

Solutions to Exercises in “Optical WDM Networks”

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ANNOUNCEMENT

All solutions to exercises in “Optical WDM Networks” are only *suggested* for readers.

We welcome email from readers who wish to provide any sort of feedback: errors, comments, criticisms, and suggestions for the solutions.

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Optical Networking: Principles and Challenges

- 1.1. What are the advantages of fiber optic technology in communication systems?

Answer:

Fiber optics provides high bandwidth, low signal attenuation, low signal distortion, low power requirement, low material usage, small space requirement, and low cost.

- 1.2. In order to take full advantage of the huge bandwidth available on fiber, various multiplexing techniques such as WDM, TDM, and CDM can be used which allow multiple users to share the bandwidth on a single fiber. Compare and contrast these multiplexing techniques. Why is WDM the most promising choice for optical communication networks?

Answer:

TDM and CDM both require synchronization at either the bit level (TDM) or at the chip level (CDM). Also, TDM and CDM systems may require bit rates (or chip rates) which are much higher than electronic processing speeds. Thus, WDM is currently the most promising multiplexing technology for optical networks.

- 1.3. Consider two regions, 1200-1400 nm and 1450-1650 nm, in a fiber low-loss spectrum. Calculate the actual bandwidth provided by each region. (Hint: Use the identity $f = v/\lambda$ where $v = 2.0 \times 10^8$ m/s. Note that velocity of light in vacuum is approx. 3.0×10^8 m/s, and the velocity of signals in fiber is approx. two-thirds of this value.)

Answer:

Using the formula $f = v/\lambda$, and differentiating both sides with respect to λ we have, $df/d\lambda = -v/\lambda^2$. Rearranging terms and considering over a wider frequency range, the absolute frequency range for a given wavelength range can be expressed as, $\Delta f = v\Delta\lambda/\lambda^2$. The center frequency in the 1200-1400 nm region is 1300 nm, so for this region the bandwidth is given by,

$$\Delta f_{1200-1400} = 2 \times 10^8 \text{ m/s} \times 200 \text{ nm} / (1300 \text{ nm})^2 = 23.67 \text{ THz}.$$

Similarly for 1450-1650 nm region the bandwidth is given by,

$$\Delta f_{1450-1650} = 2 \times 10^8 \text{ m/s} \times 200 \text{ nm} / (1550 \text{ nm})^2 = 16.65 \text{ THz}.$$

So, total bandwidth = $(23.67 + 16.65) \text{ THz} = 40.3 \text{ THz}$.

- 1.4. What is the bandwidth of a 1-nm signal at 1500 nm? At 1350 nm? Give an approximate relation for the bandwidth of a $\Delta\lambda$ nm signal at λ nm.

Answer:

The explanation remains the same as in the previous solution.

$$\Delta f_{1500} = 2 \times 10^8 \text{ m/s} \times 1 \text{ nm} / (1500 \text{ nm})^2 = 88.89 \text{ GHz}.$$

$$\Delta f_{1350} = 2 \times 10^8 \text{ m/s} \times 1 \text{ nm} / (1350 \text{ nm})^2 = 109.74 \text{ GHz}.$$

- 1.5. Consider the three solutions for upgrading the transmission capacity of a link from OC-48 to OC-192. Suppose the cost of installing additional fiber is \$100 per meter, the cost of each transceiver is \$1000, and the cost of a WDM multiplexer/demultiplexer is \$10,000. Determine the maximum length for which you would want to use the multi-fiber solution.

Answer:

Find x , such that cost of multi-fiber over length x is equal to \$10,000. $100 \times x + 6 \times 1000 = 10000$, since we need 6 new filters, three new ones on each end. Solution: $x = 4000/100 = 40$ m. This is because for the multifiber case the extra cost is due to laying 3 more fibers and 6 more filters whereas in the WDM case the extra cost is due to the multiplexer/demultiplexer. If we use OC-192 transceiver then, assuming that the cost for such a transceiver is 4 times the cost of an OC-48 transceiver, we have cost of upgrade as $100 \times x + 4 \times 1000$ and for this case $x = 60$ m compared to the WDM case. So, up to 40 meters of fiber length, the multi-fiber solution gives minimum cost.

Note: variations in assumptions may lead to other solutions also.

- 1.6. Give the advantages and disadvantages of the following wavelength crossconnects:

- (a) passive star,
- (b) passive router, and
- (c) active switch.

Answer:

Passive star: Provides broadcast capabilities, no wavelength reuse.

Passive router: Wavelength reuse, static configuration.

Active switch: Wavelength reuse and reconfigurable, but active device.

- 1.7. Consider the passive star of Fig. 1.11, the passive router of Fig. 1.12, and the active switch of Fig. 1.13. Which of these devices can support the following simultaneous connections? (Assume that TDM is not used, but multicasting is allowed.)

- (a) Wavelength λ_1 from input fiber 1 to output fiber 1,
Wavelength λ_1 from input fiber 1 to output fiber 2,
Wavelength λ_2 from input fiber 2 to output fiber 1.
- (b) Wavelength λ_2 from input fiber 1 to output fiber 2,
Wavelength λ_2 from input fiber 2 to output fiber 1,
Wavelength λ_3 from input fiber 3 to output fiber 1.
- (c) Wavelength λ_1 from input fiber 1 to output fiber 1,
Wavelength λ_2 from input fiber 2 to output fiber 1,
Wavelength λ_3 from input fiber 1 to output fiber 3.

Answer:

- a. The passive star allows a multicast on λ_1 from input fiber 1 to output fibers 1 and 2. The other architectures do not allow this multicast.
- b. The passive star does not provide any wavelength reuse; thus, it cannot support two separate

connections on wavelength λ_2 . The passive router also cannot support these connections, since the connection pattern for the passive router is fixed. Thus, the active switch is the only device which can support the required connections.

c. The passive star, passive router, and the active switch can all support these connections.

1.8. The routing matrix for an $N \times N$ passive router is called an $N \times N$ Latin Square.

(a) Identify which of the following 4×4 matrices below are Latin Squares.

$$\Lambda_1 = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_2 & \lambda_3 & \lambda_4 & \lambda_1 \\ \lambda_4 & \lambda_1 & \lambda_3 & \lambda_2 \\ \lambda_3 & \lambda_4 & \lambda_2 & \lambda_1 \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_2 & \lambda_3 & \lambda_4 & \lambda_1 \\ \lambda_3 & \lambda_4 & \lambda_1 & \lambda_2 \\ \lambda_4 & \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}$$

(b) How many distinct 3×3 Latin Squares are there?

Answer:

(a) Λ_2 is a Latin Square since each wavelength appears only once in each row and in each column.

(b) For the first row, there are $\binom{3}{1} \times \binom{2}{1}$ possible combinations. For the second row, there are 2 combinations. The total number of combinations is then $3 \cdot 2 \cdot 2 = 12$.

1.9. Consider the active switch of Fig. 1.13.

(a) What is the size of each switching element in the center?

(b) Is it possible to construct a 4×4 switch out of 2×2 switches?

Answer:

(a) Since there are four input fibers and four output fibers, the required switch size is 4×4 .

(b) Yes. An example is shown in Fig. 1.17.

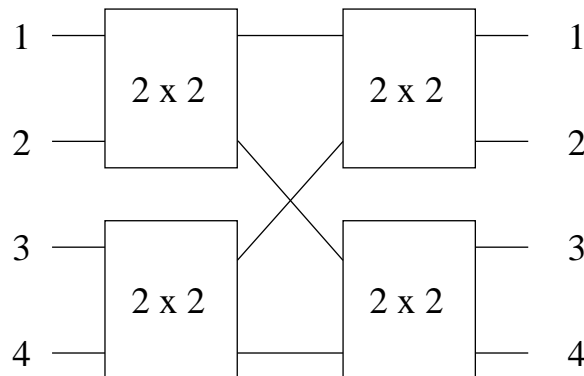


Figure 1.17 A 4×4 switch.

1.10. Consider the network of Fig. 1.15. Suppose we replace the passive star by a TDM switch. Compare this new architecture with the previous architecture.

Answer:

Synchronization is required at transmitters and receivers.

Switching intelligence is centralized in the TDM case.

Multicasting is only possible through “copying.”

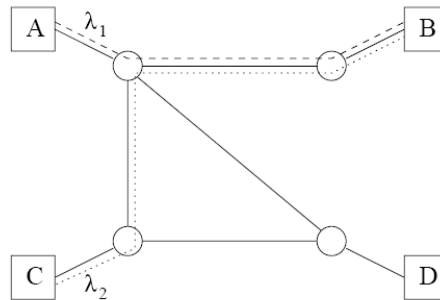


Figure 1.18 A wavelength-routed WDM network.

- 1.11.** Consider the simple wavelength-routed optical WDM network shown in Fig. 1.18. Two connections have been established: A-B on wavelength λ_1 , and C-B on wavelength λ_2 . Establish the connections D-B and C-D while using the minimum number of wavelengths. How would your solution change if wavelength conversion is available at each node?

Answer:

Establish connection D-B using wavelength λ_3 , and connection C-D using wavelength λ_1 .

If wavelength conversion is available, the solution does not change.

- 1.12.** Suppose the optical switching fabric in Fig. 1.16 is replaced by a passive-star coupler. What is the minimum number of wavelengths required to maintain the connections shown in the figure?

Answer:

Four. Since a passive star does not offer wavelength reuse, a separate wavelength is required for each connection.

Enabling Technologies: Building Blocks

- 2.1.** An IP-based network application will be built on top of a fiber-optic communication network. The application programmer considers two options for error correction. One option is to simply use the TCP/IP protocol. The other option is to use the UDP/IP protocol, which the programmer calculates will have a 10% higher bandwidth when transmissions are error free. The programmer has included a cyclic redundancy check in the UDP packets, but does not have a good scheme for retransmitting individual packets. Thus, when an error is detected, an average of 125 megabytes of data will have to be retransmitted. Which scheme will yield the higher average bandwidth if the fiber bit error rate is: (a) 10^{-9} ? (b) 10^{-15} ?

Answer:

125MB = 10^9 bits, thus for every lost bit, 1 in 10^{-9} , 10^9 bits must be retransmitted. So, approximately every billion bits, an additional billion bits must be transmitted. This reduces the bandwidth to one half its error free estimate. On the other hand the TCP/IP protocol will not be slowed noticeably by so few errors. Thus, TCP/IP would be more efficient. With a bit error rate of 10^{-15} , however, 10^9 extra bits must be transmitted every 10^{15} bits, this is a decrease in the bandwidth of $1 + (10^9/10^{15})$, or one ten thousands of one percent. With this low error rate, even this ludicrous error recovery procedure has a higher aggregate bandwidth than the TCP/IP protocol.

- 2.2.** Consider a step-index fiber which has a core refractive index of 1.495. What is the maximum refractive index of the cladding in order for light entering the fiber at an angle of 60 degrees to propagate through the fiber? Air has refractive index of 1.0.

Answer:

$$\begin{aligned}
 n_{air} * \sin(\theta_{air}) &= n_{core} * \sin(\theta_{core}) \\
 1.0 * \sin(60^\circ) &= 1.495 * \sin(90^\circ - \theta_{core}) \\
 \theta_{core} &= 54.6^\circ \\
 \theta_{core} &> \sin^{-1}(n_{clad}/n_{core}) \\
 n_{clad} &< n_{core} * \sin(\theta_{core}) \\
 n_{clad} &< 1.495 * \sin(54.6^\circ) = 1.219
 \end{aligned}$$

- 2.3.** Find the numerical aperture in a graded-index fiber with two layers shown in Fig. 2.41. Compare the answer with the numerical aperture of the step-index fiber shown in Fig. 2.40. Can we use

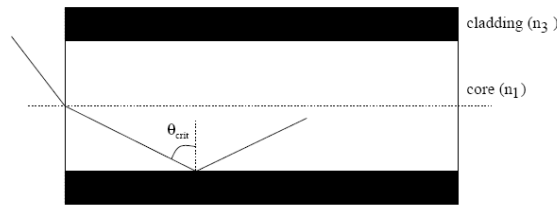


Figure 2.40 Critical angle in a step index fiber.

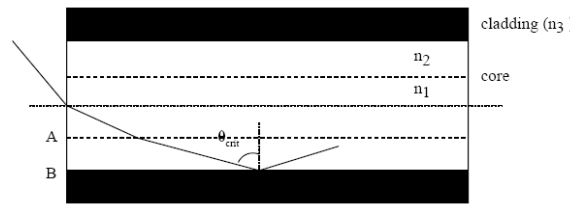


Figure 2.41 Critical angle in a graded index fiber.

geometric optics to deal with situations where the wavelength and core diameter are of the same order of magnitude (e.g., single-mode fiber)?

