplicative type of technical change, in which the efficiencies of capital and labor are different, is applicable to empirical data. This is the so-called factor augmenting technical change.

Although factor augmenting technical change was known to include Hicks neutral, Harrod neutral as well as Solow-Ranis-Fei neutral technical change, the theoretical grounds under which it maintained neutrality were completely unknown. To put it another way, it was not known under what conditions the distributive shares of capital and labor would remain constant, as in the three other types of neutrality.

The First answer to this question was the article that the author and Professor Beckmann published in the Review of Economic Studies (1968). The note that Professor Rose published a year later in the Economic Journal showed the same results as the Sato-Beckmann paper. This is the theory that was later called Sato-Beckmann-Rose technical change. It is also an explanation of the above mentioned factor augmenting technical change. Hicks, Harrod and Solow-Ranis-Fei neutral are all defined as special cases of Sato-Beckmann-Rose neutral. Hicks neutral is the case in which the efficiencies of capital and labor remain the same; Harrod neutral is one in which only the efficiency of labor goes up; and Solow-Ranis-Fei neutral is one in which only the efficiency of capital goes up.

16. What is the neutrality principle that justifies Sato-Beckmann-Rose? It is this: technical change takes the form of Sato-Beckmann-Rose factor augmenting technical change if it is neutral in the sense that it has no effect on the distributive shares of labor and capital, as long as the elasticity of factor substitution remains constant. Technical change of this type is most frequently used in empirical studies; its suitability for theoretical use is explained by the fact that it is derived from the neutrality principle of technical change.

Production functions that include Sato-Beckmann-Rose factor augmenting technical change, as well as estimates of technical change and an empirical analysis had first been carried out by the author in an article in the International Economic Review of 1963. The main point of this paper was to demonstrate that, by measuring the elasticity of factor substitution on using Kendrick data and then using that value, it was possible to measure the technical growth rates of both capital and labor in two equations. In contrast to the paper by Kendrick and Sato, it explained that there is a large gap between the rate of technical change for labor and the rate of technical change for capital; hence, the Hicks' technical change hypothesis did not apply to the US economy.

It also explained that the rate of technical change for labor is higher than that of capital, and that the latter is not zero as had been assumed by Harrod neutral technical change. It might be noted in passing that the rate of technical change for labor was 2.7 percent and the rate of technical change for capital was 0.7 percent (see Sato, International Economic Review 1963).

What is noteworthy here is that, except in cases where neutrality in one model is combined with a special production function (like the Cobb-Douglas type), it does not mean neutrality in the other models. In other words, if the model adopts Hicks neutral, then by definition, technical change of the Harrod, Solow-Ranis-Fei and Sato-Beckmann-Rose types are all not neutral; in short, they are biased technical change. The technical dealt with in this book will, for the most part, be of the Sato-Beckmann-Rose type.

17. An analysis of Sato-Beckmann-Rose technical change using Lie group transformations.

Sato-Beckmann-Rose neutral factor augmenting type of technical change under a neo-classical production function of constant returns to scale is expressed as follows:

$$Y(t) = F[A(t)\ K(t),\ B(t)\ L(t)]$$
$$\bar{Y}(t) = F[\bar{K}](t),\ \bar{L}(t)]$$

Capital and labor, which we here express in terms of the efficiency of technical change, are

$$\bar{K}(t) = A(t)\ K(t)\bar{L}(t) = B(t)\ L(t)$$

This can be seen as the transformation of $K(t) \to \bar{K}(t)$

$$L(t) \to \bar{L}(t)$$

If $A(t) = A_o e^{\alpha t}$, where $\alpha \geq 0$ and $B(t) = B_o e^{\beta t}$, where $\beta \geq 0$, the transformation of K(t) and L(t) is a magnificent type of a Lie group (Sato [1981, 1999]). I will not go more deeply into this question here.

18. A short history of economic conservation Laws. Ramsey's article in the Economic Journal of 1928 was the first in the long history of economics to introduce a dynamic method, i.e. a calculus of variation. Ramsey examined the question of how much a country would need to save and invest in order to maximize welfare, which he measured in terms of a mature economy's long-run rate of consumption. This was the first attempt at an optimum savings theory, which neo-classical growth theories have often dealt with from the 1960's on.

Ramsey intentionally did not introduce the concept of the discount rate. This marks a clear distinction from the optimum savings theories of today. Consequently, he was able to use a concept similar to the one mentioned earlier, the Hamilton energy conservation law used in physics. In the law of energy conservation, at each point of a movement in time kinetic energy + potential energy = constant; similar to this, in Ramsey's case, at each point in time the sum of net welfare and the value of saving is always constant, i.e. net welfare + value of saving = constant. Moreover, this value is the highest that nation's economic system can achieve. Ramsey called this value "bliss." Total energy in the energy conservation law and Ramsey's Bliss are the values of a maximized Hamilton function in a dynamic system. In short, Ramsey knew the conservation law that, assuming a discount rate of zero, a maximized Hamilton discount rate at each point in time is always constant. Although the concept itself was not used, for all practical purposes, this was the first use of a conservation law in economics.

