Errata and *Addenda*—Howland, *Intermediate Dynamics*

Though the book has undergone a major amount of proofing, some mistakes were overlooked pre-publication. There follow those I have discovered subsequently in presenting the class; others were pointed out to me by a colleague (Bill Goodwine) who is also used the text in our London Program Fall of 2005, and even (to my astonishment) some attentive students! Line numbers indicated are those of *text*; equations are not counted in such enumerations.

My thanks to all!

p. 65, last line before §2.2.1

"vectors in the <u>domain</u> are ..."

should be

"vectors in the <u>range</u> are ..."

p. 99, Property "RM4"

"$\mathbf{A}\underline{\mathbf{C}} + \mathbf{AC}$"

should be

"$\mathbf{A}\underline{\mathbf{B}} + \mathbf{AC}$"

p. 102, a *n*ew Theorem 3.1.5 (original "3.1.5" moved to "3.1.6"):

Theorem 3.1.5. (*Inverse of a Transpose*): If $\mathbf{A}^{-1}$ exists, so does $(\mathbf{A}^{\mathsf{T}})^{-1}$, and it is just $(\mathbf{A}^{-1})^{\mathsf{T}}$. *Proof.* $\mathbf{AA}^{-1} = \mathbf{1}$. Taking the transpose of [both sides of] this equation, $(\mathbf{A}^{-1})^{\mathsf{T}}\mathbf{A}^{\mathsf{T}} = \mathbf{1}^{\mathsf{T}} = \mathbf{1}$. But this says that $(\mathbf{A}^{-1})^{\mathsf{T}}$ is the left inverse, so inverse (by Theorem 3.1.1), of $\mathbf{A}^{\mathsf{T}}$; *i.e.*, $(\mathbf{A}^{\mathsf{T}})^{-1} = (\mathbf{A}^{-1})^{\mathsf{T}}$.

p. 108, equation (3.2-6)

"$(\mathbf{E}^*_{ij}(c))^{-1} = \underline{\mathbf{E}^*_{ij}(-c)}$"

should be

"$(\mathbf{E}^*_{ij}(c))^{-1} = \underline{\begin{cases} \mathbf{E}^*_{jj}(\frac{1}{c+1}) & i = j \\ \mathbf{E}^*_{ij}(-c) & i \neq j \end{cases}}$"

p. 109, last line

"$|\mathbf{A}_{ech}| = |\underline{\mathbf{I}}| = 1.$"

should be

"$|\mathbf{A}_{ech}| = |\underline{\mathbf{1}}| = 1.$"

p. 122, Homework 2.

"and eigenvalue of a matrix, <u>then so are, respectively, $c\boldsymbol{\xi}$ and $c\lambda$.</u>"

should be

"and eigenvalue of a matrix $\mathbf{A}$<u>, then $c\boldsymbol{\xi}$ is also an eigenvector of $\mathbf{A}$ and $c\lambda$ an eigenvalue of $c\mathbf{A}$.</u>"

p. 129 (statement of Cayley-Hamilton Theorem, Theorem 3.5.3)

"$p(\lambda) \equiv |\mathbf{A} - \lambda\mathbf{1}|$, then $p(\mathbf{A}) = \mathbf{0}$."

should be

"$p(\lambda) \equiv |\mathbf{A} - \lambda\mathbf{1}|\underline{= 0}$, then $p(\mathbf{A}) = \mathbf{0}$."

p. 136, l. 4

Might be more clear if there was added:

"the other three linearly independent <u>[generalized]</u> eigenvectors predicted by"

p. 182, equation (4.1-8): The "hat" on the $\boldsymbol{u}_1$'s "drifted". There's no significance to this, except that I need to type better!

p. 207, second line of expressions for $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$

"$(\ (\dot{\psi}\hat{\boldsymbol{K}} + p(\sin\theta\hat{\boldsymbol{\imath}}_\psi + \cos\theta\hat{\boldsymbol{K}})$"

should be

"($(\dot{\psi}\hat{\boldsymbol{K}} + p(\sin\theta\hat{\boldsymbol{\imath}}_\psi + \cos\theta\hat{\boldsymbol{K}})\underline{)}$"

pp. 221-224 (non-slip condition): The "Example" imposing this condition on the motion of a spherical rigid body is OK, but then applying *it*—particularly eqs (4.4-14)—to Example 4.4.2 at the end is *not*: the latter deals with a *disk*, whose central motion need *not* remain at a fixed distance above the surface as it is assumed the sphere's does. *Mea culpa.*

Equation (4.4-14a) can be derived more generally, and even more simply, however—at least under the additional (albeit reasonable) assumption that the body *does not lose contact* with the surface: if it does, "non-slip" becomes meaningless anyway! This means that, in addition to having $\boldsymbol{v}^t_{C'/C} \equiv \mathbf{0}$ (the non-slip condition itself), we also have $\boldsymbol{v}^n_{C'/C} \equiv \mathbf{0}$, so that the contact point does not "jump up" off the surface. Then, since the body is rigid so $\boldsymbol{v}_{rel} \equiv 0$,:

$$\boldsymbol{v}_O = \boldsymbol{v}_{C'} + \boldsymbol{\omega} \times \boldsymbol{r}_{O/C'} + \boldsymbol{v}_{rel} = (\boldsymbol{v}_C + \boldsymbol{v}_{C'/C}) + \boldsymbol{\omega} \times \boldsymbol{r}_{O/C'} = \boldsymbol{v}_C + \boldsymbol{\omega} \times \boldsymbol{r}_{O/C'},$$

by virtue of the fact that both parts of $\boldsymbol{v}_{C'/C} = \boldsymbol{v}^t_{C'/C}\hat{\boldsymbol{t}} + v^n_{C'/C}\hat{\boldsymbol{n}}$ must vanish under the non-slip and assumed non-loss of contact. No assumption is made regarding the motion of O. (4.4-14b) can hold, but only in the case that $\boldsymbol{r}_{O/C}$ is perpendicular to the surface (so that it has no component tangent to the surface—the operative assumption in (4.4-14b)).