Answer:

For the step index fiber the critical angle is given by

$$\theta_{crit} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$$

Thus, the numerical aperture is given by

$$NA = \cos(\sin^{-1}(\frac{n_3}{n_1}))$$

In the graded index fiber, at critical refraction, let θ_c^A and θ_c^B be the angles corresponding to the interfaces A and B. Now,

$$\theta_c^B = \theta_{crit} = \sin^{-1}\left(\frac{n_3}{n_2}\right)$$

and

$$\theta_c^A = \sin^{-1}\left(\frac{n_2 \sin(\theta_c^B)}{n_1}\right) = \sin^{-1}\left(\frac{n_3}{n_1}\right)$$

Hence, the numerical aperture is given by

$$NA = \cos(\sin^{-1}(\frac{n_3}{n_1}))$$

Thus, we observe that the numerical aperture is the same for the step index fiber and the graded index fiber.

Each mode of a fiber can be viewed as a family of rays traveling at an angle to the fiber axis. Light traveling straight along the fiber axis would move at a speed of c/n —that is, the speed of light in vacuum divided by the refractive index. Because the light rays of different modes have different propagation angles, they have different path lengths. Thus, a pulse traveling through a fiber in different modes spreads out, and the amount of spreading is proportional to the length

of the fiber. Typical bandwidths of $100\mu\text{m}$ silica-core step-index multimode fibers are about 20 MHz-km.

Graded index fibers have smaller cores than step-index multimode fibers. Also, the path the light rays travel in a graded fiber depends on refraction rather than total internal reflection, so light rays entering the fiber at different angles travel essentially the same distances through the fiber, thus decreasing the modal dispersion.

We cannot use geometric optics to deal with situations where the wavelength and core diameter are of the same order of magnitude.

- 2.4. Consider a step-index multimode fiber in which the refractive indices of the cladding and core are 1.35 and 1.4, respectively. The diameter of the core is $50\mu\text{m}$. Approximately how many modes are supported by the fiber for a signal at a wavelength of 1550 nm ?

Answer:

$$\begin{aligned} V &= k_0 a \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2} \\ &= \frac{2\pi}{1550\text{nm}} \cdot 25\mu\text{m} \cdot \sqrt{1.4^2 - 1.35^2} \\ &= 37.58. \\ m &\approx \frac{1}{2} 37.58^2 = 706 \text{ modes.} \end{aligned} \tag{2.1}$$

- 2.5. Find the approximate number of modes in a $100\mu\text{m}$ core step-index multimode fiber at 850 nm . Assume the refractive index of glass to be 1.5 and that of the cladding to be 1.47.

Answer:

$$\begin{aligned} V &= k_0 a \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2} \\ &= \frac{2\pi}{850\text{nm}} \cdot 50\mu\text{m} \cdot \sqrt{1.5^2 - 1.47^2} \\ &= 110.32. \\ m &\approx \frac{1}{2} 110.32^2 = 6085 \text{ modes.} \end{aligned} \tag{2.2}$$

- 2.6. Consider an optical link in which power at the transmitter is 0.1 mW and the minimum power required at the receiver is 0.08 mW . The attenuation constant for the fiber material is 0.033 dB/km . What is the maximum length of the optical link, assuming that there are no amplifiers?

Answer:

$$L_{\text{max}} = \frac{10}{A} \log_{10} \frac{P_0}{P_r} = \frac{10}{0.033\text{ dB/km}} \log_{10} \frac{0.1}{0.08} = 29.4\text{ km}.$$

- 2.7. Describe the various types of dispersion and explain how the effects of each type of dispersion can be reduced.

Answer:

Material dispersion: Caused by the refractive index being a function of the wavelength. Can be reduced through dispersion shifted fiber.

Waveguide dispersion: Caused by index and shape of the core and cladding.

Modal dispersion: Caused by different velocities of different modes. Can be eliminated by using single-mode fiber, or reduced by using graded-index fiber.

- 2.8.** Consider a single mode optical fiber with an attenuation of 0.2 dB/km and a dispersion limit of 200 Gbps·km. The transmitter power is 1 mW and the receiver sensitivity is 10^{-5} mW. The link operates at a rate of 2.4 Gbps. Assume a 10 dB power margin for losses in connectors. Calculate the maximum length of the optical link.

Answer:

If we consider the power limitation, the signal can travel at most $\frac{10}{0.2} \log_{10} \frac{1.0}{10^{-5}} = 250$ km. If we consider dispersion, the signal can travel at most $200 \text{ Gbps} \cdot \text{km} / 2.4 \text{ Gbps} = 83.3$ km. We are thus limited by dispersion, and $L_{max} = 83.3$ km.

- 2.9.** Suppose we have a system which has 3 channels operating at 1549.32 nm, 1554.13 nm, and 1558.98 nm. At which frequencies will we have sidebands as a result of four-wave mixing? (Use $c = 2.998 \times 10^8$.)

Answer:

Using the formula $f = v/\lambda$, we have:

1549.32nm \rightarrow 193.5 THz f1

1554.13nm \rightarrow 192.9 THz f2

1558.98nm \rightarrow 192.3 THz f3

Sidebands due to four-wave mixing will then occur at the following frequencies:

$2 * f1 - f2 = 194.1$ THz

$2 * f2 - f1 = 192.3$ THz

$2 * f1 - f3 = 194.7$ THz

$2 * f3 - f1 = 191.1$ THz

$2 * f2 - f3 = 193.5$ THz

$2 * f3 - f2 = 191.7$ THz

- 2.10.** Consider a broadcast star network with $N = 2^k$ nodes where k is an integer. The network is built out of 2×2 couplers with excess loss β and coupling coefficient α . Each transmitter has a laser with power P_t .

- (a) Find the power levels received by the receivers when a single transmitter (say transmitter T_1) transmits. That is, determine how many different power levels are received by the N receivers, and how many receivers receive each of these levels.

Hint: Construct a tree with the transmitter at the root.

- (b) Suppose your goal as the network designer is to maximize the minimum power received by any of the receivers from a transmitter. Find the optimal value of α for this design criterion. Explain your answer.

- (c) Now suppose you have a different design criterion. Your new goal is to maximize the expected value of the power between a random transmitter/receiver pair. Assume that each such pair is equally likely. How would you select your couplers? Explain your answer.

Hint: You can exploit the symmetry to fix the transmitter.

Answer:

Open for readers.

- 2.11.** Suppose a 1 mW, 1550 nm signal is transmitted across a 5 km fiber, through an 8×8 passive star coupler, and through another 15 km of fiber before reaching its destination. No amplifiers are used. What is the power of the signal at the destination?

Answer:

Attenuation is approximately 0.2 dB/km.

$$P(5) = 10^{-0.1} * 1 \text{ mW} = 0.794 \text{ mW}$$

$$P(\text{star}) = 0.794/8 \text{ mW} = 0.0993 \text{ mW}$$

$$P(\text{dest}) = 10^{-0.3} * 0.0993 \text{ mW} = 0.0498 \text{ mW}.$$

- 2.12.** A 16×16 passive-star hub has been constructed from combiners, couplers, and splitters as in Fig. 2.7. Each combiner, coupler, and splitter results in a 3 dB power loss. Each host is up to 10 km away from the star, with a signal attenuation of 0.2 dB/km. If each host must receive signal power of at least 0.01 mW to clearly recognize signals, how strong must each host's transmission signal be?

Answer:

Total attenuation is: $2 \cdot 10 \cdot 0.2 + 6 \cdot 3 \text{ dB} = 22 \text{ dB}$.

$$P = \frac{0.01}{10^{-22/10}}.$$

$$P = 1.58 \text{ mW}.$$

- 2.13.** Consider a unidirectional fiber bus with N nodes. (Assume that N is even.) All the couplers have an excess loss β ($\ll 1$). (Let $\gamma = 1 - \beta$ and use γ in your solution instead of β .) The i^{th} coupler has a coupling coefficient α_i for $1 \leq i \leq N$. The coupling coefficients can be independently selected. Optimize the coupling coefficients to maximize the worst-case power transfer between a transmitter and a receiver. Compare the resulting worst-case power with the case where all couplers are identical and optimized. Hint: First assume some transmitter k is the worst-off. Argue why, in the optimal solution, all the receivers to the right of k will see the same attenuation from transmitter k . Similarly all the transmitters to the left of k should be equally badly off. Obtain a recursion for α_i assuming k is known and find the value of k .

Answer:

Open for readers.

- 2.14.** Consider the following simplified model of a direct detection binary FSK system. By using a pair of optical filters and a pair of photodetectors in the receiver, we observe two Poisson distributed photon counts: X_0 and X_1 . When the data bit is 0, the parameter of X_0 is $\lambda + \lambda_d$ while that of X_1 is λ_d . (Here λ_d models the dark current.) Conversely, when the data bit is 1, X_0 has parameter λ_d and X_1 has $\lambda + \lambda_d$. X_0 and X_1 are statistically independent when conditioned on the value of the data bit.

- Obtain the Maximum Likelihood processing of X_0 and X_1 explicitly.
- Find the probability of a bit error as a function of λ and λ_d . You may leave your answer as a double series.
- Repeat part (b) when there is no dark current ($\lambda_d = 0$). Now your answer must have a simple form.

Answer:

Open for readers.

- 2.15.** In this problem, you will investigate the relationship between the finesse F and the reflection coefficient R of a Fabry-Perot filter.

- Show that for $R \simeq 1$, the finesse can be approximated as

$$F \simeq -\frac{\pi}{\ln R}$$

- (b) Find the exact and the approximate expression for R in terms of F . Evaluate the accuracy of the approximation for $F = 10$ and $F = 100$.

Answer:

Open for readers.

- 2.16.** In this problem, you will consider the worst-case crosstalk in a WDM environment with Fabry-Perot (FP) filters. Assume that the filter is lossless ($A = 0$).

- (a) Show that the power transfer function $T(f)$ of the FP can be written as

$$T(f) = \frac{1-R}{1+R} \sum_{m=-\infty}^{\infty} R^{|m|} e^{j2\pi mf/P}$$

where P is the free spectral range.

- (b) Using the result in (a), show that the worst-case interference

$$C_{\max} = \sum_{i=1}^{M-1} T\left(\frac{iP}{M}\right)$$

is given by

$$C_{\max} = M \frac{1-R}{1+R} \frac{1+R^M}{1-R^M} - 1$$

- (c) Using the approximation in Problem 2.15, show that when $F \gg 1$

$$C_{\max} \simeq \frac{\pi M}{2F} \coth\left(\frac{\pi M}{2F}\right) - 1$$

where $\coth(x) = (e^x + e^{-x})/(e^x - e^{-x})$. Find, numerically, the maximum value of M/F such that $C_{\max} \leq 1$. Comment on your result.

Answer:

Open for readers.

- 2.17.** Consider a Mach-Zehnder filter chain of K cascaded filters with

$$\Delta L_i = 2^{i-1} \Delta L \quad i = 1, 2, \dots, K$$

Show that the power transfer function of this chain is given by

$$T(f) = \frac{\sin^2(\pi M f / P)}{M^2 \sin^2(\pi f / P)}$$

where $M = 2^K$ and $P = c/\Delta L$.

Answer:

Open for readers.

- 2.18.** Optical amplifiers saturate at high levels of output power. Suppose the saturation power of an erbium-doped fiber amplifier is 20 mW, and the amplifier gain is 5 dB/mW of pump power. The pump power is set to 5 mW. What is the largest amount of input power that can be amplified without driving the amplifier into saturation.

Answer:

Total gain $G = 5 \cdot 5 = 25$ dB
 Final output power $P_o \leq 20$ mW = 13 dB
 $P_i + G = P_o$
 $P_i = P_o - G \leq 13 - 25 = -12$ dB
 $P_i \leq 0.06$ mW.

- 2.19.** An optical amplifier delivers an output power P_{out} in response to an input power P_{in} as described by the following equation

$$P_{out} = a(1 - \exp(-bP_{in}))$$

where a and b are constants.

- What is the saturation power P_{sat} of this amplifier?
- Find the power gain of the amplifier in the linear operating region (i.e., small input power).
- Suppose this amplifier is to be placed in a transmission link of length L . The fiber has an attenuation factor of α per unit length, i.e., after a distance l , a factor $e^{-\alpha l}$ of the original power remains. The transmitter has a laser with power P_t . The goal is to maximize the received power P_r . Find the optimal position x (measured from the transmitter) of the amplifier. Comment on your result.

Hint: The only root of the equation $u = e^u - 1$ is at $u = 0$.

Answer:

- Saturation power of an amplifier is the output power when the input power tends to be very high. Following this concept, we have:

$$P_{sat} = \lim_{P_{in} \rightarrow \infty} P_{out} = \lim_{P_{in} \rightarrow \infty} a(1 - \exp(-bP_{in})) = a.$$

- In the linear operating region, the input power is small. Hence, the differential small-signal power gain is given by:

$$G = [dP_{out}/dP_{in}]_{P_{in} \rightarrow \text{small}} = ab(1 - bP_{in})$$

(Expanding the power series in exponential and neglecting higher order terms in P_{in}).

- Let the distance at which the amplifier be placed is x km from the transmitter and the original launched power be unity without any loss of generality.