19. Economists did not revisit Ramsey's conservation law until the 1970s. Samuelson's paper in 1970 was the first in that era to introduce an economic conservation law into modern theoretical economics. Like Ramsey, Samuelson analyzed a von Neumann optimum problem using the analogy of the total energy law. The conclusion derived from this was the discovery that an economic conservation law is at work: aggregate capital-output ratio = constant. This was demonstrated by Sato [1981, 1999] with a detailed derivation.

Although, unlike the present book, Samuelson's article did not make use of the Noether theorem using Lie groups as his principle methodology, it was an outstanding accomplishment for both its accuracy of intuition and its economic significance.

20. Ramsey's and Samuelson's achievements were not generally known at the time the author received grants from the National Science Foundation and the Guggenheim Foundation to engage in a study of Lie groups and economic conservation laws. That was in the mid 1970s.

The author's comprehensive survey of the field, The Theory of Technical Change and Economic Invariance: Application of Lie Groups (Academic Press, 1981; revised edition, Elgar, 1999), not only made a representation of technical change using Lie groups, it conducted a thorough, full-scale analysis of economic conservation laws using the Noether theorem. It showed that Ramsey's Bliss conservation law and Samuelson's capital-output ratio conservation law are derived as special cases of the Noether theorem. (The Noether theorem, simply put, is that if a dynamic system including integral calculus is constant under Lie transformation groups that have r-parameters, r conservation laws exist in it).

The Ramsey model and other neo-classical long-run growth models were analyzed in detail in Chapter 7 of the above mentioned book. Samuelson's conservation law was also thoroughly analyzed there, and it was discovered to be the only conservation law with a von Neumann model. In addition, using the Noether theorem in standard neo-classical growth theory led to the discovery that, when consumption or the utility of consumption is maximized from the present into the infinite future, the "income-wealth ratio is constant" (for further details see Journal of Econometrics 1985; also found in The Selected Essay of Ryuzo Sato [Elgar, 1999], vol. II, chapter 18). This income-wealth conservation law was also derived by M. Weitzman, Kemp and others. But Weitzman's law was obtained using Bellman's principle of optimality not the general Noether theorem. Weitzman never mentioned the possibility that conservation laws other than the income-wealth law might also exist. In addition, unlike Sato (1985), he gave no answer to the question of whether the income-wealth conservation law would be found in a more realistic model such as one in which the discount rate changes over time.

As a matter of fact, the question of whether the income-wealth ratio is constant when the discount rate changes with time was first raised by Samuelson.

The author's analysis discovered the law that the rate of income to adjusted wealth is always constant when the discount rate is changing. This is the variable discount rate income-wealth conservation law.

The study of conservation laws had only just begun. Efforts were now made to discover new conservation laws in a form that was a further development of the Noether theorem. A representative example is the recent achievement on Nôno and Mimura. The latest achievements in this field were made public at a symposium held in April 2003 at New York University where papers by Samuelson, Weitzman, Mino, Hartwick, Cioppri, Russell-Cooper-Samuelson, and Sato were discussed. (For further details see Japan and the World Economy, vol. 16, no. 3 (august 2004), "Special Issue: Economic Conservation Laws and Optimizing Behavior," ed. T. Russell, pp. 234–415).

2. Survey of Chapters

Part One: Biased Technical Change

The Stability of the Solow-Swan Model with Biased Technical Change

21. Let me give an overview of each chapter in this book. In Chapter 2 it is shown that, by introducing endogenous technical change to the Solow-Swan model as discussed in part I of the introduction, a stable balanced growth path can be achieved under biased technical change, in short, under Sato-Beckmann-Rose factor augmenting technical change as well as under Harrod neutral technical change.

First, when the production function is given as a function of capital, labor and technical change, it can be written as follows

$$Y(t) = F[K(t),\ L(t),\ t] = F[A(t)\ K(t),\ B(t)\ L(t)] = \bar{Y}(t).$$

F is the linear and homogeneous neo-classical production function of K(t) and L(t). For simplification's sake, let us assume that B(t) and L(t) are exogenous. In the Solow-Swan model it is assumed that the capital increment = investment is determined by savings of only s% of Y, namely:

$$\dot{K}(t) = S(t) = sY(t), \text{ where } s = const,\ 0 < s < 1$$

If it is assumed that there is absolutely no technical change of capital, i.e. if A(t) = const = 1, i.e., Harrod neutral, then this economic system is able to achieve a stable, long-run growth path.

22. If the technical change rate of capital is exogenous in this Solow-Swan model, $A(t) = A_o e^{\alpha t}$, where α = const. > 0 capital measured by its efficiency, i.e. A(t) K(t) (defined as $\bar{K}(t)$), always grows faster at an annual rate of α% than K(t), which is determined endogenously by Y. In short, because the efficiency of capital

is higher than the efficiency of the economy as a whole, the shares of capital and labor cannot be constant, even in the infinitely long run t.