Note, however, that the use of $\dot{\boldsymbol{A}} = \boldsymbol{\omega} \times \boldsymbol{A} + \dot{\boldsymbol{A}}_{rel}$ is still best to find accelerations!

p. 242, first [unnumbered] equation To prevent confusion,

"

$$\boldsymbol{\omega} \cdot \sin\theta\hat{\boldsymbol{I}} = -\omega_X \sin\theta \equiv 0,$$

"

should be

"

$$\boldsymbol{\omega}_{\underline{OA'B'}} \cdot \sin\theta\hat{\boldsymbol{I}} = -\omega_X \sin\theta \equiv 0,$$

"

p. 245, Equations 4.4-30a-b: I forgot subscripts on the angular accelerations, and the origin point is wrong in (4.4-30.b):

$$\begin{aligned}\boldsymbol{a}_C &= \boldsymbol{a}_A + \boldsymbol{\alpha}_{\underline{AC}} \times \boldsymbol{r}_{C/A} + \boldsymbol{\omega}_{AC} \times (\boldsymbol{\omega}_{AC} \times \boldsymbol{r}_{C/A}) \\ &\qquad + 2\boldsymbol{\omega}_{AC} \times \boldsymbol{v}_{rel\,AC} + \boldsymbol{a}_{rel\,AC} && (4.4\text{-}30a)\\ &= \boldsymbol{a}_{\underline{B}} + \boldsymbol{\alpha}_{\underline{BC}} \times \boldsymbol{r}_{C/B} + \boldsymbol{\omega}_{BC} \times (\boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{C/B}) \\ &\qquad + 2\boldsymbol{\omega}_{BC} \times \boldsymbol{v}_{rel\,BC} + \boldsymbol{a}_{rel\,BC} && (4.4\text{-}30b)\end{aligned}$$

p. 251, just before Summary: Not really an error: insert *Addendum* on p. 14

p. p. 289, l. -8

"$\boldsymbol{\Xi}^{\mathsf{T}}\boldsymbol{\Xi} = \underline{\mathbf{I}}$"

should be

"$\boldsymbol{\Xi}^{\mathsf{T}}\boldsymbol{\Xi} = \underline{\mathbf{1}}$"

p. p. 293, l. -3 (exclusive of equations at bottom)**:**

"we merely rotate about the $\underline{x'}$-axis"

should be

"we merely rotate about the $\underline{\xi}$-axis"

p. 294, final answer for $\mathsf{I}_O^{(p)}$: I left out the m. It probably could better be written

"

$$\begin{pmatrix} \left(\frac{13}{3} - 2\sin\theta\right) & 0 & 0 \\ 0 & \left(\frac{1}{3} - 2\sin\theta + 4\sin^2\theta\right) & (4\sin\theta - 1)\cos\theta \\ 0 & (4\sin\theta - 1)\cos\theta & 4\cos^2\theta \end{pmatrix} ma^2.$$

"

p. 300, second sentence of last paragraph

"easy: since AC has length $2a$ and $Z_A = \sqrt{3}a$ while $Z_{\underline{B}} \equiv 0$,"

should be

"easy: since AC has length $2a$ and $Z_A = \sqrt{3}a$ while $Z_{\underline{C}} \equiv 0$,"

And, **at the end of that paragraph:**

"$\hat{\boldsymbol{\imath}} \equiv \hat{\boldsymbol{K}} \times \boldsymbol{r}_{AC} = a\sin\theta\hat{\boldsymbol{I}} + a\cos\theta\hat{\boldsymbol{J}}$—a vector of magnitude a, and

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{I}} \equiv \ldots"$$

should read:

$\boldsymbol{i} \equiv \hat{\boldsymbol{K}} \times \boldsymbol{r}_{AC} = a\sin\theta\hat{\boldsymbol{I}} + a\cos\theta\hat{\boldsymbol{J}}$—*not* a unit vector, but one of magnitude a, and

$$\boldsymbol{i} \cdot \hat{\boldsymbol{I}} \equiv \ldots"$$

p. 301, bottom of page: The determinant for $\hat{\boldsymbol{\jmath}}$,

$$\text{“}\hat{\boldsymbol{\jmath}} \equiv \hat{\boldsymbol{k}} \times \hat{\boldsymbol{\imath}} = \begin{vmatrix} \hat{\boldsymbol{I}} & \hat{\boldsymbol{J}} & \hat{\boldsymbol{K}} \\ -\frac{\cos\theta}{2} & \frac{\sin\theta}{2}\hat{\boldsymbol{J}} & +\frac{\sqrt{3}}{2} \\ \sin\theta & \underline{\sin\theta} & 0 \end{vmatrix}\text{”}$$

should be

$$\text{“}\hat{\boldsymbol{\jmath}} \equiv \hat{\boldsymbol{k}} \times \hat{\boldsymbol{\imath}} = \begin{vmatrix} \hat{\boldsymbol{I}} & \hat{\boldsymbol{J}} & \hat{\boldsymbol{K}} \\ -\frac{\cos\theta}{2} & \frac{\sin\theta}{2}\hat{\boldsymbol{J}} & +\frac{\sqrt{3}}{2} \\ \sin\theta & \underline{\cos\theta} & 0 \end{vmatrix}\text{”}$$

p. 319, [text] line 10

“. . . three constraints at both O and $\underline{B}$:”

should be

“. . . three constraints at both O and $\underline{A}$:”

p. 323, first figure: To make this consistent with the notation of the rest of this example, there should be a first-level subscript A on all six components, *e.g.*, “$\boldsymbol{f}_{A_{ABX}}$”.

p. 324, fourth line above eq. (5.2-31h)

“The three scalar components of <u>this equation</u>. . . ”

should be

“The three scalar components of <u>these equations</u>. . . ”

p. 330, l. -5 I believe this *is* true in general: both $\bar{\mathbf{I}}(\boldsymbol{\Omega} \times \boldsymbol{p}) = (\bar{\mathbf{I}}\boldsymbol{\Omega}) \times \boldsymbol{p}$ can be represented as *matrix* products(Example 2.2.7), which, in turn, are associative (eq. 2.1-14).