Then, input power to the amplifier is $e^{-\alpha x}$.

The output power of the amplifier is then, $ab(1 - be^{-\alpha x})e^{-\alpha x}$.

This power, after traveling the remaining $L - x$ distance gets attenuated to

$$ab(1 - be^{-\alpha x})e^{-\alpha x} \times e^{-\alpha(L-x)} = ab(1 - be^{-\alpha x})e^{-\alpha L}.$$

So received power

$$P_r = ab(1 - be^{-\alpha x})e^{-\alpha L} = K(1 - be^{-\alpha x}), \text{ where } K = ab \times e^{-\alpha L} = \text{constant}$$

P_r is maximum in the range $[0, L]$, for $x = L$. Therefore, the amplifier must be placed right before the receiver as pre-amplifier to maximize the received power.

- 2.20.** (a) Use four 2×2 optical crosspoint elements to construct a 4×4 Banyan interconnect.
- How many rows and how many columns of 2×2 crossbars would be required for an $N \times N$ Banyan interconnect.
 - Suppose we have an 8×8 space-division Banyan optical routing switch. Label the inputs from 0 to 7 and label the outputs from 0 to 7. Suppose we need to simultaneously route Input 5 to Output 2 and Input 7 to Output 0. Can this routing be accomplished? If yes, give the routes through the switch, otherwise explain why the routing isn't possible.

Answer:

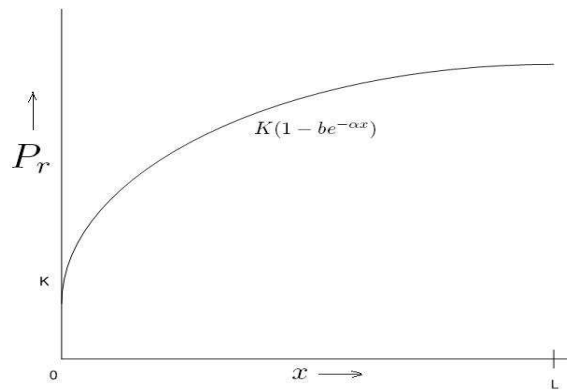


Figure 2.42 Solution to Problem 2.19.

- (a) The structure is shown in Fig. 2.44 (b) $\lceil N/2 \rceil$ rows, and $\lceil \log_2 N \rceil$ columns.
 (c) Yes. See Fig. 2.45.

2.21. What are the uses of wavelength conversion in a WDM network? Consider the two wavelength-convertible switch (WCS) architectures shown in Fig. 2.43. Construct a set of connection requests that can be routed by the share-per-node WCS and cannot be routed by the share-per-link WCS, and vice-versa.

Answer:

Wavelength conversion has two important applications in WDM networks: (1) conflict resolution and wavelength reuse, and (2) seamless integration of heterogeneous WDM networks. An example of the first application is shown in Fig. 2.46. We observe that without wavelength conversion, the connection requests could not be satisfied, while with wavelength conversion, the connections requests were satisfied. We define two networks A and B to be heterogeneous WDM networks, if A and B use different set of wavelengths for communication. By using wavelength conversion, we can seamlessly integrate such networks. Figure 2.47 shows how wavelength conversion supports the distribution of network control and management functionalities into smaller subnetworks by allowing flexible wavelength assignments within each subnetwork.

If we want to perform wavelength conversion on a total of four wavelengths (two wavelengths on each link), then we need to employ the share-per-link wavelength conversion switch. On the other hand, if we want to perform wavelength conversion on three wavelengths, all of which are going out on the same link, then we have to employ the share-per-node wavelength conversion switch.

2.22. Suppose we want to design a system with 16 channels, each channel with a rate of 1 Gbps. How much bandwidth is required for the system?

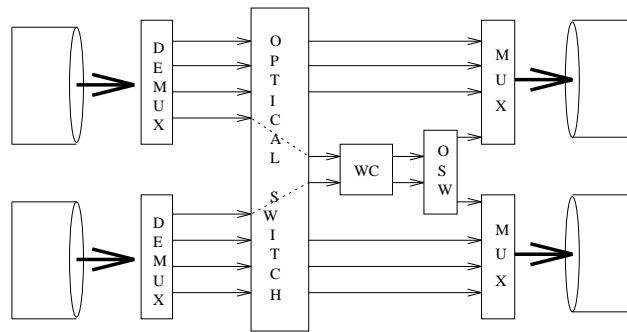
Answer:

Each channel requires 2 GHz of bandwidth. The channel spacing should be 6 times the channel bit rate to minimize crosstalk. Thus we require $2 \text{ GHz} * 8 \text{ channels} + 6 * 1 \text{ GHz} * 7 \text{ spacings} = 58 \text{ GHz}$ of bandwidth.

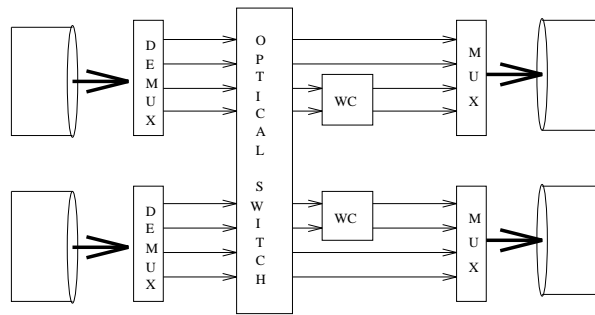
2.23. Suppose we have a fiber medium with a bandwidth of about 20 nm. The center wavelength is $0.82 \mu\text{m}$. How many 10 GHz channels can be accommodated by the fiber? Calculate the maximum number of channels for a center wavelength of $1.5 \mu\text{m}$.

Answer:

112 channels for a center wavelength of $0.82 \mu\text{m}$. 34 channels for a center wavelength of $1.5 \mu\text{m}$.



(a) Share-per-node wavelength-convertible switch architecture



(b) Share-per-link wavelength-convertible switch architecture

Figure 2.43 Two architectures for wavelength convertible routers: (a) share-per-node, (b) share-per-link.

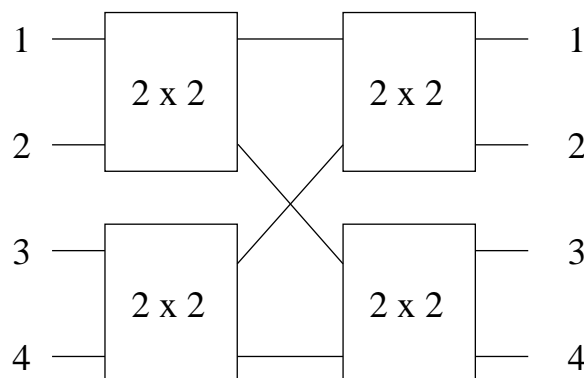


Figure 2.44 A 4×4 Banyan interconnect.

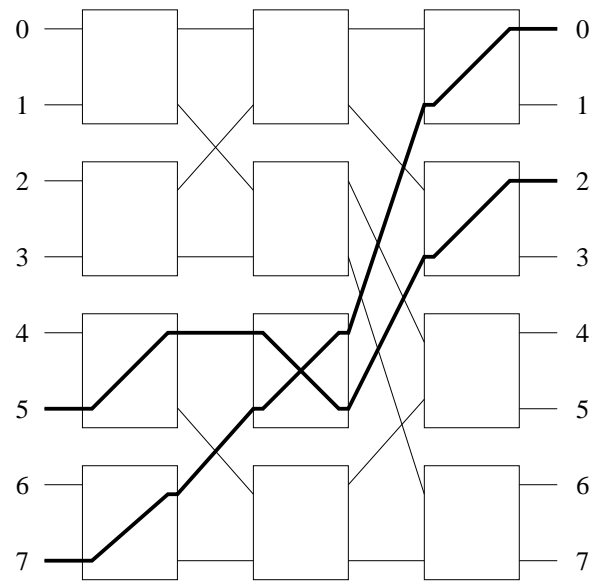
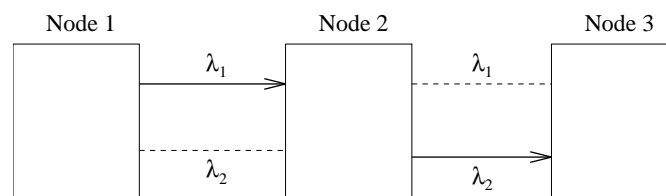
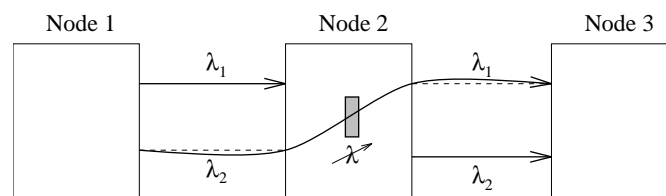


Figure 2.45 Routes in an 8×8 Banyan switch.



(a) without converter



(b) with converter

Figure 2.46 Conflict resolution and wavelength reuse in a wavelength-routed network. Existing connections: (a) Node 1 \rightarrow Node 2 on wavelength λ_1 , (b) Node 2 \rightarrow Node 3 on wavelength λ_2 . Connection request: Node 1 \rightarrow Node 3.

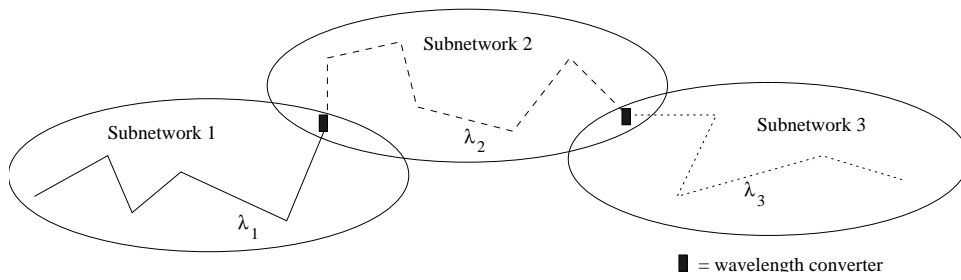


Figure 2.47 Seamless integration of heterogeneous WDM networks.

- 2.24.** Consider an optical communication system in which the transmitter tuning range is from 1450 nm to 1600 nm, and the receiver tuning range is from 1500 nm to 1650 nm. How many 1 Gbps channels can be supported in the system?

Answer:

Available bandwidth = 100 nm

$\Delta f = 8.324$ THz

$C_{max} = \frac{8324+6}{8} = 1041$

- 2.25.** Consider a WDM passive-star-based network for N nodes. Let the tuning range of the transmitters be 1550 nm to 1560 nm, and let the tuning range of the receivers be 1555 nm to 1570 nm. Assume that the desired bit rate per channel is 1 Gbps. Also assume that a channel spacing of at least 10 times the channel bit rate is needed to minimize crosstalk on a WDM system. Find the maximum number of resolvable channels for this system.

Answer:

The available transceiver bandwidth is given by

$$BW_T = \Delta R \cap \Delta T = 5nm \quad (2.3)$$

The frequency needed for BW_T is $\Delta f = cBW_T/\lambda^2 = 412.23GHz$. If we want W channels, we need $2W + 10(W - 1)GHz$ of bandwidth. Hence, the number of resolvable channels for this system is

$$\frac{412.23 + 10}{12} = 35.18, \quad (2.4)$$

i.e. 35 channels.

- 2.26.** In which type of network, single-hop or multihop, is a smaller tuning latency more critical? Why?

Answer:

Any protocol for single-hop networks would require the source's transmitter and the destination's receiver to be tuned to the same channel during the length of the connection. Moreover, in order to communicate with different nodes, the transmitter and/or receivers will have to be tuned to other channels. However, for multihop networks, one can come up with protocols that require tunability only for infrequent reconfigurations in the network. Hence, smaller tuning latency is more critical in single-hop networks.

- 2.27.** In a WDM network node, if two signals on the same wavelength arriving from different input ports need to go to the same output port, then a conflict may occur. Describe two or more methods for resolving this conflict. Discuss the advantages and disadvantages of each solution.

Answer:

- (1) Use wavelength conversion. Expensive.
- (2) Use delay lines. May result in large switch size.
- (3) Reassign wavelengths for each connection or reroute one of the connections around the node. Requires more complex routing and wavelength assignment algorithms.

2.28. Figure 2.48 shows a WDM WAN constructed using AWGs at each node. Assume that there are sufficient number of fibers between the node pairs (not shown in Fig. 2.48). Show how the following connection requests can be satisfied (you may have to write a program which tries out various possibilities).

