Preface

The subject of this book is the theory and applications of cooperative stochastic differential games. Game theory is the study of decision making in an interactive environment. It draws on mathematics, statistics, operations research, engineering, biology, economics, political science and other disciplines. The theory's foundations were laid some sixty years ago by von Neumann and Morgenstern (1944). One particularly complex and fruitful branch of game theory is dynamic or differential games (originated from Isaacs (1965)), which investigates interactive decision making over time. Since human beings live in time and decisions generally lead to effects over time, it is only a slight exaggeration to maintain that "life is a dynamic game". Applications of differential game theory in various fields have been accumulating at an increasing rate over the past fifty years.

Cooperative games hold out the promise of socially optimal and group efficient solutions to problems involving strategic actions. Formulation of optimal player behavior is a fundamental element in this theory. In dynamic cooperative games, a particularly stringent condition on the cooperative agreement is required: In the solution, the optimality principle must remain optimal at any instant of time throughout the game along the optimal state trajectory chosen at the outset. This condition is known as *dynamic stability* or *time consistency*.

Since interactions between strategic behavior, dynamic evolution, and stochastic elements are considered simultaneously, cooperative stochastic differential games represent one of the most complex forms of decision making. In the field of cooperative stochastic differential games, little research has been published to date owing to difficulties in deriving tractable solutions. In the presence of stochastic elements, a very stringent condition – *subgame consistency* – is required for a credible cooperative solution. A cooperative solution is subgame consistent if an extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal. Conditions ensuring time consistency of cooperative solutions are generally analytically intractable.

This book expounds in greater detail the authors' recent contributions in research journals. In particular, Yeung and Petrosyan (2004) offered a generalized theorem for the derivation of the analytically tractable "payoff distribution procedure" leading to the realization of subgame consistent solutions. This work is not only theoretically interesting in itself, but would enable the hitherto intractable problems in cooperative stochastic differential games to be fruitfully explored.

In the case when payoffs are nontransferable in cooperative games, the solution mechanism becomes extremely complicated and intractable. A subgame consistent solution is developed for a class of cooperative stochastic differential games with nontransferable payoffs in Yeung and Petrosyan (2005). The previously intractable problem of obtaining subgame consistent cooperative solution has now been rendered tractable for the first time.

This book draws on the results of this research and expands on the analysis of cooperative schemes to dynamic interactive systems with uncertainty. It is the first volume devoted to cooperative stochastic differential games and complements two standard texts on cooperative differential games, one by Leitmann (1974) and the other by Petrosyan and Danilov (1979). We provide readers a rigorous and practically effective tool to study cooperative arrangements of conflict situations over time and under uncertainty. Cooperative game theory has been successfully applied in operations research, management, economics, politics and other areas. The extension of these applications to a dynamic environment with stochastic elements is likely to prove even more fruitful, so that the book will be of interest to game theorists, mathematicians, economists, policy-makers, corporate planners and graduate students.

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