But a more direct demonstration can be made:

“

$$\sum \bar{\boldsymbol{M}} = \boldsymbol{\omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} + \bar{\mathbf{I}}\boldsymbol{\alpha}$$

But

$$\boldsymbol{\omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} = (\boldsymbol{\Omega} + \boldsymbol{p}) \times \bar{\mathbf{I}}\boldsymbol{\omega} = \boldsymbol{\Omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} + \boldsymbol{p} \times \bar{\mathbf{I}}\boldsymbol{\omega}$$

in which

$$\begin{aligned} \boldsymbol{p} \times \bar{\mathbf{I}}\boldsymbol{\omega} &= \boldsymbol{p} \times \bar{\mathbf{I}}(\boldsymbol{\Omega} + \boldsymbol{p}) = \boldsymbol{p} \times \bar{\mathbf{I}}\boldsymbol{\Omega} + \boldsymbol{p} \times \bar{\mathbf{I}}\boldsymbol{p} \\ &= -\bar{\mathbf{I}}\boldsymbol{\Omega} \times \boldsymbol{p} + \mathbf{0} \end{aligned}$$

since $\boldsymbol{p}$ is along a principal axis of $\bar{\mathbf{I}}$ so $\bar{\mathbf{I}}\boldsymbol{p} \parallel \boldsymbol{p}$. Thus

$$\boldsymbol{\omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} = \boldsymbol{\Omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} - \bar{\mathbf{I}}\boldsymbol{\Omega} \times \boldsymbol{p} \tag{a}$$

Further, relative to the $\boldsymbol{\Omega}$-rotating coordinate system by Theorem 4.3.1,

$$\begin{aligned}\boldsymbol{\alpha} \equiv \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_{rel} + \boldsymbol{\Omega} \times \boldsymbol{\omega} = \dot{\boldsymbol{\omega}}_{rel} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} + \boldsymbol{p}) \equiv \boldsymbol{\alpha}_{rel} + \boldsymbol{\Omega} \times \boldsymbol{\Omega} + \boldsymbol{\Omega} \times \boldsymbol{p} \\ = \boldsymbol{\alpha}_{rel} + \boldsymbol{\Omega} \times \boldsymbol{p},\end{aligned}$$

so that

$$\bar{\mathbf{I}}\boldsymbol{\alpha} = \bar{\mathbf{I}}\boldsymbol{\alpha}_{rel} + \bar{\mathbf{I}}\boldsymbol{\Omega} \times \boldsymbol{p}, \tag{b}$$

Thus, adding (a) and (b),

$$\sum \bar{\boldsymbol{M}} = \boldsymbol{\Omega} \times \bar{\mathbf{I}}\boldsymbol{\omega} + \mathbf{I}\boldsymbol{\alpha}_{rel},$$

as desired."

p. 332, bottom line: There is no need for the "O" subscript in the expression for the form of both $\mathbf{I}\boldsymbol{\omega}$ and $\mathbf{I}\boldsymbol{\alpha}$.

p. 334 diagram: The diagram defining the angles α and β for the moment-free gyroscope on **page** should have θ going all the way to the z-axis.

p. 336, top of page

All "$\underline{\theta}$" should be "$\underline{\theta^*}$."

And the **last bulletted item**:

"$Ip\dot{\psi}\sin\theta = 0$ <u>and, assuming $p \neq 0$—$\theta \neq 0$ by definition of the entire problem, recall—$\dot{\psi}$ must vanish: there *is* no precession!</u>"

should be

"$Ip\dot{\psi}\sin\theta = 0$<u>—$p$ must vanish: there *is* no spin (as above).</u>" This really makes sense considering the equation

$$\dot{\psi}\cos\theta = \Omega_z = \frac{Ip}{I_O - I}:$$

For all quantities except θ and p fixed, $p \to 0$ as $\theta \to 90°$. This, then, becomes the "break-even point" between positive and negative values of θ^*.

p. 364, l. 3 (though it's really never used)

"constant height of <u>210</u> mm"

should be

"constant height of $\underline{105}$ mm"

p. 365, ll. 8-9 (not counting equations)**:** Not really a mistake, but I don't see where I actually calculated $\boldsymbol{r}_{B/A_2}$ to get $\boldsymbol{t}_2$. But again using the technique of Example 4.4.7, it is easy to get that $Y_{B_2} = 80\,\text{mm}$ so that $X_{B_2} = 60\,\text{mm}$; thus $\boldsymbol{r}_B = 0.06\hat{\boldsymbol{I}} + 0.08\hat{\boldsymbol{J}}\,\text{m}$, leading to $\boldsymbol{r}_{B/A}$.

p. 369, second equation

"$\bar{\mathsf{I}}\boldsymbol{\omega} = \bar{I}_{xx}(-\omega\hat{\boldsymbol{I}}\underline{\ }$"

should be

"$\bar{\mathsf{I}}\boldsymbol{\omega} = \bar{I}_{xx}(-\omega\hat{\boldsymbol{I}}\underline{)}$"

p. 369, end of Example 5.4.3: Add the *Addendum* on p. 15

p. 374, first [unnumbered] equation The end of the equation,

"$-Ip(\dot{\psi}\cos\theta + p) = 0$"

should be

"$-Ip\dot{\theta} = 0$"

p. 375, eq. (5.4-13)

"

$$\frac{1}{2}\left(I_O\dot{\theta}^2 + I_O\left(\frac{H_Z - I\omega_z\cos\theta}{I_O\sin^{\underline{2}}\theta}\right)^2\sin^2\theta + I\omega_z^2\right)$$