- *node 3* → *node 1*
- *node 3* → *node 1*
- *node 1* → *node 2*
- *node 1* → *node 2*
- *node 2* → *node 3*
- *node 2* → *node 3*

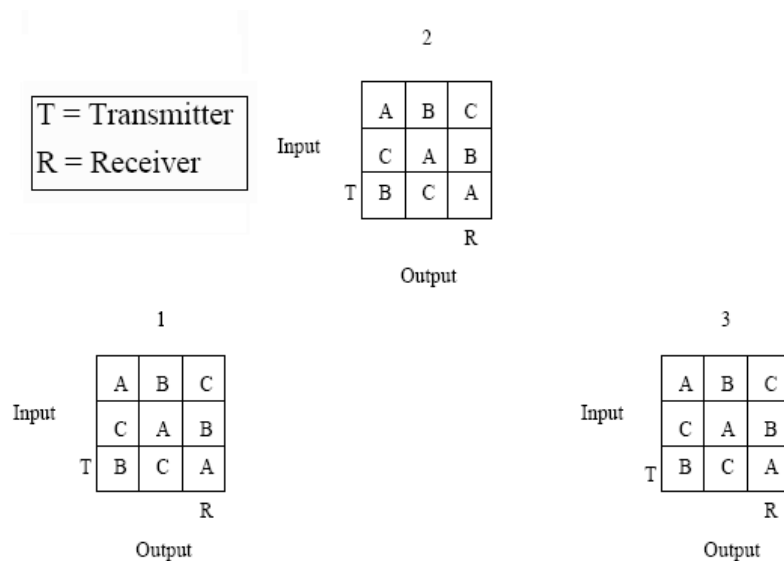


Figure 2.48 A WDM WAN constructed using AWGs at each node. (All connections begin at transmitters and end at receivers.)

Answer:

A connection pattern which satisfies the above set of connection requests is shown in Fig. 2.49.

2.29. Suppose we are given the network in Fig. 2.37 and have two wavelengths available. We wish to set up the following connections:

- i. H-2-3-4-8-9-E
- ii. C-7-8-4-F
- iii. B-6-7-8-9-E
- iv. D-10-7-C

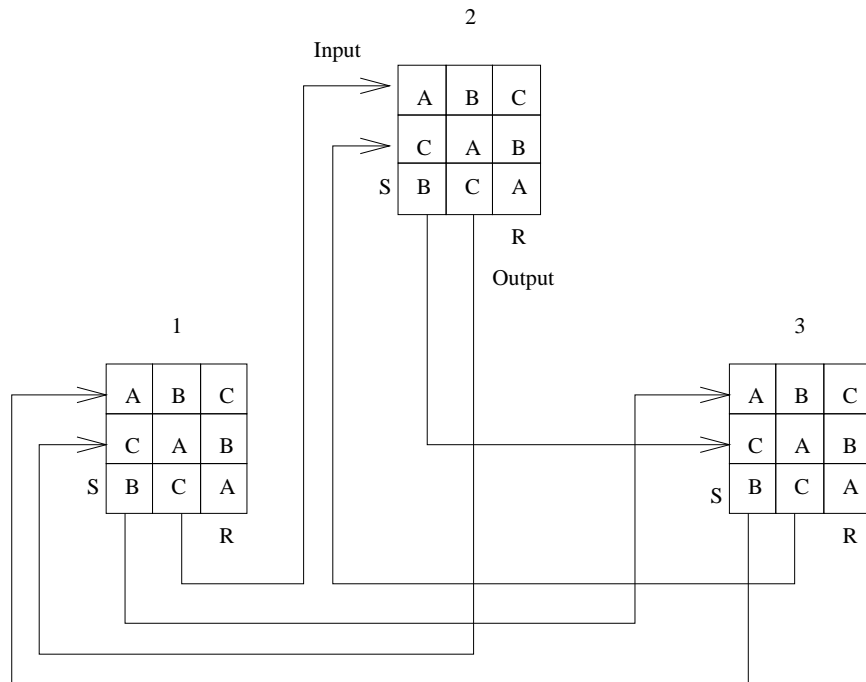


Figure 2.49 Connection pattern for Question 4.

At which nodes are wavelength converters required, and how many conversions are required at these nodes? Explain.

Answer:

One wavelength conversion is required at node 8.

Single-Hop Networks

- 3.1.** Compare the physical topologies: star, bus, and tree with respect to: (a) number of simultaneous connections, (b) scalability, and (c) delay.

Answer:

(a) In star and bus topologies, we can have only one connection (unique source-destination pair) per wavelength. In a tree, we may be able to have multiple connections per wavelength.

(b) The scalability of a star topology is limited by the size of the device used to implement the star, as well as the power loss of $1/N$, where N is the number of nodes. Scalability of the bus topology is limited by the power attenuation of $(1/2)^N$. A bus topology is also limited by propagation delays. A tree is scalable as long as the internal nodes employ repeaters, but propagation delays may also limit scalability. In a tree, the propagation delays increase logarithmically with N as opposed to linearly in the bus topology.

(c) Delay depends on the physical hop distance between nodes. In a star topology, the physical hop distance is one, thus the star potentially provides the lowest delay. In a linear bus, the average physical hop distance is $N/2$, while in a tree, the average hop distance is $\log(N)$.

- 3.2.** Consider the following two designs of a passive-star coupler using 2×2 couplers.

- (a) One approach is shown in Fig. 3.1. Determine the number of 2×2 couplers needed for this design, and prove that each output gets $1/N^2$ of the input power.
- (b) The second approach is the butterfly arrangement (also called a multistage interconnection network) mentioned in the text. Determine the number of 2×2 couplers needed for this design, and prove that each output gets $1/N$ of the input power.
- (c) What are the trade-offs between the two designs.

Answer:

Open for readers.

- 3.3.** Compare the maximum power loss in an N node passive-star coupler to the maximum power loss in an N node linear bus. Assume that the transmitted power is P . Consider only losses from splitting and coupling.

Answer:

In a passive-star coupler, the received power is P/N since the star coupler splits the power evenly among its output ports. In a linear bus, the minimum received power is $P/2^N$ at the farthest station since each station on the bus has a tap which typically reduces the power by $1/2$. However, the situation can be improved by using taps with unequal splitting ratios. (Open Problem: What are the optimal tap ratios?)

- 3.4. Compare protocols with pretransmission coordination to those protocols without pretransmission coordination.

Answer:

In protocols based on pre-transmission coordination, the system has several data channels and a single, shared control channel which is used to coordinate the data transmission. The control channel may become a bottleneck in the system if there are a large number of nodes or if the arrival rate of messages to the system is high. The advantage of this type of protocol is that it is dynamic and flexible.

Protocols without pre-transmission coordination rely on preassigned schedules or random-access methods. Thus, such protocols are somewhat fixed, and tend to be less scalable. Also, random-access protocols tend to have low performance.

- 3.5. Compare single-hop WDM systems which employ fixed transceivers to single-hop WDM systems which employ tunable transceivers.

Answer:

Properties of typical WDM single-hop networks employing only fixed transceivers are:

- Not scalable. Without TDM, N wavelengths are required for a system with N nodes for communication between all node pairs. To add a node in the network, the number of transceivers at a node may have to be changed.
- Cannot adapt to changes in traffic patterns. Bandwidth assignment is usually fixed.
- Simple medium access control protocol. Since there are no tunable components, medium access control protocol is usually very simple.
- Only channel collisions can occur. Depending on channel assignment strategy, two nodes may transmit on the same channel at the same time, resulting in a packet collision.
- Multicasting not possible. In general, single-hop multicasting may not be possible because receivers at nodes are not tunable.

Properties of typical WDM single-hop networks employing tunable transceivers are:

- Scalable. We can devise protocols which can easily add and delete a node from the network.
- Can adapt to changes in traffic patterns. Since nodes have tunable transceivers, arbitrary bandwidth assignment can be performed, e.g., a group of nodes may share a single channel while two other nodes may have a single dedicated channel between them.
- Complex medium access control protocol. Since there are tunable components, the control protocol has to track which channel each transceiver is tuned to.
- Channel and receiver collisions. Due to tunable transceivers, there is a possibility of receiver collisions.
- Multicasting possible. Multicasting can be achieved if multiple receivers tune to the same channel simultaneously.

3.6. Consider a single-hop network with four nodes and three channels.

- (a) Give a schedule for the fixed channel assignment technique.
- (b) Give a schedule for the Destination Allocation protocol.

Answer:

(a)

Channel #	t	t+1	t+2	t+3
0	(1,2)	(1,3)	(3,2)	(1,4)
1	(2,3)	(3,1)	(2,4)	(2,1)
2	(3,4)	(4,2)	(4,1)	(4,3)

(b)

Channel #	t	t+1	t+2
0	(1,2) (3,2)	(3,1) (2,1)	(2,3)
1	(2,4)	(1,3) (1,4)	(3,4)
2	(4,1) (4,3)	(4,2)	

3.7. Consider an optical LAN consisting of three nodes connected to a PSC. Assume two channels. Consider the following three scheduling protocols:

- (a) fixed assignment,
- (b) destination allocation, and
- (c) source allocation.

Show the schedules for three time slots. Assume uniform traffic conditions. Calculate the maximum throughput per channel that can be achieved by employing the protocols. Now assume that a packet corresponding to every *(source, destination)* pair arrives with probability p every three time slots. Calculate the throughput by taking account of channel and receiver collisions. Calculate the maximum expected throughput for the protocols.

When can source/destination assignment protocols perform better than fixed assignment?

Answer:

Fixed Assignment:

Channel #	t	t+1	t+2
0	(1,2)	(1,3)	(2,1)
1	(2,3)	(3,1)	(3,2)

Destination Allocation:

Source Allocation:

In the schedules shown above, let T = number of time slots, P = maximum number of concurrent packet transmissions, and C = number of channels. Then the normalized maximum throughput is given by $S = P/CT$. Thus, for all scheduling disciplines, the maximum throughput is given by $S_{max} = \frac{6}{2 \cdot 3} = 1$.

Channel #	t	t+1	t+2
0	(1,2)	(1,3)	(2,1)
	(3,2)	(2,3)	(3,1)
1	(2,3)	(3,1)	(3,2)
	(1,3)	(2,1)	(1,2)

Channel #	t	t+1	t+2
0	(1,2)	(1,3)	(2,1)
	(1,3)	(1,2)	(2,3)
1	(2,3)	(3,1)	(3,2)
	(2,1)	(3,2)	(3,1)

The expected throughput is calculated as follows. For fixed assignment, we have $S_{fixed} = \frac{6 \cdot p}{2 \cdot 3} = p$. Hence, the maximum expected throughput is achieved at $p = 1$, and is equal to $S_{max} = 1$.

For destination assignment, each $(source, destination)$ pair occurs exactly twice in the schedule. Thus the probability of transmission for each $(source, destination)$ occurrence is $p/2$. Now, in a time slot on a particular channel, the probability for successful transmission is approximately given by $2 \cdot p/2 \cdot (1 - p/2)$. Hence, the throughput is $S_{destination} = p \cdot (1 - p/2)$. Thus, the maximum expected throughput is achieved at $p = 1$, and is given by $S_{destination} = 1 - (1/2) = 0.5$.

For source assignment, through a similar argument, we get $S_{source} = p \cdot (1 - p/2)$, and the maximum expected throughput is achieved at $p = 1$, and is given by $S_{source} = 1 - (1/2) = 0.5$.

Source/destination assignment protocols may perform better than fixed the assignment protocol under non-uniform loads.

- 3.8.** A single-hop $TT-TR$ network with 10 nodes connected via a passive star coupler is to use a fixed channel assignment. Each transmitter and receiver is capable of tuning to three noninterfering channels. How many TDM slots are required? Show that there is no interference. Assume negligible tuning times.

Answer:

$10 \times 9 = 90$ connections are required. The best possible answer would then be $90/3 = 30$. This can be realized by dividing the 30 cycles into 3 parts, such that in the first part (given nodes 0 through 9) each node i communicates with nodes $i + 1$ through $i + 3 \bmod 10$. In the next two parts, node i communicates with nodes $i + 4$ through $i + 6$ and nodes $i + 7$ through $i + 9$. To formalize this, in time slot j , for $j = 0, \dots, 29$, we have the following sets of 3 communications:

$$\lambda_0 : (3j) \bmod 10 \Rightarrow (1 + 3j + \lfloor (3j)/10 \rfloor) \bmod 10$$

$$\lambda_1 : (1 + 3j) \bmod 10 \Rightarrow (2 + 3j + \lfloor (1 + 3j)/10 \rfloor) \bmod 10$$

$$\lambda_2 : (2 + 3j) \bmod 10 \Rightarrow (3 + 3j + \lfloor (2 + 3j)/10 \rfloor) \bmod 10$$

- 3.9.** A partial fixed-assignment protocol is used in a passive star network. There are 10 nodes in the network and three channels available. What is the shortest time slot scheme (i.e., the scheme with the shortest period)? Avoid receiver collision, but not necessarily channel collision. When might such a scheme be desirable, and when might it not?

Answer:

The best possible scheme is to allow each of the ten to receive during a different time/frequency slot. This gives a period of four, and a total of 12 slots. If traffic is very light, this scheme would be ok. Traffic with many connect requests might suffer, as there would be potentially many channel collisions.

- 3.10.** A fixed-assignment protocol is designed for a system with N nodes and W channels. The time slot duration is T . Find a lower bound on the expected packet delay in the system. Compare this with the lower bound on a partial fixed-assignment protocol. By what factor do these values differ?

