Preface

This book grew out of a one-semester course given by the second author in 2001 and a subsequent two-semester course in 2004-2005, both at the University of Missouri-Columbia. The text is intended for a graduate student who has already had a basic introduction to functional analysis; the aim is to give a reasonably brief and self-contained introduction to classical Banach space theory.

Banach space theory has advanced dramatically in the last 50 years and we believe that the techniques that have been developed are very powerful and should be widely disseminated amongst analysts in general and not restricted to a small group of specialists. Therefore we hope that this book will also prove of interest to an audience who may not wish to pursue research in this area but still would like to understand what is known about the structure of the classical spaces.

Classical Banach space theory developed as an attempt to answer very natural questions on the structure of Banach spaces; many of these questions date back to the work of Banach and his school in Lvov. It enjoyed, perhaps, its golden period between 1950 and 1980, culminating in the definitive books by Lindenstrauss and Tzafriri [138] and [139], in 1977 and 1979 respectively. The subject is still very much alive but the reader will see that much of the basic groundwork was done in this period.

We will be interested specifically in questions of the following type: given two Banach spaces X and Y, when can we say that they are linearly isomorphic, or that X is linearly isomorphic to a subspace of Y? Such questions date back to Banach's book in 1932 [8] where they are treated as *problems of linear dimension.* We want to study these questions particularly for the classical Banach spaces, that is, the spaces c_0, ℓ_p $(1 \leq p \leq \infty)$, spaces $\mathcal{C}(K)$ of continuous functions, and the Lebesgue spaces L_p, for $1 \leq p \leq \infty$.

At the same time, our aim is to introduce the student to the fundamental techniques available to a Banach space theorist. As an example, we spend much of the early chapters discussing the use of Schauder bases and basic sequences in the theory. The simple idea of extracting basic sequences in order to understand subspace structure has become second-nature in the subject, and so the importance of this notion is too easily overlooked.

It should be pointed out that this book is intended as a text for graduate students, not as a reference work, and we have selected material with an eye to what we feel can be appreciated relatively easily in a quite leisurely two-semester course. Two of the most spectacular discoveries in this area during the last 50 years are Enflo's solution of the basis problem [54] and the Gowers-Maurey solution of the unconditional basic sequence problem [71]. The reader will find discussion of these results but no presentation. Our feeling, based on experience, is that detouring from the development of the theory to present lengthy and complicated counterexamples tends to break up the flow of the course. We prefer therefore to present only relatively simple and easily appreciated counterexamples such as the James space and Tsirelson's space. We also decided, to avoid disruption, that some counterexamples of intermediate difficulty should be presented only in the last optional chapter and not in the main body of the text.

Let us describe the contents of the book in more detail. Chapters 1-3 are intended to introduce the reader to the methods of bases and basic sequences and to study the structure of the sequence spaces ℓ_p for $1 \le p < \infty$ and c_0. We then turn to the structure of the classical function spaces. Chapters 4 and 5 concentrate on $\mathcal{C}(K)$-spaces and $L_1(\mu)$-spaces; much of the material in these chapters is very classical indeed. However, we do include Miljutin's theorem that all $\mathcal{C}(K)$-spaces for K uncountable compact metric are linearly isomorphic in Chapter 4; this section (Section 4.4) and the following one (Section 4.5) on $\mathcal{C}(K)$-spaces for K countable can be skipped if the reader is more interested in the L_p-spaces, as they are not used again. Chapters 6 and 7 deal with the basic theory of L_p-spaces. In Chapter 6 we introduce the notions of type and cotype. In Chapter 7 we present the fundamental ideas of Maurey-Nikishin factorization theory. This leads into the Grothendieck theory of absolutely summing operators in Chapter 8. Chapter 9 is devoted to problems associated with the existence of certain types of bases. In Chapter 10 we introduce Ramsey theory and prove Rosenthal's ℓ_1-theorem; we also cover Tsirelson space, which shows that not every Banach space contains a copy of ℓ_p for some p, $1 \le p < \infty$, or c_0. Chapters 11 and 12 introduce the reader to local theory from two different directions. In Chapter 11 we use Ramsey theory and infinite-dimensional methods to prove Krivine's theorem and Dvoretzky's theorem, while in Chapter 12 we use computational methods and the concentration of measure phenomenon to prove again Dvoretzky's theorem. Finally Chapter 13 covers, as already noted, some important examples which we removed from the main body of the text.

The reader will find all the prerequisites we assume (without proofs) in the Appendices. In order to make the text flow rather more easily we decided to make a default assumption that all Banach spaces are real. That is, unless otherwise stated, we treat only real scalars. In practice, almost all the results in the book are equally valid for real or complex scalars, but we leave to the reader the extension to the complex case when needed.

There are several books which cover some of the same material from somewhat different viewpoints. Perhaps the closest relatives are the books by Diestel [39] and Wojtaszczyk [221], both of which share some common themes. Two very recent books, namely, Carothers [23] and Li and Queffélec [126], also cover some similar topics. We feel that the student will find it instructive to compare the treatments in these books. Some other texts which are highly relevant are [10], [78], [149], and [56]. If, as we hope, the reader is inspired to learn more about some of the topics, a good place to start is the *Handbook of the Geometry of Banach Spaces*, edited by Johnson and Lindenstrauss [90, 92] which is a collection of articles on the development of the theory; this has the advantage of being (almost) up to date at the turn of the century. Included is an article by the editors [91] which gives a condensed summary of the basic theory.

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