"

should be

"

$$\frac{1}{2}\left(I_O\dot{\theta}^2 + I_O\left(\frac{H_Z - I\omega_z\cos\theta}{I_O\sin\theta}\right)^2\sin^2\theta + I\omega_z^2\right)$$

"

389, fourth and fifth lines [of text]: Should read:
"in the *forces*—the *kinetics*<u>—</u>and the [vector] coordinates used to describe this—the <u>*kinematics*</u>."

p. 413, l. 5

> “constraint, even though holonomic, is *__scleronomic__*, work *is* done”

should be

> “constraint, even though holonomic, is *__rheonomic__*, work *is* done”

Chap. 8, development of Euler-Lagrange Equations: Though it is necessary to assume all constraints have been eliminated, it is *not* necessary, as stated in the Introductory material, to assume such constraints be holonomic: all that is used is the linear independence of the δq_i!

p. 430, l. 12 of Example 8.1.1

> “__counter__clockwise”

should be

> “clockwise”

p. 435, second equation block

$$“\frac{\boldsymbol{v}}{\underline{|}\boldsymbol{v}\underline{|}}”$$

should be

“

$$\frac{\boldsymbol{v}}{\underline{\|}\boldsymbol{v}\underline{\|}}$$

”

Also, l-5:

$$“v \equiv \underline{|}\boldsymbol{v}\underline{|} = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}”$$

should be “

$$v \equiv \underline{\|}\boldsymbol{v}\underline{\|} = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

”

Also, l-3 (in line starting with “$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) - \frac{\partial T}{\partial r}$”):

$$“m\ddot{r}\underline{-}mr\dot{\theta}^2”$$

should be

$$\text{“}m\ddot{r}\underline{\pm}mr\dot{\theta}^2\text{”}$$

p. 440, third [unnumbered] equation: The last term of the first [general] equation for vitrual work should have “$\delta\theta_2$”, not “$\delta\theta_1$”!

Also: calculation of $\delta U(m_1 g)$::

$$\text{“} - m_1 g\hat{\boldsymbol{J}} \cdot (\ldots \underline{-} l_1 \sin\theta_1\, \delta\theta_1 \hat{\boldsymbol{J}})\text{”}$$

should be “

$$-m_1 g\hat{\boldsymbol{J}} \cdot (\ldots \underline{\ \ } l_1 \sin\theta_1\, \delta\theta_1 \hat{\boldsymbol{J}})\text{”}$$

p. 448, expression for L in Example 8.2.2

“ $-(-m_1 g l_1 \cos\theta_1 - m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2)\underline{\ }$”

should be

“ $-(-m_1 g l_1 \cos\theta_1 - m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2)\underline{)}$”

Also: line before final equations: It probably would have been clearer to write “$\boldsymbol{\alpha} \equiv -\alpha\hat{\boldsymbol{k}} \equiv -\ddot{\theta}\hat{\boldsymbol{k}}$, and $\boldsymbol{\omega} \equiv -\omega\hat{\boldsymbol{k}} \equiv -\dot{\theta}\hat{\boldsymbol{k}}$”.

p. 451, very last equation: Leave it out. The *previous* (next-to-the-bottom) equation should equal 0.

p. 463, last [unnumbered] equation: In the equation , the *text* “*tan*” should be [*math operator*] “tan.”

p. 476, [unnumbered] equation following (9.2-2): In the unnumbered equation following (9.2-2), “**l**” should be “**1**.”

p. 484, l. 8 (just a fonts issue)

“$\dot{\boldsymbol{q}}(\boldsymbol{q},\boldsymbol{p};\underline{\mathrm{t}})$”

should be

“$\dot{\boldsymbol{q}}(\boldsymbol{q}, \boldsymbol{p}; \underline{t})$”

p. 488: Equation (10.2-4) should have parentheses around the Hessian; *i.e.*, it should read
"
$$\left|\left(\frac{\partial^2 F}{\partial u_i \partial u_j}\right)\right| \neq 0, \tag{10.2-4}$$
"
The same thing happens in the **second of the four equations at the bottom of p. 489**, and the **second [unnumbered] equation on p. 490**.

pp. 498, last paragraph, *through* 499, Theorem 10.4.1: This is not so much an "error" as an alternative derivation: it is more terse and puts the condition itself in bolder relief relative to the normal transformation of differential equations. *Replace this by the Addendum on page 17.*

p. 499, Statement of Theorem 10.4.1

"Jacobian matrix J"

should be

"Jacobian matrix T"

p. 501, l. 4: I originally had this right but thought I had a quick-and-dirty method of proving this result. But I made a mistake: mere transposition of (10.4-6) simply returns that condition again! Rather it is necessary to be a little more circumspect: From that equation we get immediately that $\mathsf{T}^\mathsf{T} = \mathsf{J}^{-1}\mathsf{T}^{-1}\mathsf{J}$. Combining this with Problem 1a) on page 502 gives us $\mathsf{T}^\mathsf{T} = -\mathsf{J}\mathsf{T}^{-1}\mathsf{J}$, so

$$\mathsf{T}^\mathsf{T}\mathsf{J}\mathsf{T} = (-\mathsf{J}\mathsf{T}^{-1}\mathsf{J})(\mathsf{J}\mathsf{T}) = -\mathsf{J}\mathsf{T}^{-1}\mathsf{J}^2\mathsf{T} = +\mathsf{J}\mathsf{T}^{-1}\mathsf{T} = \mathsf{J}.$$

p. 501, second block of equations

"$(\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{q}})^\mathsf{T}(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{p}})$- $(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{q}})^\mathsf{T}(\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{p}})=\underline{\mathsf{I}}$"

should be

"$(\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{q}})^\mathsf{T}(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{p}})$- $(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{q}})^\mathsf{T}(\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{p}})=\underline{\mathbf{1}}$"

p. 501, eq. (10.4-10c)