Answer:

Assuming that an optimum scheme for allocating time slots and channels for communication is possible, the number of time slots $\times W$ is greater than or equal to the number of connections needed, i.e., $N \times (N - 1)$. Thus, the number of time slots needed is $N \times (N - 1) / W$. The expected number of time slots delay is then one half of this, $N(N - 1) / (2W)$. Therefore, a lower bound on the actual expected packet delay, is $N(N - 1)T / (2W)$. By similar reasoning, the partial fixed assignment protocol would have a lower bound of $NT / (2W)$. They differ by a factor of N .

- 3.11.** Consider a two-channel, four-node broadcast-and-select WDM network. Each node is equipped with a single fixed transmitter and a single tunable receiver. Design a fixed assignment schedule for the following traffic matrix. Assume that the tuning time is equal to one time slot. Be sure to indicate which node is tuning its receiver during which slot.

$$A = \begin{bmatrix} 0 & 1 & 4 & 3 \\ 2 & 0 & 1 & 2 \\ 3 & 2 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Answer:

Schedule is shown in Table 3.1.

Table 3.1 Fixed assignment schedule .

Ch	t	t+1	t+2	t+3	t+4	t+5
0	(1,3)	(1,3)	(1,3)	(1,3)	(4,3)	(4,3)
1	(2,4)	(2,4)	(3,4)	(3,4)	(2,1)	(2,1)
Ch	t+6	t+7	t+8	t+9	t+10	t+11
0	(1,2)	(4,2)	(1,4)	(1,4)	(1,4)	(4,1)
1	(3,1)	(3,1)	(3,1)	(2,3)	(3,2)	(3,2)

- 3.12.** Consider a system using a fixed assignment protocol. Each node is equipped with a fixed transmitter and a tunable receiver. For arbitrary values of N (number of nodes), W (number of channels), L , (tuning time for a receiver), and traffic matrix a_{ij} where $i, j = 1, 2, \dots, N$, find a lower bound on the optimal schedule length.

Answer: In this problem, we have two cases to consider.

Case 1:

If $N \geq W$, a node i will take up all the channels to set up connections to all other nodes. In

that case, total time to schedule:

$$= \sum_{l=0}^{W-1} a_{il} + \text{tuning time} = \sum_{l=0}^{W-1} a_{il} + WL, \quad \forall i \in \{0, 1, \dots, N-1\}$$

Note that, it is assumed that tuning time of all the nodes cannot be absorbed in the schedule. So, this is the worst case solution. However, different assumptions may lead to different solutions. We should consider the maximum of this scheduling time.

Thus,

$$T_{\text{schedule,opt.}} \geq \max_i \left(\sum_{l=0}^{W-1} a_{il} \right) + WL$$

Case 2:

If $N < W$, then all the nodes can establish connections between each other using all the W channels.

Therefore, the total schedule time should be greater than the average schedule time per wavelength.

Thus,

$$T_{\text{schedule,opt.}} \geq \left\lceil \left(\frac{1}{W} \right) \sum_{i=0}^{N-1} \sum_{l=0}^{W-1} a_{il} \right\rceil$$

Therefore, the optimum scheduling time is given by:

$$\max \left(\max_i \left(\sum_{l=0}^{W-1} a_{il} \right) + WL, \left\lceil \left(\frac{1}{W} \right) \sum_{i=0}^{N-1} \sum_{l=0}^{W-1} a_{il} \right\rceil \right)$$

Our assumption in this problem is that the lowest granularity of the traffic matrix is one wavelength channel capacity. [Note: The lower bound given in this solution can be improved more. It is left as an open problem to the readers to find out a more tighter lower bound].

- 3.13.** Suppose we have a system with four nodes. Each node is equipped with a single tunable transmitter and two fixed receivers. Assign channels to each of the transmitters and receivers ($T(i)$ and $R(i)$) for the random TDM protocol. What must we consider when assigning channels?

Answer:

$$R(1) = \{1, 2\}, R(2) = \{1, 3\}, R(3) = \{2, 3\}, R(4) = \{1, 2\}$$

$$T(1) = \{1, 3\}, T(2) = \{2\}, T(3) = \{1\}, T(4) = \{2, 3\}$$

In assigning channels, we should consider:

- The intersection of $T(i)$ and $R(j)$ is not null, if node i wants to transmit to node j .
- For each given channel, set as few transmitters as possible to be able to tune to it, thus reducing the possibility of transmitter collision.
- If more receivers share a given channel, there is a higher potential for supporting multicasting.
- If the system has unbalanced traffic, links with heavier traffic may be assigned its own channel if possible.

- 3.14.** Consider a packet network in which the packet arrival process is a Poisson process with rate λ . In this network, users can start packet transmissions at times $T, 2T, 3T, \dots$, where T is the length of a packet. All the packets that arrive during the time interval $[(n-1)T, nT)$ are transmitted

during the time “slot” $[nT, (n+1)T)$. When a single packet is transmitted in a time slot, that packet is successfully received by the receiver. When two or more packets are transmitted in the same slot, a collision occurs and all the packets are lost.

- (a) Find the probability P_k that k packets are transmitted in a time slot.
- (b) The *throughput* S of this network is defined as the fraction of time slots which result in successful packet transfer. Find S as a function of λ and T .
- (c) Find the value of λ that maximizes the throughput and the resulting throughput.

Answer:

Open for readers.

3.15. Consider the following WDM protocol. The network has a large number of nodes and W channels. Each node has W fixed-tuned receivers, one per channel, and a tunable transmitter. The time is slotted, and packets arrive according to a Poisson process with rate G packets/slot. When a node has a packet to transmit, it selects a wavelength randomly and transmits the packet in the first time slot on that wavelength.

- (a) Find the probability that a wavelength is successfully utilized in a given time slot.
- (b) Find the traffic rate that maximizes the throughput per channel and the resulting throughput.
- (c) Find the average number of packets successfully carried by this network per time slot.

Answer:

There are two cases we should consider in this problem.

- Assume G be the overall network arrival rate.

(a) For each node, it can pick any wavelength randomly, so the arrival rate on each wavelength is G/w packets/slot.

Each packet arrival is a Poisson process with parameter G/w , so

$$P\{k \text{ packets arriving on a specific channel}\} = \frac{\left(\frac{G}{w}\right)^k e^{-\frac{G}{w}}}{k!}.$$

$P_s = P\{\text{a wavelength is successfully utilized in a given time slot}\} = P\{\text{only 1 packet arriving on a specific channel}\} = \frac{G}{w} e^{-\frac{G}{w}}$

(b) To maximize the throughput per channel, $dP_s/dG = 0$.

Therefore the traffic rate is $G = w$ packets/slot and the maximum throughput is

$$P_{max} = P_s(G = w) = 1/e.$$

(c) The average number of packets successfully carried by this network per time slot is $M = w \times P_{max} = w/e$.

- Assume G be the arrival rate to each node.

(a) Let the number of nodes in the network be N .

$$P\{k \text{ packets arriving on a specific channel}\} = \frac{(G)^k e^{-G}}{k!}.$$

$$P\{\text{no packet arrive on a specific channel}\} = e^{-G}.$$

$$P\{\text{a node has a packet to transmit}\} = 1 - e^{-G}.$$

$$P_a = P\{\text{a node selects a specific wavelength for transmitting}\} = \frac{1 - e^{-G}}{w}.$$

If we assume that a node does not transmit two packets on the same channel in the same time slot, collisions will only occur when two or more packets arrive from different nodes. Thus,

$$P_s = P\{\text{a wavelength is successfully utilized in a given time slot}\}$$

$$= P\{\text{only one of the } N \text{ nodes attempts to transmit on a specific channel}\} \\ = \binom{N}{1} P_a (1 - P_a)^{N-1}.$$

(b) To maximize the throughput per channel, $dP_s/dG = 0$.

Therefore the traffic rate is $G = \ln\left(\frac{N}{N-M}\right)$ packets/slot

and the maximum throughput is $P_{max} = P_s(G = w) = (1 - 1/N)^{N-1}$.

$$\lim_{N \rightarrow \infty} P_{max} = 1/e.$$

(c) The average number of packets successfully carried by this network per time slot is $M = w \times P_{max} = w(1 - 1/N)^{N-1}$.

$$\lim_{N \rightarrow \infty} M = w/e.$$

- 3.16.** Consider a random-access $TT - FR$ system in which the home channel for node j is specified by $\lambda_{j \bmod 8}$. There are 64 nodes in this slotted ALOHA network, and the capacity of each channel is 2.5 Gbps. What is the maximum bandwidth of the network assuming uniform traffic, i.e., if every node has packets to send to every other node, what is the total of all the links bandwidth. Also, under these circumstances, what is the maximum average bandwidth of a single communication link?

Answer:

Max utilization of a channel is $1/e$. So, the max bandwidth of a channel is $2.5/e = 0.920$ Gbps. There are eight channels, hence the max. network bandwidth is 8×0.920 Gbps = 7.36 Gbps. The max. average bandwidth of a single link is then, the network bandwidth, 7.36 Gbps, divided by the number of links, $8 \times 7 = 56$. That is, 0.131 Gbps = 131 Mbps.

- 3.17.** In the ALOHA/ALOHA protocol, calculate the throughput as a function of the normalized offered load and calculate the maximum throughput. Assume that there are N stations, and packets arrive to each station according to a Poisson process with rate λ . Traffic is uniformly distributed across all stations. (Normalized offered load is defined as the rate of arrivals per packet transmission time.)

Answer:

The arrival rate to a specific channel (or station) is $g = (N - 1) * \lambda$. The normalized offered load is then $g * L$, where L is the packet length. The throughput is the fraction of time during which useful information is transmitted on the channel. Thus, we need to calculate the probability that a transmitted packet is successful. The vulnerable period in the ALOHA protocol is of length $2L$. If another arrival occurs during this time, a collision will occur. The probability of a collision is then $P(\text{collision}) = 1 - e^{-2gL}$, and the probability of a successful transmission is $P(\text{success}) = e^{-2gL}$. If messages arrive at rate g , and $P(\text{success})$ of them are successful, then the rate of successful arrivals is $g * e^{-2gL}$. A successful transmission occupies L amount of time on the channel. Therefore, throughput is equal to $S = g * L * e^{-2gL}$ or $S = x * e^{-2x}$, where $x = \text{normalized load}$. If we take the derivative of S with respect to x , and set the result equal to zero, we can find the maximum throughput. $dS/dx = e^{-2x} - 2x * e^{-2x} = 0$ $x = 1/2$ $S_{max} = 1/2 * e^{-1} = 1/2e$.

- 3.18.** In slotted-ALOHA/ALOHA, what is the length of the vulnerable period for the control packet? For the data packet? Assume control packets to be one time unit in length, and data packets to be L time units in length.

Answer:

The vulnerable period for the control packet is one time unit, from $t_0 - 1$ to t_0 , where t_0 is the starting time of the control packet's transmission. The vulnerable period for the data packet is of length $2L - 1$ time units, and lasts from time $t_0 - L$ to time $t_0 + L - 1$.

- 3.19.** In the ALOHA/ALOHA partial random-access protocol, a node transmits its data packet immediately after it transmits its control packet, regardless of whether the control packet succeeds or not. Suppose a node can transmit its data packet only when the corresponding control packet is successful. Compute the throughput per channel for the improved protocol. Assume N data channels in the system.

Answer:

Suppose node i successfully transmits the control packet and chooses Channel 1 as the intended data channel. If some control packet from another node succeeds in the $(2L - 1)$ vulnerable period and also chooses Channel 1, these two data packets will collide. Thus, we can get the probability (P_s) that no other node whose control packet succeeds during the vulnerable period chooses Channel 1 for its data transmission.

Let us define events B_k and C_k as follows:

$B_k = \{\text{successful transmission of control packet in } k\text{th slot of the } (2L - 1) \text{ slot vulnerable period}\}$

$C_k = \{\text{the intended data channel of the control packet in slot } k \text{ is Channel 1}\}$

$$\begin{aligned} \text{Then, } P_s &= \Pi[1 - P(B_k \cap C_k)] \\ &= \Pi[1 - P(B_k) * P(C_k)] \\ &= \Pi[1 - \frac{1}{e} \frac{1}{N}] \\ &= (1 - (eN)^{-1})^{2L-1} \end{aligned}$$

Also, P_s is the probability of successful transmission of a data packet on Channel 1. Thus,

$$\begin{aligned} \text{Throughput/channel} &= P_s * \text{average number of data packets per slot per channel} \\ &= P_s * \frac{L}{Ne} \\ &= \frac{L}{Ne} * (1 - (eN)^{-1})^{2L-1} \end{aligned}$$

- 3.20.** Explain the reason behind bimodal throughput seen in a slotted-ALOHA /delayed-ALOHA protocol. What is the optimal number of channels for a system with data packet length equal to 15 slots.