If the elasticity of factor substitution is greater or less than 1, the shares of capital and labor will both be zero. This means that a balanced growth path cannot be achieved.

23. In Chapter 2, A(t) is also assumed to be endogenously determined. This is a more realistic assumption in the macroeconomy. Moreover, both A(t) and K(t) are thought to be dependent on Y(t). As for the technical change rate of capital, it is assumed that a certain percentage of income will be allocated to technological development. In short, it is assumed that a nation will make appropriate investments in and allocations to both kinds of technical change to improve the quantity and quality of its physical capital. This means

$$\dot{\bar{K}}(t) = \frac{d\,(A(t)\,K(t))}{dt} = s\bar{Y}(t) = (s_1 + s_2)\bar{Y}(t)$$

Hence,

$$\frac{\dot{\bar{K}}(t)}{\dot{\bar{K}}(t)} = \frac{\dfrac{d\ (A(t)\ K(t))}{dt}}{A(t)\bar{K}(t)} = \frac{s\bar{Y}(t)}{\bar{K}(t)} = s_1\frac{\bar{Y}(t)}{\bar{K}(t)} + s_2\frac{\bar{Y}(t)}{\bar{K}(t)}$$

i.e.

$$\frac{\dot{A}(t)}{A(t)} + \frac{\dot{K}(t)}{K(t)} = s_1\frac{\bar{Y}(t)}{\bar{K}(t)} + s_2\frac{\bar{Y}(t)}{\bar{K}(t)}$$

The efficiency of capital rises only $\frac{\dot{A}}{A}$ and the amount of capital rises only $\frac{\dot{K}(t)}{K(t)}$. Moreover, is financed by the investment of s_1% of income $\bar{Y}$ of per unit capital (measured by efficiency), and $\frac{\dot{K}(t)}{K(t)}$ is created similarly by the investment of s_2%. The efficiency of labor B(t) as well is essentially unchanged, even though it assumed that it too is created in the same way by endogenous technical change. This is because labor L(t) is exogenous.

24. In long-run equilibrium, the shares of capital and of labor each preserves a constant value; as for the economy, $\bar{Y}(t)$ and $\bar{K}(t)$ continues to grow at the same rate, and $\frac{A(t)K(t)}{B(t)L(t)}$ slowly approaches a constant value.

Next, given the fact that return to capital is actually relatively stable and that interest rates and profit rates are also relatively stable, isn't there a contradiction in the model introduced here? The answer is that there is no contradiction. Why not? The reason is that the real economy is always on a path to long-run equilibrium but not at long-run equilibrium itself. This is the theory of endogenous technical growth shown in Sato [1963] where $\frac{\dot{B}(t)}{B(t)}$ = an average of 2.7% and $\frac{\dot{A}(t)}{A(t)}$ = an average of 0.7%; more over, it is explained as a path to a stable path. It is not in fact on a long-run balanced growth path itself, but merely on a path to reach that path.

A Model of Optimal Economic Growth and Endogenous Bias

25. Here, optimal technical change is introduced into the model described above. An analysis is made of how an economy can achieve balanced growth under biased technical change by combining optimal technical change and optimal growth.

The main themes here are the introduction of an investment function and a technical change function. In the case of the latter, moreover, technical change of capital and technical change of labor are treated separately. As an extension of the way of thinking in the preceding chapter, the technical change function with diminishing returns is assumed.

The model shows how a central planner can implement a plan that can most efficiently maximize welfare of per capita consumption by investing a certain amount of resources Y in three categories: increasing the amount of physical capital and raising both its technical change rate and that of labor.

26. Given the general factor-augmenting type (or biased type) of technical change under a neo-classical constant returns to scale production function, $Y = F[AK, BL]$, the society chooses three control variables m_1, m_2 and m_3 so as to maximize the discounted value of the sum of utility of consumption over an infinite time. Here rates of growth of efficiency of inputs K and L are given by technical progress functions,

$$\frac{\dot{A}(t)}{A(t)} = h_1\left(\frac{m_1 Y}{AK}\right)$$

$$\frac{\dot{B}(t)}{B(t)} = h_2\left(\frac{m_2 Y}{AK}\right)$$

It is also assumed that the transformation of income into capital goods (capital accumulation) is non-linear, with the function taking a form similar to that of the technical progress function:

$$\frac{\dot{K}(t)}{K(t)} = h_3\left(\frac{m_3 Y}{AK}\right)$$

Labor grows at a constant proportional rate,

$$\frac{\dot{L}(t)}{L(t)} = n.$$

Then the maximization problem can be written as

$$\underset{(m_1,\, m_2,\, m_3)}{\text{Max}} \int_0^\infty e^{-(\rho+n)t} F[AK,\, BL](1 - m_1 - m_2 - m_3)dt$$