"$\sum_k^f \left(\frac{\partial Q_k}{\partial q_{\underline{k}}}\frac{\partial P_k}{\partial p_j} - \frac{\partial P_k}{\partial p_i}\frac{\partial Q_k}{\partial p_j}\right) = 0.$"

should be

“$\sum_k^f \left(\frac{\partial Q_k}{\partial \underline{q_i}} \frac{\partial P_k}{\partial p_j} - \frac{\partial P_k}{\partial p_i} \frac{\partial Q_k}{\partial p_j} \right) = 0.$”

p. 501, next-to-last equation block

“$[u, v]_{\underline{q,p}} \equiv \sum_k^f \left(\frac{\partial q_k}{\partial u} \frac{\partial p_k}{\partial v} - \frac{\partial p_k}{\partial u} \frac{\partial q_k}{\partial v} \right)$”

should be

“$[u, v]_{\underline{\boldsymbol{q},\boldsymbol{p}}} \equiv \sum_k^f \left(\frac{\partial q_k}{\partial u} \frac{\partial p_k}{\partial v} - \frac{\partial p_k}{\partial u} \frac{\partial q_k}{\partial v} \right)$”

p. 505 l. 5 (exclusive of equations)**:**

“$\frac{\partial B_j}{\partial p_i} = \frac{\partial B_i}{\partial \underline{q}_j}$”

should be

“$\frac{\partial B_j}{\partial p_i} = \frac{\partial B_i}{\partial \underline{p}_j}$”

p. 508: The idea here is to use the results of Theorem 10.2.1, which applies to *Lagrangians*, to get the results for time-dependent *Hamiltonians*. But, in order to do that, *we must use* $\tilde{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{p}; t)$, and the independent variables $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\boldsymbol{p}$, not $L(\boldsymbol{q}, \dot{\boldsymbol{q}}; t)$ itself (see section 10.2.3, particularly Example 10.2.1). Thus all the “L’s” on this page should really be “$\tilde{L}$’s”:

“$\underline{L}(\boldsymbol{q}, \boldsymbol{p}; t) = \boldsymbol{p} \cdot \dot{\boldsymbol{q}}(\boldsymbol{q}, \boldsymbol{p}; t)$”

should be

“$\underline{\tilde{L}}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{p}; t) = \boldsymbol{p} \cdot \dot{\boldsymbol{q}};$”

“$\underline{L^*}(\boldsymbol{Q}, \boldsymbol{P}; t) = \boldsymbol{P} \cdot \dot{\boldsymbol{Q}}(\boldsymbol{Q}, \boldsymbol{P}; t)$”

should be

“$\underline{\tilde{L}^*}(\boldsymbol{Q}, \dot{\boldsymbol{Q}}, \boldsymbol{P}; t) = \boldsymbol{P} \cdot \dot{\boldsymbol{Q}};$”

“*we add* $\dot{W}$ *to* $\underline{L^*}$; *i.e.*, rather than using the above $\underline{L^*}$, these equations remain unchanged if we use”

should be

“*we add* $\dot{W}$ *to* $\underline{\tilde{L}^*}$; *i.e.*, rather than using the above $\underline{\tilde{L}^*}$, these equations remain unchanged if we use”

“$\underline{L^{*\prime}} \equiv \boldsymbol{P} \cdot \dot{\boldsymbol{Q}} - H^* + \frac{dW}{dt} =$”

should be

“$\underline{\tilde{L}^{*'}} \equiv \boldsymbol{P} \cdot \dot{\boldsymbol{Q}} - H^* + \frac{dW}{dt} =$”

and

“if such a transformation transforms $\underline{L}$ above to $\underline{L^{*'}} = L$”

should be

“if such a transformation transforms $\underline{\tilde{L}}$ above to $\underline{\tilde{L}^{*'}} = L$”

p. 511, paragraph starting before eq. (10.6.3)

I inadvertently switched the sign of the generating function in the text, but not the equations themselves:

“$p = {}_{-}\frac{\partial W}{\partial x}$” and “$P = \underline{-}\frac{\partial W}{\partial X}$”

should be

““$p = \underline{-}\frac{\partial W}{\partial x}$” and “$P = {}_{-}\frac{\partial W}{\partial X}$”

p. 581, equations

The capital “P’s” and Greek subscripts should, of course, all be *small* “p’s” and Greek:

“

$$P_\Theta^2 \equiv \left(\frac{\partial W}{\partial \theta}\right)^2$$
$$P_\Phi^2 \equiv \left(\frac{\partial W}{\partial \phi}\right)^2 + \frac{P_\Theta^2}{\sin^2 \phi}$$
$$R \equiv \frac{1}{2m}\left(\left(\frac{\partial W}{\partial r}\right)^2 + \frac{P_\Phi^2}{r^2} - \frac{\mu}{r}\right)$$

or

$$\frac{\partial W}{\partial \theta} = \frac{\partial W_\theta}{\partial \theta} = P_\Theta$$
$$\frac{\partial W}{\partial \phi} = \frac{\partial W_\phi}{\partial \phi} = \sqrt{P_\Phi^2 - \frac{P_\Theta^2}{sin^2 \phi}}$$
$$\frac{\partial W}{\partial r} = \frac{\partial W_r}{\partial r} = \sqrt{2mR - \frac{P_\Phi^2}{r^2} + \frac{\mu}{r}}.$$

”

should be

“

$$p_\theta^2 \equiv \left(\frac{\partial W}{\partial \theta}\right)^2$$
$$p_\phi^2 \equiv \left(\frac{\partial W}{\partial \phi}\right)^2 + \frac{p_\theta^2}{\sin^2\phi}$$
$$R \equiv \frac{1}{2m}\left(\left(\frac{\partial W}{\partial r}\right)^2 + \frac{p_\phi^2}{r^2} - \frac{\mu}{r}\right)$$

or

$$\frac{\partial W}{\partial \theta} = \frac{\partial W_\theta}{\partial \theta} = p_\theta$$
$$\frac{\partial W}{\partial \phi} = \frac{\partial W_\phi}{\partial \phi} = \sqrt{p_\phi^2 - \frac{p_\theta^2}{sin^2\phi}}$$
$$\frac{\partial W}{\partial r} = \frac{\partial W_r}{\partial r} = \sqrt{2mR - \frac{p_\phi^2}{r^2} + \frac{\mu}{r}}.$$