Answer:

The reason for bimodal throughput is as follows. If the number of data channels is small, the data channel bandwidth is under-dimensioned and the data channels are the bottleneck, while if there is a large number of data channels, then the control channel's bandwidth is under-dimensioned and it is the bottleneck.

The optimal number of channels for a system with data packet length = 15 slots is given by

$$\begin{aligned} N_{\text{optimal}} &= \lfloor \frac{2L-1}{e} \rfloor \\ &= 10 \end{aligned} \tag{3.1}$$

- 3.21.** Why are CSMA protocols usually not very attractive for a single-hop WDM optical network?

Answer:

In CSMA protocols, the channel is sensed for activity, and if activity is detected, the node defers its transmission until the activity has ended. In case of optical fiber, each tap results in a 3 dB power loss. Therefore, with a power margin of a few tens of dB, only a small number of nodes could be accommodated if CSMA protocols are used.

- 3.22.** Describe the procedure for receiving a packet in the Receiver Collision Avoidance (RCA) protocol.

Answer:

Let a control packet from node i to node j arrives at node j at time t . Node j will first check for receiver collision on data packet by consulting its reception scheduling queue (RSQ). Let $c(RSQ)$ be the number of entries in the RSQ. If $c(RSQ) = 0$, no receiver collision will occur. If $c(RSQ) > 0$, then let the scheduled reception time of the last element in RSQ be t_r . If $t - (t_r - R) \leq T + W$, then a receiver collision is detected and the control packet is ignored (this is simultaneously detected by the sending node). Finally, node j starts tuning its receiver to the assigned data channel at time t_r and receives a data packet from $t_r + T$ to $t_r + T + W - 1$. After this, node j tunes its receiver back to the control channel or to another data channel if the next scheduled reception time follows immediately.

- 3.23.** In the Receiver Collision Avoidance (RCA) protocol, we set the size of the Node Activity List (NAL) to $2T + L$ slots. Explain why.

Answer:

The purpose of using NAL is to avoid receiver collision. Suppose the control packet from i to j arrives at time t without receiver collision. Another control packet arrives from k to j at time $t_1 < t$ without receiver collision. Then, the following conditions must exist:

$$t_1 + D + T + L + T < t + D$$

or

$$t_1 < t - (2T + L).$$

Therefore, $(2T+L)$ is the smallest time duration in which another control packet to j will not incur receiver collision.

- 3.24.** Describe possible scenarios in which receiver collisions may occur in the RCA protocol. Assume all nodes are D slots away from the hub, T is the tuning time, and L is the data packet length.

Answer:

Suppose transmitter i wants to send a packet to Receiver j at time t . Receiver collision may happen in the following situations:

1) Control packet with destination j is found in NAL. Suppose the control packet found in the NAL is from transmitter k and is recorded at time $t_1 < t$. Then j tuned to receive a data packet from $t_1 + T$ to $t_1 + 2T + D + L$. In this case, $t - t_1 < 2T + L$, then $t + D < t_1 + 2T + L + D$, and j cannot read the control packet from i .

2) A successful control packet to j is detected on the control channel at time t_1 between $t + D - T + L$ and $t + R - 1$. In this case, j is busy reading data from k between $t_1 + D$ to $t_1 + D + T + L$. If the control packet is successful, its data packet will reach j at time $t + D + D$. Because $t_1 > t + D - T + L$ and $t_1 + D + T + L > t + D + D$, receiver collision will occur during the data packet transmission.

- 3.25.** Consider the DT-WDMA protocol with N stations. Suppose that a packet arrives to a station in a given time slot with probability p , and no packets arrive to the station in the time slot with probability $(1 - p)$. Calculate the probability that a transmitted packet will encounter a receiver collision. Assume tuning time is included in the time slot.

Answer:

$\Pr(\text{a given station has an arrival for same destination}) = p * 1/(N - 1)$. Since the original source node and the destination node will not transmit a colliding packet, a collision will only occur if one of the remaining $N - 2$ stations transmits a packet to the same destination. $\Pr(\text{no collision}) = (1 - p * 1/(N - 1))^{N-2}$. Hence, $\Pr(\text{collision}) = 1 - (1 - p * 1/(N - 1))^{N-2}$.

- 3.26.** Show that under uniform traffic conditions and a large user population, the peak throughput in a DT-WDMA system is given by $1 - 1/e$.

Answer:

Let the arrival process be Bernoulli with parameter p . Thus, in each control slot, the probability that a packet will be scheduled is p . Consider an arbitrarily chosen receiver in the system, say receiver J . Let K be the number of packets scheduled to node J in a control frame. K is binomially distributed random variable. For each input, the probability that a packet is scheduled and is destined to node J is $p/(N-1)$. Thus, for any integer k between 0 and N , the probability that the number of packets, K , is equal to k is given by

$$P(K = k) = {}^{N-1}C_k \left(\frac{p}{N-1}\right)^k \left(1 - \frac{p}{N-1}\right)^{N-k-1}$$

For a large N , the binomial distribution approaches that of the Poisson, and thus, for large N , the probability that the number of packets, K , is equal to k is given by

$$P(K = k) = \frac{e^{-p} p^k}{k!}$$

Collision occurs if more than one packet is scheduled to the same destination in any control frame. Thus, if k packets are scheduled to a destination in a control slot, $k-1$ packets will be lost. Hence, the expected number of lost packets in a control frame per destination node is given by

$$\begin{aligned} \text{Packet Loss} &= \sum_{k=2}^{k=N-1} (k-1) P(K = k) \\ &= \sum_{k=2}^{k=N-1} (k-1) \frac{e^{-p} p^k}{k!} \\ &= \sum_{k=2}^{k=N-1} k \frac{e^{-p} p^k}{k!} - \sum_{k=2}^{k=N-1} e^{-p} p^k \text{ over } k! \\ &= p + e^{-p} - 1 \end{aligned}$$

Thus, the throughput is given by

$$\begin{aligned} \text{Load} - \text{Packet Loss} &= p - (p + e^{-p} - 1) \\ &= 1 - e^{-p} \end{aligned}$$

Hence, the maximum achievable throughput occurs at $p = 1$, and is given by $S_{max} = 1 - 1/e$.

- 3.27.** In DT-WDMA, nodes can be made more intelligent by storing an $N \times N$ matrix B , called the “backlog” matrix at each node. Element b_{ij} denotes the number of packets at node i destined for node j . We can find an optimal algorithm which constructs a transmission schedule T , where $t_{ij} = 1$ denotes that node i should transmit a data packet to node j in the next slot. An optimal algorithm maximizes the number of transmission in a slot. What is the maximum throughput that can be achieved using this algorithm? (A qualitative justification is sufficient.)

Answer:

If we use an intelligent scheduling algorithm that avoids receiver collisions (by employing a “backlog matrix”), then in case of a receiver contention only one packet is transmitted and other

packets are queued at the sender's node. If we assume a buffer of L packets at each node, the packet loss is given by

$$\text{Packet Loss} = \sum_{k=L+1}^{k=N-1} (k - L)P(K = k)$$

Hence, we see that by increasing the value of L , we can reduce the packet loss to any arbitrary value. Thus, in this case, the maximum throughput will approach 1.

- 3.28.** Given a STARNET network of 100 nodes, and control packets of length 100 bytes, with an average distance between nodes of 100 m and negligible processing time, how long will a simple broadcast take, given that the information in the broadcast signal is embedded in the control packets?

Answer:

The STARNET network transmits control information in a ring, thus, 99 hops are required to send the information in a packet to all nodes. First, we compute the total propagation delay. 9900 m, at $\frac{2}{3}c = 2 * 10^8 \text{m/sec}$ requires $9.9 * 10^3 \text{m} / (2 * 10^8 \text{m/sec}) = 4.45 * 10^{-5} \text{sec} = 44.5 \mu\text{sec}$. 800 bits at 125 Mbps requires $800/125000000 \text{sec} = 6.4 \mu\text{sec}$ per transmission. There are 99 such transmissions requiring $99 * 6.4 \mu\text{sec} = 633.6 \mu\text{sec}$. Thus, the total time required is $(633.6 + 44.5) \mu\text{sec} = 678.1 \mu\text{sec}$.

- 3.29.** For LAMBDANET, approximately how much of the low-loss region of bandwidth was utilized?

Answer:

LAMBDANET used 18 channels of 2 Gbps each.

Bandwidth needed = $2Bw + 6B(w - 1)$ GHz

= $2 * 2 * 18 + 6 * 2 * 17$

= 276 GHz

utilization = $\frac{276}{50 \times 10^{12}} \times 100 = 0.55$.

- 3.30.** Professor W. D. Myer receives a small grant to set up an experimental WDM testbed. He decides to have four nodes in his network, each node with a fixed transmitter and tunable receiver.

(a) Should Professor Myer go for pretransmission control? Why or why not?

(b) Suddenly Professor Myer gets a lot of money and decides to construct a large network with pretransmission coordination, with each node having two tunable transceivers. How would this network be represented using the notation introduced in this chapter?

Answer:

(a) Pre-transmission coordination requires an extra control channel. Also, it is preferable to use pre-transmission coordination for networks with a large number of nodes. Since Prof. W. D. Myer's network is small (and so is his budget!), not using pre-transmission coordination makes more sense.

(b) $CC - TT^2 - TR^2$.

- 3.31.** Consider the linear bus with attempt and defer nodes. Suppose the bus has five nodes over 5 km, and we require the received power at each node to be at least 30% of the transmitted power. How many amplifiers are required? Assume that amplifiers provide a gain of 25 dB.

Answer:

Assume each station is placed 1.2 km apart. Also, assume that each node contains regeneration circuitry. Then, power loss over a 1.2 km stretch is 3 dB due to tap + $1.2 \text{ km} \times 0.2 \text{ dB/km} = 3.24 \text{ dB}$.

$$10 \log_{10} \frac{P(T)}{P(R)} = 3.24$$

$$\frac{P(T)}{P(R)} = 2.1$$

$$\frac{P(R)}{P(T)} = 0.47.$$

Thus, no amplifiers are required.

- 3.32.** An engineer is asked to come up with a transmission protocol for a single-hop network in which each node has a tunable transmitter and a tunable receiver. There are N nodes in the network and W channels. His solution employs a control channel. A frame consists of $N + L$ slots, where the length of a data packet is L . The first N slots in the frame correspond to control slots. Host i puts the number of the destination host to which it wants to transmit in slot i , $1 \leq i \leq N$. When more than one host wants to transmit to the same destination, the station with the lowest index wins. Further, the choice of channel is implicit – the first winner transmits on Channel 1, the second winner transmits on Channel 2, etc. Note that some winners may not be able to transmit because of the constraint on the number of channels. Comment on the characteristics of the protocol and suggest ways of improving it.

Answer:

- i) The protocol is unfair, i.e., the nodes with lower index have higher priority. This can be handled by rotating the priority. In the first cycle, Node 1 has highest priority, followed by Node 2, etc. In the second cycle, Node 2 has the highest priority, followed by Node 3, etc.
- ii) The cycle length is always taken to be $N + L$, which is unnecessary in the case when none of the nodes wants to transmit. In this case, to increase throughput, the cycle can be terminated at N slots and the next cycle allowed to start immediately thereafter.
- iii) The protocol will work better for packet-switched traffic, because in the case of circuit-switched traffic, the overhead associated with the control packets for each data packet may become significant.

- 3.33.** The Rainbow testbed utilized Fabry-Perot filters for the tunable receivers. In regards to the Rainbow medium-access control (MAC) protocol, how would you justify this choice of receiver?

Answer:

Since the setup time for transmission is large due to the polling mechanism, it doesn't help too much to use a fast tunable receiver. On the other hand, since this receiver has a wide tuning range, it can accommodate more nodes in the system.

- 3.34.** Which kind of traffic is more well-suited to the Rainbow protocol – packet-switched traffic or circuit-switched traffic? Why?

Answer:

The Rainbow protocol has a long set-up and acknowledgement delay. Therefore, it may not be very suitable for packet-switched traffic, whereas it would work well for circuit-switched traffic with long holding times.

- 3.35.** Suppose station A is trying to set up a connection with station B . Draw simple state diagrams for stations A and B that illustrate this process.

Answer:

State diagrams for connection set-up are shown in Fig. 3.17.

- 3.36.** Why is the timeout mechanism necessary in the Rainbow protocol? Show by example how, in the absence of a timeout mechanism, the system can become deadlocked. Give an alternate method of avoiding deadlocks in the Rainbow protocol.