Subject to

$$\frac{\dot{A}}{A} + \frac{\dot{K}}{K} = h_1(\cdot) + h_3(\cdot)$$

and

$$\frac{\dot{B}}{B} + \frac{\dot{L}}{L} = h_2(\cdot) + n.$$

There exists an interior solution such that, in a steady state, the effective capital growth rate is identical with that of effective labor i.e.

$$\frac{\dot{A}}{A} + \frac{\dot{K}}{K} = \frac{\dot{B}}{B} + \frac{\dot{L}}{L}\ .$$

27. This implies that Y, AK and BL are all growing at the same rate and the relative shares of capital and labor approach constant values. One of the paradoxes of the neo-classical model of growth is that it has a steady state only if technological progress is Harrod-neutral; a positive rate of labor augmenting has the same net consequences as a higher rate of growth of labor except that the per-capita output in steady state increases at the rate of increase in labor efficiency.

In contrast, the model presented in this chapter, has an interior solution and a steady state in which there is positive accumulation of capital and increases in efficiency of both capital and labor. A comparison of the two models seems to indicate that the crucial difference is in the assumption about the effect of expenditure investment and in increasing the efficiency of capital.

At steady state, the ratio of inputs measured in efficiency terms will remain constant and with it the factor shares. If one assumes a CEDD production function, then the elasticity of substitution is also a constant at steady state. However, before the system converges to the steady state, both factor shares and the elasticity of substitution will vary. Thus, it provides a theoretical growth model that is consistent with the empirical work of Sato (1970).

A Three Sector Model of Endogenous Hicksian Bias

28. This chapter uses the model of biased technical progress presented in the previous chapter and explores it in further detail. The economy is assumed to comprise of three sectors. The first sector produces a consumption good, the second produces a capital good while the third sector determines the innovation possibility frontier. The total quantity of labor at any time, L(t), is allocated among the three sectors:

$$L = L_C + L_K + L_R = L_0 e^{nt},$$

Or, dividing by L,

$$1 = l_C + l_K + l_R. \tag{1}$$

The consumption good is produced using capital and labor. The constant-returns to scale production function is

$$C = F(AK, BL_C).$$

Where A and B are the efficiency factors of the inputs. Capital good is produced using only labor:

$$\frac{\dot{K}}{K} = \left(\frac{DL_K}{K}\right) - \delta = \left(\frac{Dl_k}{k}\right) - \delta \tag{2}$$

where δ is the rate of depreciation, D is the efficiency of labor in the capital-good-producing sector, and $k = \frac{K}{L}$.

The rate of factor augmentation is determined in the research sector according to a concave innovation possibility frontier,

$$G(A, \dot{A}, B, \dot{B}, D, \dot{D}, E, \dot{E}, K, L_R) = 0 \tag{3}$$

where E is the efficiency of labor in the research sector. As E is the only input into this sector, changes in E can be thought of as Harrod-neutral technical progress.

Under the assumption of factor-augmenting technical progress, capital enters the production function in efficiency terms, AK. An economy can choose between increasing the number of physical units of inputs or their efficiency, and the decision will be determined by marginal considerations. The question then is how these marginal conditions are affected by the formulation of the innovation possibilities frontier. The intertemporal optimization can now be stated as follows.

$$\max \int_0^{\infty} e^{-\rho t} \frac{1}{L} F(AK, BL_C) dt$$

subject to (1), (2) and (3).

29. The solution to the above problem will yield optimal bias and the derived bias in this model agrees with Sato (1970), which estimated that the growth in the efficiency of labor exceeds that of capital though both are positive. However, while Sato (1970) assumed that the economy produces one malleable good, in this model the economy is divided into three sectors with the assumption that the growth in productivity of labor in the capital good sector is less than that in the consumption goods sector.

If productivity growth is to be achieved only by the allocation of scarce resources to research, the question whether or not it is efficient to allocate some resources to increasing the efficiency of capital cannot be avoided. In microeconomic models, where the firm is the price-taker in the input market, the relative growth of input prices can be taken as the driving force. In macroeconomic models, the relative growth in input prices has to be derived from the model itself in terms of the intertemporal paths of resources and technology.

Formulating a model to address the question of optimal bias poses many problems. In Sato *et al.* (1999) and in this chapter, an attempt has been made to develop tractable models to analyze the optimal intertemporal allocation of resources leading to a steady-state bias. These papers reveal the crucial role of the technology of the production and research sectors in determining the allocation of resources for physical capital accumulation and for the optimal innovation frontier.

Estimation of Biased Technical Progress

30. Empirical works on neo-classical growth models led to the recognition that technological progress is the dominant factor in the growth of *per capita* income. There are four questions: (1) How important is technological and technical progress in the process of economic growth? (2) What is the cause of technical progress -is it exogenous or endogenous to the system? (3) How is technological change transmitted into technical progress in the macroeconomy? and (4) If technical progress can be classified as labor saving, neutral or capital saving, is there any systematic bias in an economy towards any particular kind of technical progress and, if so, why?