”

Addendum to Section 4.4.3 (before the Summary)

Actually working Example 4.4.6 using the present approach illustrates, however, the efficacy of combining all the kinematic equations into a single system, even when the problem seems amenable to "chaining":

In the present notation, $\hat{\boldsymbol{u}}_{OA'} = \hat{\boldsymbol{I}}$ in that Example; thus $\boldsymbol{\omega}_{AO} \times \hat{\boldsymbol{u}}_{OA'} = \omega\hat{\boldsymbol{J}} \times \hat{\boldsymbol{I}} = -\hat{\boldsymbol{K}}$. But then the condition 4.4-33a implies merely that $\omega_Z \equiv 0$—precisely the result derived from the rotating coordinate systems equations for $\boldsymbol{v}_{A'}$ in that Example—and with no means of determining ω_X as we had before! Indeed, careful examination of the "four" equations—three components from the equations for $\boldsymbol{v}_{A'}$ and the one "additional condition" give, respectively,

$$\begin{aligned} 0 &= 0 \\ 0 &= a\omega_Z \\ -a\omega &= -a\omega_Y \\ \omega_Z &= 0 \end{aligned}$$

—only *two* [independent] equations in the four unknowns. This is just another manifestation of the special orientation of $\boldsymbol{r}_{A'/O}$: had it lain *off* the X-axis with a non-0 Z-component, there would have been a non-0 X-component for $\boldsymbol{v}_{OA'}$ relative to $OA'B'$, and ω_X would have appeared in the "condition" $\boldsymbol{\omega}_{OA'B'} \cdot \boldsymbol{\omega}_{AO} \times \hat{\boldsymbol{u}}_{OA'} \equiv 0$, making the above system of rank 4. Incorporating the fact that the original equations do give us that $\omega_Y = \omega$, then, we have at this point in the Example only that

$$\boldsymbol{\omega}_{OA'B'} = \omega_X\hat{\boldsymbol{I}} + \omega\hat{\boldsymbol{J}}.$$

We can only hope that equations involving OB will resolve this issue!

In fact they do: The equation for $\boldsymbol{v}_{B'}$ relative to $OA'B'$ now becomes, upon substitution of the above $\boldsymbol{\omega}_{OA'B'}$,

$$\begin{aligned} \boldsymbol{v}_{B'} &= \boldsymbol{0} + (\omega_X\hat{\boldsymbol{I}} + \omega\hat{\boldsymbol{J}}) \times a(\sin\theta\hat{\boldsymbol{J}} + \cos\theta\hat{\boldsymbol{K}}) + \boldsymbol{0} \\ &= a\omega\cos\theta\hat{\boldsymbol{I}} + a(-\omega_X\cos\theta\hat{\boldsymbol{J}} + \omega_X\sin\theta\hat{\boldsymbol{K}}) \\ &\vdots \\ &= a\omega_{OB}(\cos^2\theta + \sin^2\theta)\hat{\boldsymbol{I}}. \end{aligned}$$

Thus the Y- and Z- components *both* give $\omega_X = 0$; this would hold even at $\theta = 0$ or $180°$. The Z-component gives $\omega_{OB} = \omega\cos\theta$ as before.

Note that we were able to determine $\boldsymbol{\omega}_{OA'B'}$ before from the analysis of $\boldsymbol{v}_{A'}$ alone because our "condition" considered the motion of OB right from the start; here we looked only at the spider. At the same time, however, the present approach would *include* OB in the *complete* system (rather than being used in a separate, second step); and, in any event, also covers the acceleration analysis.

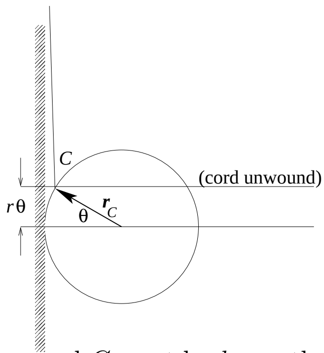
Addendum to Example 5.4.3

There is the interesting question of what happens when the cords have completely unwound from the cylinder. While we could use the angular momentum already considered to get $\theta(t)$ from the integration of $\omega(t)$ and, with the value of θ from the non-slip condition to get to the bottom, find the necessary value of t—(whew!)—it is easier simply to use energy:

Though there is tension in the cords, which might be expected to do work, in fact it doesn't: due to non-slip, the point of contact with the fixed wall has zero velocity, so $d\boldsymbol{r} \equiv 0$, so does no work. Even if we *didn't* have non-slip, the cords could be considered part of the "system," "internalizing" the work. The only other force, which *does* do work, is gravity; it is conservative, however, so energy $E = T + V$ is a constant, for $V = mg\bar{Z}$. But we must still calculate T:

$$\begin{aligned} T &= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{\mathsf{I}}\omega^2 \\ &= \frac{1}{2}m(-r\omega)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 \\ &= \frac{3}{4}mr^2\omega^2 \end{aligned}$$[1]

Perhaps even more interesting is consideration of what happens [just] *after* the cords are fully extended: the body cannot continue to move "without slipping"! For assume it *did*: If the body rotates an additional angle θ from that point at which the cords are extended against the wall, then the center of the cylinder has moved an additional $y = -r\theta$ below that point, while the point C where the cords join have moved the same distance $r\theta$ around the circumfrence; thus $\boldsymbol{r}_C = (-r(1 - \cos\theta), r\sin\theta)$ relative to the center of the disk—a distance $y = -r(\theta - \sin\theta)$ *below* the point of full extension. Now $\sin\theta < \theta$; thus this coordinate is negative, and C must be *lower* than this point. But the end of the cords can only move *up* as they swing away from the wall; thus the body *cannot* move without slipping!