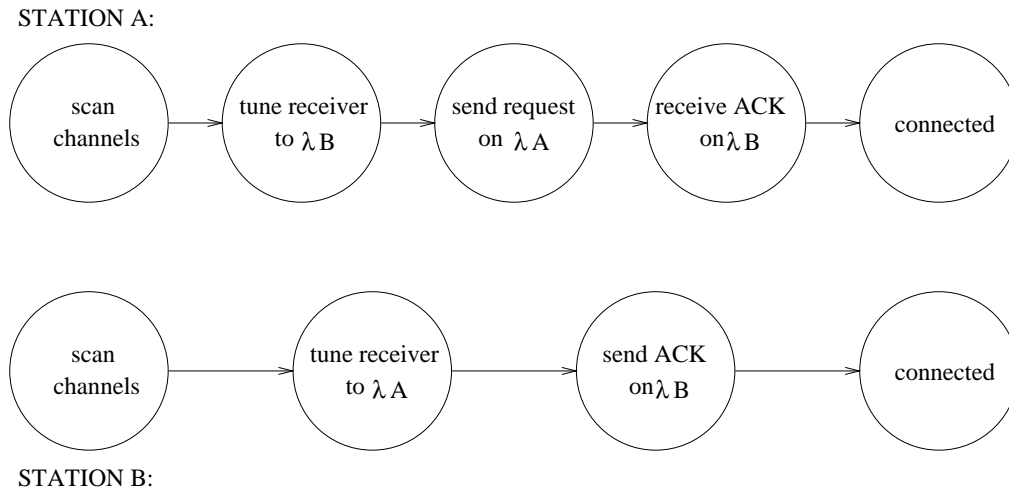


Figure 3.17 State diagrams.

Answer:

Consider three hosts numbered 1 through 3. At about same time, Host 1 wants to set up a connection to Host 2, Host 2 wants to set up a connection to Host 3, and Host 3 wants to set up a connection to Host 1. Host 1's receiver is tuned to Host 2's channel, Host 2's receiver is tuned to Host 3's channel, and Host 3's receiver is tuned to Host 1's channel. None of the hosts will ever get an acknowledgement, thus leading to deadlock.

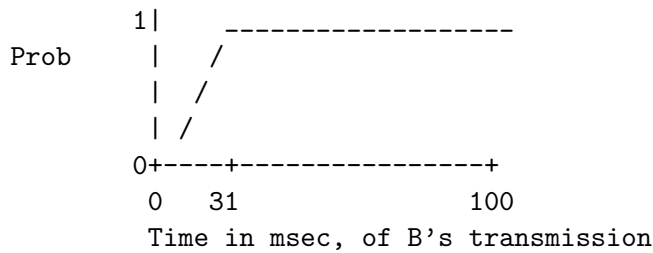
A less elegant way of preventing deadlock would be to maintain a global state of all connections and avoid sending a request if it results in a cycle. All requests can be ordered to prevent simultaneous requests.

- 3.37.** Consider a Rainbow network with 32 stations. Each station is 10 m from the star coupler (i.e., propagation time is negligible), and the receiver tuning time is 1 ms. Two stations, *A* and *B*, wish to send a message to each other at the same time. What will happen assuming the protocol is as indicated in the text? Now suppose that station *A* transmits its message first. Station *B* will then transmit its message at some random time chosen from an uniform random distribution of duration over the range 0 to 100 ms. What is the probability that station *B*'s message will be successfully transmitted? Assume that the timeout is sufficiently large to allow the scanning of all channels.

Answer:

The first attempt will end in failure since each will attempt to send a request at the same time, and neither will be listening at the correct time. The text suggests a timeout, which effectively means that each message is blocked. If *B* chooses to transmit its message before it has completed a full scan of all 31 channels (that it scans), there is a possibility that the message will not be received, due to the same "deadlock" phenomena. But, with each progressing millisecond that *B* waits before it transmits, the probability of a successful transmission increases by $1/31$. Anything beyond 31 milliseconds has probability of 1 of successfully transmitting the message. Thus, if we integrate this probability curve, from 0 to 100 msec, and divide by 100msec, we obtain the probability.

This is $(100 - 31/2)/100$, (subtract half the rectangle from 0 to 31), which is $169/200 = 84.5\%$



3.38. In this exercise, we will simulate the Rainbow protocol. Consider a Rainbow system with four nodes, four channels numbered 1 through 4, and a tuning time of 10 slots. Propagation delay is 5 slots. At time = 0, all receivers are parked on channel 1. Connection hold times are long (assume infinity for this exercise), and the timeout duration is finite. The connection requests are:

- Connections $(1 \rightarrow 2)$ and $(2 \rightarrow 3)$ at time = 0.
- Connections $(1 \rightarrow 3)$ and $(4 \rightarrow 3)$ at time = 10.

Which connection(s) will succeed?

Answer:

The time diagram is shown below. Let R_i be the channel receiver i is tuned to. Now,

- At time $t = 10$, $R_1 = R_3 = R_4 = 2$, and $R_2 = 3$.
- At time $t = 20$, $R_1 = R_3 = 2$, $R_2 = 3$, and $R_4 = 3$.
- At time $t = 30$, node 3 finds request $(4 \rightarrow 3)$ and sends an acknowledgement to node 4.
- At time $t = 40$, node 4 establishes the connection and begins to transmit.
- All other connections will eventually timeout.

3.39. Consider a Rainbow network with 100 nodes, an average scan time of $100 \mu s$ for each channel (including tuning time as well as signal detection time), and a round-robin channel scanning algorithm on each receiver. What is the expected time required to broadcast a packet of 1000 bytes from one node to all of the other nodes? Propagation time, as well as acknowledgement transmission time, is negligible. Also assume that the network has no traffic when the broadcast occurs. Bandwidth per channel is 250 Mbps.

Answer:

The RAINBOW protocol uses a full duplex handshake to establish a link. Expected time for acknowledgement is half the time of one scan cycle; in this case, $50 * 100 \mu s = 5 ms$. $8000 \text{ bits} / (250 * 10^6 \text{ bits/s}) = 32 \mu s$ is the transmission time for the 1000 byte packet. Thus, $5.032 ms * 99$ nodes leaves a total time of $498.2 ms = 0.4982$ seconds to broadcast a message.

3.40. If we allow synchronization among the receivers and senders in the Rainbow network with 100 nodes, can you find a scheme to significantly reduce the time of a broadcast from one node to all other nodes, given the same network and assumptions of the previous problem? What is the total broadcast time required by your scheme?

Answer:

$8000 \text{ bits} / (250 * 10^6 \text{ bits/s}) = 32 \mu s$ is the time of transmission of the 1000 byte packet. Since each receiver takes $100 \mu s$ to tune and check a channel, we make each receiver wait $132 \mu s$ on each channel, so that sufficient tuning time is allowed, and a packet may also be received. Thus, the time to transmit all the packets is at best, $132 \mu s * 99$ nodes, which is approximately 13ms. This can be accomplished if all the receivers scan the channels in a staggered fashion. Thus, at time t , node i will be scanning channel $i + t \bmod 99$.

3.41. What is EPA? How does the EPA technique simplify the analysis? What is lost in the simplification?

Answer:

EPA is a technique of analyzing complex systems by assuming that the system is always at an equilibrium point.

The system state in an “exact” Markov model can be very large. In EPA, we only consider system states which satisfy “equilibrium” conditions. Thus, EPA simplifies the system by reducing the number of states.

Information pertaining to the exact distribution of nodes is lost. For example, if we want to find the probability distribution of the number of nodes in a particular state, then EPA technique may not be the appropriate method.

3.42. Model the following system. There are N jobs and one server. A job can be in two states only: either it is being serviced (or queued), or it is idle. The service time is exponentially distributed with parameter μ , while the “idle” time (of the jobs) is exponentially distributed with parameter λ . Develop and solve the Markov chain for the system. Using EPA, find the average number of idle nodes.

Let $N = 10, \mu = 1$ and vary λ from 0.05 to 1 in steps of 0.05. Compare the average number of idle jobs calculated from EPA analysis with those computed from the Markov chain. Explain your results.

Answer:

The exact Markov chain is shown in Fig. 3.18, and the corresponding steady state probabilities for the system are as follows:

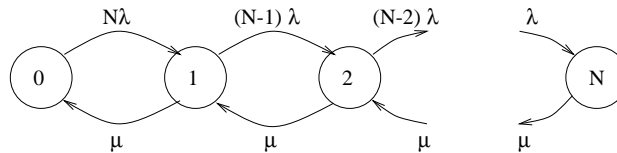


Figure 3.18 A Markov chain.

$$\pi_0 = \frac{1}{\sum_{i=0}^N \frac{N!}{(N-i)!} \left(\frac{\lambda}{\mu}\right)^i}$$

$$\pi_i = \frac{N!}{(N-i)!} \left(\frac{\lambda}{\mu}\right)^i \pi_0$$

The average number of jobs in the queue is $\sum_{i=0}^N i\pi_i$.

The EPA state diagram is shown in Fig. 3.19, and the corresponding equations are shown below:

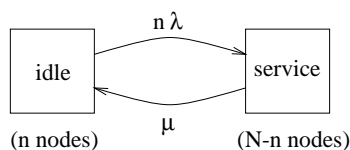


Figure 3.19 EPA state diagram.

At equilibrium, $n\lambda = \mu$, or $n = \mu/\lambda$. The number of jobs in the queue is then $N - n = N - \mu/\lambda$. The average number of idle nodes, as calculated from the exact Markov chain, and as calculated from EPA are shown in Table 3.2.

Table 3.2 Average number of idle nodes.

λ	Markov	EPA
0.05	0.75	-10
0.10	2.14	0.00
0.15	3.77	3.33
0.20	5.09	5.00
0.25	6.02	6.00
0.30	6.67	6.67
0.35	7.14	7.14
0.40	7.50	7.50
0.45	7.77	7.77
0.50	8.00	8.00
0.55	8.18	8.18
0.60	8.33	8.33
0.65	8.46	8.46
0.70	8.57	8.57
0.75	8.66	8.66
0.80	8.75	8.75
0.85	8.82	8.82
0.90	8.88	8.88
0.95	8.94	8.94
1.00	9.00	9.00

Explanation: In EPA, we assume that the rate of jobs leaving the server is μ regardless of the number of jobs in the server. This assumption leads to the inaccuracies in the average number of idle nodes, because if there are no jobs in the server, then the rate of jobs leaving the server is 0.

3.43. Why is the analysis of the Rainbow protocol inaccurate for large timeout values?

Answer:

In the analytical model, the deadlock probability is included in the timeout probability. A request times out when the target of the request is either in request mode or engaged in a connection. The connections that are in request mode will have a high probability of being acknowledged, especially as the timeout duration increases. However, if a deadlock occurs, the nodes involved in the deadlock and those waiting for an acknowledgement from nodes involved in a deadlock will timeout. This situation is not modelled in the analytical model, and hence gives rise to inaccuracies for high timeout values.

3.44. Prove that if N connections are attempted in an N node Rainbow system, *all* nodes will be “locked up”, i.e., either a node will be involved in a deadlock or will “wait” for a node involved in a deadlock.

Answer:

Let each node represent a node in a directed graph G . A connection will be represented by a directed edge. Since there can be only one connection per source, the out-degree of each node in this graph is at most 1. If there are N connections, i.e., N directed edges, then we have to show that there will always exist at least one cycle, and all nodes will be “locked up.” The proof is very simple. There are N connections. Thus there is an outgoing link from every node. Let us start from any node and follow the outgoing link. Each time we enter a new node, we will always be able to leave via an outgoing edge. Since there are only finite nodes, we will eventually visit a node we visited earlier. Thus, we will always have a cycle, which may or may not contain the starting node. Since we made no assumption about the starting node, all nodes will be “locked up.”

- 3.45.** Through simulation, find out the average number of nodes “locked up” due to deadlock when E simultaneous connections (chosen randomly) are attempted in a system with N nodes.

Answer:

The plot of the probability of a node being “locked up” vs. the the number of simultaneous connections is shown in Fig. 3.20.

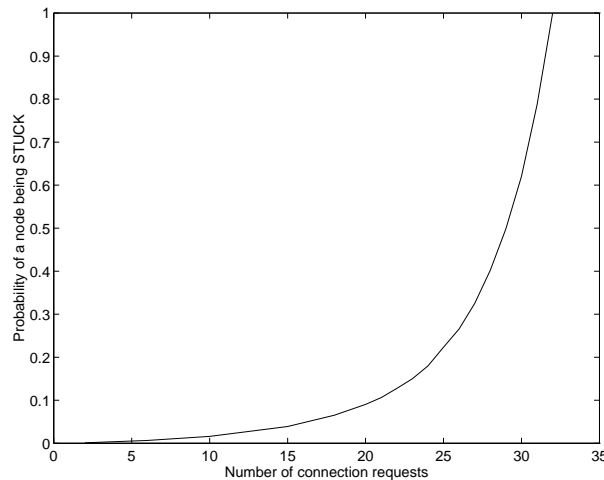


Figure 3.20 Probability of a node being locked up vs. the number of connection requests.

- 3.46.** Let $\Lambda_N(e)$ equal the average number of nodes that are “locked up” when E random connections are attempted simultaneously. Show how we can use $\Lambda_N(e)$ to improve the analytical model for the Rainbow protocol.

Answer:

If we know the function $\Lambda_N(e)$, then one way to improve the analytical model is shown in Fig. 3.21.

where $D = \Lambda_N(e)/N$. Thus, the idea is that out of e nodes attempting a connection, $D \cdot e$ are deadlocked, and $(1 - D) \cdot e$ are not. Deadlocked nodes time out after ϕ slots. ($e = \sum$ all nodes attempting connections.)