This chapter gives a partial answer to the above questions. By using the estimation equations for the growth of efficiencies of capital and labor (see Sato [1970]), we compare the growth of factor productivities in the United States and Japan. The growth rate of capital's efficiency is estimated from

$$\frac{\dot{A}(t)}{A(t)} = \frac{\sigma\frac{\dot{r}}{r} - \frac{\dot{Y}}{Y} + \frac{\dot{K}}{K}}{\sigma - 1}, \ \sigma \neq 1$$

and that of labor's, from

$$\frac{\dot{B}(t)}{B(t)} = \frac{\sigma\frac{\dot{w}}{w} - \frac{\dot{Y}}{Y} + \frac{\dot{L}}{L}}{\sigma - 1}, \ \sigma \neq 1$$

where $\sigma =$ the elasticity of factor substituting, $\frac{\dot{\mathrm{r}}}{\mathrm{r}} =$ growth rate of returns to capital and $\frac{\dot{\mathrm{w}}}{\mathrm{w}} =$ the growth rate of the wage rate.

Because of the impossibility theorem of Diamond-McFadden-Sato, we estimate average elasticity of substitution assuming Hicks neutral technical progress. Using this value, we then estimate the average growth rates of efficiencies of capital and labor. But the estimate of the elasticity based on the assumption of Hicks neutrality has a bias if technical progress is indeed biased. To test the robustness of the results, we estimate the growth rates for different elasticities of substitution.

31. For the U.S. data (1909–1993), what is striking about various estimates is that the growth rate of the efficiency of labor is consistently four to five times that of capital, i.e.

$$\frac{\dot{A}}{A} = 0.004 \ \textit{and} \ \frac{\dot{B}}{B} = 0.019 \sim 0.020$$

The same method is used to estimate the growth rates in efficiencies in Japan. There is considerable variation in the relative magnitudes of the growth rates of the two efficiencies in Japan. In fact, for elasticities of substitution greater than 0.8, $\frac{\dot{A}}{A}$ is positive and greater than $\frac{\dot{B}}{B}$ while for lower values of elasticity, $\frac{\dot{A}}{A}$ is not only less than $\frac{\dot{B}}{B}$, but turns negative.

This is in direct contrast to the United States where the growth rate of A(t) decreases as elasticity increases. This can be explained by inefficiency of the capital market in Japan, even before the collapse of the bubble economy. There were hidden problems of over investment and non-performing debt in the Japanese economy.

A Note on Modelling Endogenous Growth

32. This chapter explores the conditions for the existence of a steady state in an optimal growth model with factor-augmenting technical progress. Given a non-linear "investment" function, the consumption function is;

$$C = C\,(A, K, B, L; \dot{K}, \dot{L}, \dot{A}, \dot{B})$$

where A and B = efficiencies of capital K and labor L respectively.

There are several sufficient conditions under which the "effective" capital-" effective" labor ratio can attain a constant value in steady state.

They are;

(1) $\lambda^2 C = C\,[\lambda A, \lambda K, \lambda B, \lambda L; \lambda(A\dot{K}), \lambda(B\dot{L})]$

(2) $\lambda^2 C = C\,[\lambda A, \lambda K, \lambda B, \lambda L; \lambda(\dot{A}K), \lambda(\dot{K}A), \lambda(\dot{B}L), \lambda(\dot{L}B), \lambda(\dot{A}\dot{K}), \lambda(\dot{B}\dot{L})]$

(3) $$\lambda^2 C = C\left[\lambda A, \lambda K, \lambda B, \lambda L; \lambda(AK)\,\phi_1\left(\frac{\dot{A}}{A}\right), \lambda(AK)\,\phi_2\left(\frac{\dot{K}}{K}\right), \lambda(BL)\,\phi_3\left(\frac{\dot{B}}{B}\right), \lambda(BL)\,\phi_4\left(\frac{\dot{L}}{L}\right)\right]$$

Examples which satisfy each one of the above conditions are presented.

A special case of (3) is the additive "investment" function used in Chapter 3 of this book.

The general form is,

$$C = F\,[AK, BL] - [M_1 + M_2 + M_3]$$

or

$$C = F\,[AK, BL] - \left[AK\phi_1\left(\frac{\dot{A}}{A}\right) + AK\phi_2\left(\frac{\dot{K}}{K}\right) + BL\phi_3\frac{\dot{B}}{B}\right]$$

where M_i = the amount of investment for each category.

This implies that the "investment" functions for technical change and capital accumulation take the form;

$$\frac{\dot{A}}{A} = \phi_1^{-1}\left(\frac{M_1}{AK}\right)$$
$$\frac{\dot{K}}{K} = \phi_2^{-1}\left(\frac{M_2}{AK}\right).$$
$$\frac{\dot{B}}{B} = \phi_3^{-1}\left(\frac{M_3}{BL}\right)$$

These are exactly the functions used in chapter 3.