How then could we account for the *non*-non-slip motion of the cylinder? A kinematical analysis reveals this: For the cords of [generic] length L, connected

[1]By the discussion on page 308, the same result would be obtained by taking I about the point of contact. But this approach is more general, and less likely to succumb to abuse.

at [generic] point P and rotating with $\boldsymbol{\omega}_L = \dot{\phi}\hat{\boldsymbol{K}}$

$$\begin{aligned}
\boldsymbol{v}_C &= \boldsymbol{v}_P + \boldsymbol{\omega}_L \times \boldsymbol{r}_{C/P} + \boldsymbol{v}_{rel} \\
&= \mathbf{0} + \dot{\phi}\hat{\boldsymbol{K}} \times L(\sin\phi\hat{\boldsymbol{I}} - \cos\phi\hat{\boldsymbol{J}}) + \mathbf{0} \\
&= l\dot{\phi}(\sin\phi\hat{\boldsymbol{J}} + \cos\phi\hat{\boldsymbol{I}}) \\
&= \boldsymbol{v}_O + \boldsymbol{\omega}_O \times \boldsymbol{r}_{C/O} + \boldsymbol{v}_{rel} \\
&= -v_O\hat{\boldsymbol{J}} + \dot{\theta}(-\hat{\boldsymbol{K}}) \times r(-\cos\theta\hat{\boldsymbol{I}} + \sin\theta\hat{\boldsymbol{J}}) + \mathbf{0} \\
&= -v_O\hat{\boldsymbol{J}} + r\dot{\theta}(\cos\theta\hat{\boldsymbol{J}} + \sin\theta\hat{\boldsymbol{I}})
\end{aligned}$$

The two component equations:

$$\begin{aligned}
L\dot{\phi}\cos\phi &= r\dot{\theta}\sin\theta \\
L\dot{\phi}\sin\phi &= -v_O + r\dot{\theta}\cos\theta
\end{aligned}$$

are two equations in $\dot{\phi}$, $\dot{\theta}$, and v_O, two of which can be solved for in terms of the third. But, noting that $\phi \to 0$ and $\theta \to 0$ together just as the body starts out of non-slip motion, the first tells us that $L\dot{\phi} \equiv 0$, so the second says that $v_O = r\dot{\theta}$ [note the signs!], which *is* still non-slip! (Of course, that changes immediately.)

Alternative Derivation of Symplectic Condition

We consider only the autonomous case of transformation explicitly. The reasons for this restriction will be given at the end.

Assume an n-vector variable $\boldsymbol{x}$ satisfies the autonomous first-order ordinary differential equation

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \tag{1}$$

and consider an arbitrary time-independent [non-singular] transformation $\boldsymbol{x} \to \boldsymbol{X}$ *dependent only on the coordinates*—not *the time* (a *contact* transformation):

$$\boldsymbol{x} = \boldsymbol{\xi}(\boldsymbol{X}) \tag{2}$$

where $\boldsymbol{\xi}$ is a *function*. Then, by differentiating (2) and expressing (1) entirely in terms of $\boldsymbol{X}$,

$$\dot{\boldsymbol{x}} = \frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}} \dot{\boldsymbol{X}} = \left. \boldsymbol{f}(\boldsymbol{x}) \right|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})},$$

in which

$$\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}} \equiv \begin{pmatrix} \frac{\partial \xi_1(\boldsymbol{X})}{\partial X_1} & \cdots & \frac{\partial \xi_1(\boldsymbol{X})}{\partial X_n} \\ \vdots & & \vdots \\ \frac{\partial \xi_n(\boldsymbol{X})}{\partial X_1} & \cdots & \frac{\partial \xi_n(\boldsymbol{X})}{\partial X_n} \end{pmatrix};$$

thus $\boldsymbol{X}$ satisfies

$$\dot{\boldsymbol{X}} = \left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}} \right)^{-1} \left. \boldsymbol{f}(\boldsymbol{x}) \right|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})}. \tag{3}$$

Now consider the special case of a *Hamiltonian* system, where

$$\boldsymbol{f}(\boldsymbol{x}) = \mathsf{J} \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}} \tag{4}$$

in which J is the *symplectic matrix*

$$\mathsf{J} \equiv \begin{pmatrix} \mathbf{1}_n & \mathbf{0} \\ \mathbf{0} & -\mathbf{1}_n \end{pmatrix},$$

$\mathbf{1}_n$ being the $n \times n$ identity matrix, and we denote

$$\boldsymbol{x} \equiv \begin{pmatrix} q_1 \\ \vdots \\ q_n \\ p_1 \\ \vdots \\ p_n \end{pmatrix} \qquad \text{and} \qquad \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}} \equiv \begin{pmatrix} \frac{\partial H(\boldsymbol{x})}{\partial q_1} \\ \vdots \\ \frac{\partial H(\boldsymbol{x})}{\partial q_n} \\ \frac{\partial H(\boldsymbol{x})}{\partial p_1} \\ \vdots \\ \frac{\partial H(\boldsymbol{x})}{\partial p_n} \end{pmatrix}.$$

For $\boldsymbol{x}$ above, under what condition is a transformation (2), $\boldsymbol{x} \to \boldsymbol{X}$, where

$$\boldsymbol{X} \equiv \begin{pmatrix} Q_1 \\ \vdots \\ Q_n \\ P_1 \\ \vdots \\ P_n \end{pmatrix}$$

and $\boldsymbol{x} = \boldsymbol{\xi}(\boldsymbol{X})$, *canonical*; *i.e.*, when is the *general*

$$\dot{\boldsymbol{X}} = \left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^{-1} \mathsf{J}\, \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})} \tag{5a}$$

(by (3) and (4)) going to satisfy equation (4) in the *new* variables $\boldsymbol{X}$:

$$= \mathsf{J}\frac{\partial H^*(\boldsymbol{X})}{\partial \boldsymbol{X}}, \tag{5b}$$

where $H^*(\boldsymbol{X}) \equiv H(\boldsymbol{\xi}(\boldsymbol{X}))$?