Finally, this chapter shows that an optimal growth path under endogenous bias is stable even under more general "investment" functions than those used in chapter 3 of this book.

Technical Change and International Competition

33. This chapter is not directly related to biased technical change but is closely related to technical change resulting from R&D activities in basic and applied technologies.

A differential game model is used to study the basic nature of international competition between the technology leaders and latecomers. In this model, two monopolistic firms in the two types of countries engage in R&D activities to produce similar products and export them in the world market. The technologically advanced firm in country A engages in both basic and applied research, whereas the firm in country B imports basic technology from country A and improves on it.

There are three relevant parameters, which will affect the outcome of the competition: (1) the index of diffusion of basic technology (2) the index of relative efficiency of applied technology, and (3) the index of cost of sharing of basic research.

It is shown that the final outcome (i.e. who wins the competition?), depends on the strategies the two firms employ: an open-loop or a closed-loop strategy. Under normal conditions, the open-loop strategy may impart equal market shares among the world competition. A paradoxical case may occur when the less technologically advanced country increases the cost of sharing of basic technology. The country with no domestically produced basic research, can now win even if its efficiency in applied technology is lower than that of the country producing basic research. The closed-loop strategy may eliminate the paradox.

Part Two: Conservation Laws

Optimal Economic Growth: Test of Income/Wealth Conservation Laws in OECD Countries

34. This chapter attempts to derive several economic conservation laws and to test the validity of the optimal growth models using the income/wealth ratios for the United States, Japan, and other OECD countries.

In this traditional approach, one major concern arises from determining the nature of appropriate choices in investment and how these decisions will influence the economy's path of long-run development. This method of analysis involves typical application of "optimal" growth theory. Another application of the optimal growth theory occurs when economists and statisticians attempt to justify and estimate the single index of "national income" from a theoretical point of view. However, little empirical work has been done to test the validity of the second aspect of the growth theory, namely the testing of an appropriate measure of national income.

The purpose of this chapter is to unify both aspects of the optimal growth theory with a general theory of "economic conservation laws". The theory of conservation law involves the identification and discovery of hidden invariant quantities in a dynamic system. In this dynamic economic system, as in a dynamic physical system, it is suspected that a certain variable remains unchanged during its process of evolution, as long as the system follows an optimal trajectory. In a growing economy, the variable that is invariant is called the conservation law. By uncovering the existence of conservation laws and by formulating the operational concepts associated with them, one can test the valdity of both the optimal growht models as well as the measurement of national income. It has been shown that in an optimally controlled ecomomy, the ratio of income to wealth remains invariant. This is the income/wealth conservation law.

35. The model formulating this conservation law is presented. It is assumed that the consumption of the economy depends on output, which in turn depends on a vector of capital goods, a vector of investment and labor input, so that

$$C = C[Y(K;\ \dot{K};\ L;\ \dot{L})], \tag{1}$$

where Y = output, which depends on $\mathrm{K} = (\mathrm{K}_1, \ldots\ \mathrm{K}_n)$ = n capital goods, $\dot{\mathrm{K}} = (\dot{\mathrm{K}}_1, \ldots \dot{\mathrm{K}}_n) = \mathrm{dK/dt}$ = investment and L labor input, exogenously given by $\mathrm{L} = \lambda \mathrm{L}$. Without loss of generality, we can assume that $\frac{\mathrm{dC}}{\mathrm{dY}} = 1$ and (1) can be written as

$$C = F(K;\ \dot{K};\ \ L;\ \dot{L})$$

Consumption per capita is given by

$$\begin{aligned} c = \frac{C}{L} &= F\left(\frac{K_1}{L}, \ldots,\ \frac{K_n}{L}; \frac{\dot{K}_1}{L}, \ldots,\ \frac{\dot{K}_n}{L};\ 1; \frac{\dot{L}}{L}\right) \\ &= F(k_1, \ldots,\ k_n;\ \dot{k}_1 + \lambda k_1, \ldots,\ \dot{k}_n + \lambda k_n;\ 1;\ \lambda \end{aligned}$$

where $\mathrm{k_i} = \frac{\mathrm{K_i}}{\mathrm{L}}$ and $\dot{k}_i = \left(\frac{\dot{K}_i}{K_i} - \lambda\right) k_i \quad i = 1, \ldots, n.$

Thus we have

$$c = f(k;\ \dot{k} + \lambda k;\ \lambda).$$

The society's objective is to maximize the discounted future value of consumption per capita, c(t) = f(t), as

$$J = \int_0^{\infty} e^{-\rho t} f(k;\ \dot{k} + \lambda k);\ \lambda)dt \rightarrow \text{Max}$$

Using the Noether theorem and its invariance principle, the general expression for the conservation law can be derived as

$$\rho = \frac{\textit{Income per capita}}{\textit{Wealth per capita}}$$

36. Conservation laws vary with the type of objective function depcted, such as the maximization of the aggregate consumption of the maximization of per-capita consumption. The operational concept of "wealth-like quantity" is identified, although the Goldsmith-Kendrick standard definition of "net national wealth" should not always be used. The last section of this chapter takes up an emprical analysis to determine how different economies have achieved long-term (optimal) growth. The U.S. economy has been operating rather efficiency, whereas the Japanese economy, after the oil shocks of the 1970's, has behaved differently, leading into the bubble period of the early 1990's.

Economic Conservation Laws as Indices of Corporate Performance

37. This chapter attempts to apply economic conservatio laws to corporate behavior at the microeconomic level. At the macro level, efficient operation and performance of nations are analyzed using the income-wealth conservation law. Corporate management needs to be able to judge whether or nor a company can achieve the maximum long-run profit.

In modern finance and accounting theory several different criteria are considered for measuring corporate performance. Investors in stocks, for exmaple, will use P-E ratios etc. to judge how high or low a company's stock price is relative to it's profitability. Corporate mangers will analyze management conditions collectively by considering whether or not profitability per unit capital is at a satisfactory level, or by looking at trends in productivity. In the final analysis, the yardstick for successful corporate management is whether corporate profits are growing at a satisfactory level.

Purely from an economic perspective, a corporation may be analyzed on the assumption that is operating to maximize profits over a certain period under the given conditions. In actually, however, production costs and R&D activity etc. at such an institution may be unpredictable. Since the numerical values of crucial parameters are ambiguous, it is impossible to judge whether the company's reported profits are being maximized or not. Under such circumstances, managers regard positive growth of profits as the criterion of succss. In short, managers are regarded as successful as long as profits are growing even though profits may not be maximized.

Hence corporate management needs to be able to judge whether or not under the given conditions a company is being managed in a way that will maximize long-term profit growth. This chapter aims to apply macroeconomic methods to microeconomic organizations and to come up with an assessment of corporate performance by applying economic conservation laws to individual companies and individual industries.

38. Because theoretical analyses using economic conservation laws are not yet in general use, the paper begins with a general model and an explanation of the Noether theorem for deriving conservation laws. The forms that corporate profit maximization take often idffer greatly depending on the nature of management and the management variables that are emphasized. In order to analyze these forms, several representative corporate behaviors found in the existing literature are presented as examples. The view of long-term profit maximization shared in these examples is explored and an attempt is made to verify the workings of their profit maximization behavior using empirical data. As a theoretical assumption, the important criterion is whether or not the discount rate is fixed. Economic conservation laws under different hypotheses are then derived from this.

To explain Noether's invariance principle, a profit equation of the firm π, which is twice continuously differentiable in each of its $2n + 1$ arguments, is considered.

We have the firm's long-run profit,

$$J(x) = \int_a^b \pi(t, x(t), \dot{x}(t))\, dt$$

where x is the vector of all quantities and prices, which the firm controls to maximize the long-run profit

$$J(x),\ x(t) = (x^1(t), \ldots x^n(t)),\ and\ \dot{x}(t) = (\dot{x}^1(t), \ldots, \dot{x}^n(t)).$$

The firm's problem is to maximize the long-run profit

$$J(x) \to Max.$$

The following specialized examples of corporate behavior are taken from the literature.

1. Capital investment model

If P(k) is the profit rate generated with capital k producing output f(k), revenue is given by pf(k) = P(k). Let C(I) be cost of investment and b the constant proportional decay of capital stock, then $C(I) = C(\dot{k} + bk)$. The firm's objective is to maximize the present value of the net stream over time,

$$\Pi = P(k) - C(\dot{k} + bk),\ i.e.$$

$$\max \int_0^\infty e^{-\rho t}[P(k) - C(\dot{k} + bk)]\, dt$$

subject to the initial conditions.

2. Price-quantity adjustment model

In an oligopolistic market profit depends on price p, quantity q and $\dot{q}$, rate of change of q;

$$\Pi = (p, q, \dot{q})$$

but since other producers participate, $\dot{p} = f(p, q)$

Thus, the firm's objective is to maximize

$$\max \int_0^{\infty} e^{-\rho t} \pi(p, q, \dot{q})\, dt$$

Subject to

$$\dot{p} = f(p, q) \textit{ and the initial conditions.}$$

3. R&D profit maximization model

The typical firm in oligopolistic competition attempts to increase its profits by adjusting output and adopting cost-reducing innovations. To do this, the firm engages in both basic and applied research. The firm's objective is to maximize

$$J = \int_0^{\infty} e^{-\rho t}[R(p, y) - C(A, \dot{A}, B) - T(B, \dot{B})]\, dt$$

where p = price of output y, A = level of applied research and B = level of basic research.

Since there exists no stylized facts as to which example fits the observed data, we will use the general case of maximizing

$$J\,(x) = \int_a^b \pi(t, x(t), \dot{x}\,(t))\, dt.$$

Using the Noether theorem invariance conditions, we derive several conservation laws. There are six conservation laws that have been discovered including the one used to test macroeconomic performance in the previous chapter.