Differentiating the last definition, we know that

$$\begin{aligned}
\frac{\partial H^*(\boldsymbol{X})}{\partial \boldsymbol{X}} \equiv \begin{pmatrix} \frac{\partial H^*(\boldsymbol{X})}{\partial X_1} \\ \vdots \\ \frac{\partial H^*(\boldsymbol{X})}{\partial X_n} \end{pmatrix} &= \begin{pmatrix} \sum \frac{\partial H(\boldsymbol{x})}{\partial x_i}\Big|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})} \frac{\partial \xi_i(\boldsymbol{X})}{\partial X_1} \\ \vdots \\ \sum \frac{\partial H(\boldsymbol{x})}{\partial x_i}\Big|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})} \frac{\partial \xi_i(\boldsymbol{X})}{\partial X_n} \end{pmatrix} \\
&= \begin{pmatrix} \frac{\partial \xi_1(\boldsymbol{X})}{\partial X_1} & \cdots & \frac{\partial \xi_n(\boldsymbol{X})}{\partial X_1} \\ \vdots & & \vdots \\ \frac{\partial \xi_1(\boldsymbol{X})}{\partial X_n} & \cdots & \frac{\partial \xi_n(\boldsymbol{X})}{\partial X_n} \end{pmatrix} \begin{pmatrix} \frac{\partial H(\boldsymbol{x})}{\partial x_1} \\ \vdots \\ \frac{\partial H(\boldsymbol{x})}{\partial x_n} \end{pmatrix}\Bigg|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})} \\
&\equiv \left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^{\mathsf{T}} \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})}.
\end{aligned}$$

Multiplying this equation by J gives us (5b)

$$\mathsf{J}\frac{\partial H^*(\boldsymbol{X})}{\partial \boldsymbol{X}} = \mathsf{J}\left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^{\mathsf{T}} \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})},$$

which, to be canonical, must

$$= \left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^{-1} \mathsf{J}\, \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})},$$

expression (5a). From the latter equation, then, the terms multiplying $\frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\boldsymbol{\xi}(\boldsymbol{X})}$ must be the same; *i.e.*,

$$\left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right) \mathsf{J}\left(\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^{\mathsf{T}} = \mathsf{J} \tag{6}$$

—precisely the *symplectic condition.* If we denote $\frac{\partial \boldsymbol{\xi}(\boldsymbol{X})}{\partial \boldsymbol{X}} \equiv \mathsf{T}$, this can be written in the more compact form $\mathsf{TJT}^{\mathsf{T}} = \mathsf{J}$.

Note that T *is the Jacobian of the transformation expressing the* old $\boldsymbol{x}$*—variables* $\boldsymbol{q}$ *and* $\boldsymbol{p}$*—in terms of the* new $\boldsymbol{X}$ *(*$\boldsymbol{Q}$ *and* $\boldsymbol{P}$*)*; derivatives are taken with respect to $\boldsymbol{Q}$ and $\boldsymbol{P}$. It is more traditional to write this in terms of the transformation expressed in terms of the old. As it turns out (not altogether unexpectedly!), if either is canonical, they *both* are:

Theorem. If a transformation, assumed [as always] to be invertible, given by Jacobian T is canonical, then T^{-1} is also.

Proof. Assume T is canonical; thus $\mathsf{TJT}^\top = \mathsf{J}$, so $\mathsf{JT}^\top = \mathsf{T}^{-1}\mathsf{J}$. But $\left(\mathsf{T}^\mathsf{T}\right)^{-1} = \left(\mathsf{T}^{-1}\right)^\top$ by [the *new*!] Theorem 3.1.5, so

$$\mathsf{JT}^\top\left(\mathsf{T}^\mathsf{T}\right)^{-1} = \mathsf{T}^{-1}\mathsf{J}\left(\mathsf{T}^\mathsf{T}\right)^{-1} = \mathsf{T}^{-1}\mathsf{J}\left(\mathsf{T}^{-1}\right)^\top,$$

so, by definition, T^{-1} is also canonical. □

In the sequel, then, we will normally mean by T the [Jacobian of the] transformation expressed in terms of the old variables, derivatives being taken with respect to $\boldsymbol{q}$ and $\boldsymbol{p}$.

In the event the transformation is *time-dependent*, there will be an an additional term $\frac{\partial \boldsymbol{\xi}(\boldsymbol{X},t)}{\partial t}$ in $\dot{\boldsymbol{x}}$, which then propogates through to (3), leading to unappealing form in the symplectic condition.

References

In reviewing the manuscript, Springer asked about a Bibliography. I responded that most proofs were [I assumed] standard and that any citations—there were very few—were given directly as footnotes to the relevant material. There are, however, a small number of books I did use repeatedly; most (I'm dating myself, here) I used in college and graduate school. Those for the Analytical Dynamics are the "classical" references. While the development and many of the proofs are personal, the reference for Algebra relied on one reference for some of the more difficult proofs, particularly Theorems 2.1.1, 2.2.1, and 2.2.3. (These are probably the classical proofs; I just relied on the presentations in this book for them.)

Here are those books:

Algebra

- Herstein, I.N.,*Topics in Algebra*, 1964, Blaisdell. A very good text on a [for me!] difficult topic.

Analytical Dynamics

- Goldstein, Herbert, *Classical Mechanics, 2nd edition*, 1981, Addison-Wesley. *The* classic text in Physics (at least then).
- Lanczos, Cornelious, *The Variational Principles of Mechanics, 3d edition*, 1966, University of Toronto. Less a text than an *appreciation* of the field, and perhaps therefore more valuable to some.

Springer

http://www.springer.com/978-0-387-28059-2

Intermediate Dynamics
A Linear Algebraic Approach
Howland, R.A.
2006, XX, 542 p., Hardcover
ISBN: 978-0-387-28059-2