This conservation law states that the current value of profit and investment must be equal to the discount rate multiplied by the discounted value of the firm. We will call this the *Total Value Conservation Law of the Firm.*

39. Thomas Mitchell's comments are available at the end of the chapter as an appendix.

Empirical Tests of the Total Value Conservation Law of the Firm

40. The previous chapter shows how economic conservation laws could be applied in the evaluation of corporate behavior. This chapter applies a conservation law to individual industries and companies, and empirical tests are presented to show the possibility that there are ways to evaluate manager behavior relative to the profit-maximization goal. The results represent the first extensive investigation to apply a conservation law using Japanese corporate data.

Within the context of the *total value conservation law of the firm* derived in the previous chapter, empirical evidence using the Japanese corporate data is examined. In conducting the empirical analysis, a constant discount rate is assumed. We can test whether the conservation laws hold for firms by using data from their financial statements and market prices of their securities and shares. Because the theory predicts that if a firm optimizes profits, the derived ratio of income creation to the value of the firm should be relatively stable, the volatility of this ratio (σ_β) should be negatively correlated with some good measure of performance of the firm.

We then perform a regression analysis using the performance indicator of the firm (*PI*) and its level of leverage (*L/V*), as predictors of the volatility of its ratio, σ_β. Testing the model with the Nikkei Performance Index as *PI*, we obtain a statistically significant negative coefficient for *PI*. Using return on assets as an alternative performance index on the same selection of firms used previously, yields a statistically significant positive correlation coefficient for *PI*. Testing the model with another measure of performance, the growth of total assets over the sample period, indicates a statistically significant positive correlation for *PI*.

Thus the regression results are inconclusive. Using the Nikkei index shows the possibility that better managed firms are related to low volatility of the ratio, which is consistent with our assumption of a constant discount rate. However, indices constructed from financial statement data suggest, that greater the values of either return on assets or the growth of total assets, the higher the volatility of the ratio.

Another data issue is survivorship bias. Since a long history of company data is required for the regression analysis, companies that are short-lived because of poor management or any reasons are excluded from the sample data. Thus the tests in this chapter are not ideal for identifying specific firms with profit maximizing operations.

41. This chapter attempts to abstract the real world and construct a convincing model as a good approximation of firm behavior. In traditional valuation theory under uncertainty, firms are given their discount rate from the capital markets, as opportunity cost of their investment. Capital markets force managers to optimize and surviving firms are assumed to be optimizing. On the other hand, given the relevant value of the discount rate, it is important to examine whether a firm has been operating optimally even under the assumption of certainty. From this perspective, we are interested in longer-term trends rather than daily market fluctuations in firm valuation.

In this regard, we have shown that the assumption of a constant discount rate and profit-maximizing behavior is consistent with observed data for many top-performing Japanese firms during the period of 1980 to 2002. Although the results are tentative because of the limited data period and data availability, they suggest that it is promising to extend the notion of conservation laws to the industry or firm level.

Hartwick's Rule and Economic Conservation Laws

42. By using the theory of conservation laws we derived two investment rules for optimal resource extraction and intergenerational equity: first, the so-called Hartwick rule as a special case of a more general rule; and second, a new investment rule for Benthamite utility maximization.

We start with a simple model of optimal capital accumulation with extraction cost of resources. The aggregate income is spent by consumption expenditure, investment on physical capital, and extraction of natural resources. The production function is subject to the condition of constant returns to scale.

43. The society's problem is to find the largest constant consumption with a feasible pattern of resource use over the infinite time horizon. Hartwick [1977] derived a rule that investment must be equal to the rents from exhaustible resources in order to maintain a path of constant consumption. We examine whether Hartwick's rule can be derived from one of the conservation laws and whether there is another policy rule. It is shown that, in fact, Hartwick's rule is a special case when the conservation law (H = Hamiltonian) takes a special value, i.e. $H = 0$. There are many other rules when $H \neq 0$.

Next, we study the Hotelling differential equation. The solution of the differential equation specifies how the capital-resource ratio must follow the optimal path. A complete closed form solution is given for the Cobb-Douglas production function.

44. We then present a simple model of optimal resource extraction under the Benthamite utility function. The economy has a representative agent with a well-defined utility function. Then the economy's objective is to solve the Benthamite problem of dynamic optimization with a positive discount rate.

By using the theory of conservation laws, we derive the income-wealth invariance law which yields a new investment rule. The rule depends not only on the discount rate but also on how profit changes over time. The Benthamite investment rule coincides with Hartwick's rule, if and only if the discount rate is zero

Appendix

45. Excerpt from Ryuzo Sato [1999], Chapter 7 Dynamic Symmetries and Economic Conservation Laws (Sections I, II & III) is appended for those who are not familiar with the methodology of Lie group application.

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