- 3.47.** Using the analytical model of Rainbow, find the ratio of stations which are scanning to the stations which are requesting connections.

Answer:

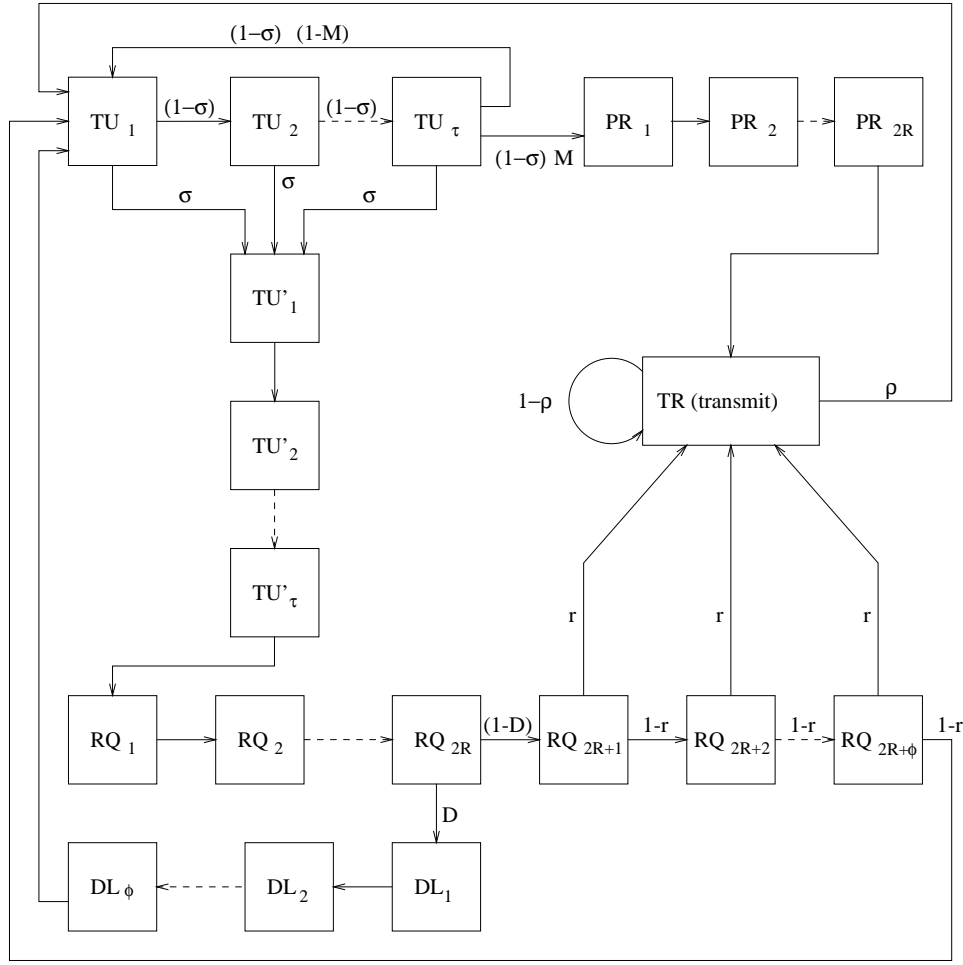


Figure 3.21 An analytical model.

From the diagram of the model, we can get the following equations:

$$\sum_{i=1}^{\tau} N_{TU_i} = \sum_{i=1}^{\tau} (1-\sigma)^{i-1} N_{TU_1} = \frac{1 - (1-\sigma)^\tau}{\sigma} N_{TU_1}$$

$$\sum_{i=1}^{2R+\phi} N_{RQ_i} = \sum_{i=1}^{2R} N_{RQ_i} + \sum_{i=1}^{\phi} N_{RQ_{2R+i}} = (1 - (1-\sigma)^\tau) N_{TU_1} \left[2R + \frac{1 - (1-r)^\phi}{r} \right]$$

So,

$$\frac{\sum_{i=1}^{2R+\phi} N_{RQ_i}}{\sum_{i=1}^{\tau} N_{TU_i}} = \sigma \left[2R + \frac{1 - (1-r)^\phi}{r} \right]$$

From this equation, we can see that, if we increase timeout duration, then there will be more stations in request state. When the ratio exceeds a certain value, r will decrease.

- 3.48.** Explain the relationship between the timeout duration and the normalized throughput in the Rainbow system. What happens to the system throughput when the timeout duration is made too large? Too small? Why?

Answer:

Open for readers.

- 3.49.** In the Rainbow state diagram, no state information is maintained regarding specific node identities. The state diagram applies to the case in which round-robin scanning is performed (i.e., scanning order is $1, 2, \dots, N, 1, 2, \dots, N$, etc.). Why can't this state diagram also apply to the protocol in which elevator-type scanning ($1, 2, \dots, N-1, N, N-1, \dots, 2, 1, 2$, etc.) is performed?

Answer:

The asymmetry in elevator-type scanning causes stations to have varying inter-visit times. For example, in an N station network, end stations will be visited every $2 * (N-1)$ stations, while stations in the middle of the scanning cycle will be visited every N stations. Longer inter-visit times to a station may lead to a higher probability that the station has a message to transmit. Therefore, the probability M in the state diagram would be different for each station.

- 3.50.** Consider the Rainbow protocol. In order to avoid deadlock, suppose the source node, upon tuning its receiver to the destination node's channel, doesn't proceed with its connection request if it finds that the destination node is busy transmitting its own connection request. It instead resumes scanning. How does this affect the performance of the protocol? How can this change be modelled in the state diagram shown in Fig. 3.10?

Answer:

The change would probably improve the performance of the protocol. This modification can be modelled in Figure 4.2 by adding a transition from state TU'_τ to state TU_1 . Some probability, q , should be associated with this link, while the probability $(1-q)$ should be associated with the link from state TU'_τ to state RQ_1 . The probability q is the probability that the selected destination is in one of the states RQ_{R+1} to $RQ_{R+\phi}$, i.e. $q = (1/N) * (N_{RQ_{R+1}} + N_{RQ_{R+2}} + \dots + N_{RQ_{R+\phi}})$.

- 3.51.** Suppose we modify the Rainbow protocol such that if a source node, upon tuning its receiver to the destination node's channel (state TU'_τ in the state diagram in Fig. 3.10), finds that the destination node is requesting a connection with the source node; and then, instead of sending a request, it sends an acknowledgement to the destination node's request. In Fig. 3.10, this transition could be modelled as a link from state TU'_τ to state PR_1 . What would be the probability associated with this link? How would the flow equations change?

Answer:

The probability associated with this link would be M .

In Equation 3.2,

$$N_{PR_1} = N_{PR_2} = \dots = (1 - \sigma)^\tau * M * N_{TU_1} + M * N_{TU'_\tau}$$

plugging in $N_{TU'_\tau}$ from Equation 3.3 yields:

$$N_{PR_1} = [(1 - \sigma)^\tau + (1 - (1 - \sigma)^\tau)] * M * N_{TU_1} = M * N_{TU_1}$$

In Equation 3.3,

$$N_{RQ_1} = N_{RQ_2} = \dots = (1 - M) * N_{TU'_\tau} = (1 - M) * (1 - (1 - \sigma)^\tau) N_{TU_1}$$

Multihop Networks

- 4.1. What are some of the advantages and disadvantages of passive-star coupler multihop networks compared to single-hop networks?

Answer:

Some advantages are:

- Requires no tuning, thus may use less expensive fixed-tuned components.
- Requires no pre-transmission coordination or connection set-up.

Some disadvantages are:

- Large delay. In each hop, the packet goes through the PSC, and thus the propagation delay is $H * R$, where H = hop distance, and R = round-trip propagation delay to the PSC.
- Opto-electronic conversion. At each hop, all traffic is converted into the electronic domain before being routed to the outgoing link, increasing the delay at each node.
- Forwarding traffic. Each link carries not only the traffic between its end-nodes, but also multihop traffic routed through it.

- 4.2. Consider an eight-node (2,2) ShuffleNet. Calculate the average delay for a uniform traffic matrix (the arrival rate for each source-destination pair is λ packets per second). Assume that the service time for a packet at a node is exponentially distributed with mean $1/\mu$ seconds. Assume shortest-path routing.

Answer:

Each packet traverses \bar{h} hops. In a (2,2) ShuffleNet, $\bar{h} = 2$. At each hop, the packet experiences a queueing delay and a transmission delay. The average transmission delay at each node is simply $\frac{1}{\mu}$. The queueing delay can be approximated using a standard M/M/1 queueing model. The aggregate arrival rate to an arbitrary node is given by: $\lambda_{tot} = 7\lambda \cdot \bar{h}$. Thus, the queueing delay is given by: $D_q = \frac{\lambda_{tot}}{\mu(\mu - \lambda_{tot})}$. The total delay at each node, including the transmission delay is: $D = \frac{1}{\mu - \lambda_{tot}}$. Since each packet experiences an average of two hops, the average delay is given by:

$$D_{avg} = 2D = \frac{2}{\mu - 14\lambda}$$

4.3. Describe how a packet gets routed from node 0 to node 7 in a (2, 2) ShuffleNet.

Answer:

Destination address: Node 7 $\Rightarrow (c^d, r^d) = (01, 11)$ based on binary code.

At Node 0, we determine the column distance X as:

$$X = (2 + 1 - 0) = 3 = 11_2.$$

Now, Node 0 can forward to either Node 4 or Node 5. The row address of Node 4 is 00 base 2, and the row address of Node 5 is 01 base 2. Since the least significant digit of the row address for Node 5 matches $r_2^d = 1$, we route the packet to Node 5.

Again, at Node 5 $\Rightarrow (01, 01)_2$, calculate X as:

$$X = k = 2$$

We can go to Node 2 or Node 3 (i.e. rows 10_2 or 11_2). Since the least significant digit of the row address for Node 3 matches $r_1^d = 1$, we route the packet to Node 3.

At Node 3 $\Rightarrow (00, 11)_2$, $X = 3$, we can go to Node 6 (row 10_2) or Node 7 (row 11_2). $r_2^d = 1$, so we route the packet to Node 7.

The route is: 0 \rightarrow 5 \rightarrow 3 \rightarrow 7.

4.4. Show how a packet gets routed from node 0 to node 6 in a (2, 3) de Bruijn graph. What is the maximum hop distance in this graph?

Answer:

Node 0 = $(000)_2$.

Node 6 = $(110)_2$.

Comparing the last few bits of 000_2 to 110_2 , we don't find an overlap. Hence, the route from Node 0 to Node 6 can be represented as: 000110_2 .

Taking three bits of the route at a time, we have the route:

$000 \rightarrow 001 \rightarrow 011 \rightarrow 110$, or $0 \rightarrow 1 \rightarrow 3 \rightarrow 6$.

The maximum number of hops occurs when there is no match in the digit representations of the source and destination. In that case, the link can be written as: $sssddd$, where sss is the binary representation of the source, and ddd is the binary representation of the destination. This route results in a maximum of four hops.

4.5. Draw a (3,2) de Bruijn graph. Compute the average hop distance.

Answer:

The de Bruijn graph is shown in Fig. 4.11.

$$\begin{aligned} \bar{h} &= \frac{\sum h_{ij}}{N(N-1)} = \frac{\sum h_{ij}}{72} \\ &= \frac{(2 + 2 \cdot 6) \cdot 3 + (3 + 2 \cdot 5) \cdot 2 + (3 + 2 \cdot 5) \cdot 4}{72} \\ &= 1.67 \end{aligned}$$

4.6. Compare and contrast the following topologies and calculate the average hop distance for each:

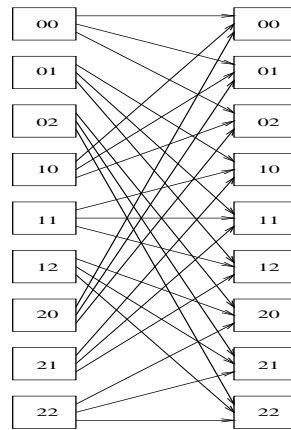


Figure 4.11 A (3,2) de Bruijn graph.

- (a) (2,2) ShuffleNet
- (b) (2,3) de Bruijn graph
- (c) 8-node binary hypercube
- (d) 8-node 4×2 Manhattan Street Network

Answer:

Average hop distances and nodal degrees:

Topology	\bar{h}	nodal degree
(2,2) ShuffleNet	2	2
(2,3) deBruijn graph	2.11	2
Binary Hypercube	1.71	3
4×2 MSN	2	2

4.7. Find the average hop distance for a 4×4 Manhattan Street Network.

Answer:

The number of nodes is 16. We will find the hop distance to all nodes from node (0,0). Let d_{ij} be the hop distance from node (0,0) to node (i,j).

$d_{01} = d_{10} = 1$, $d_{02} = d_{20} = d_{13} = d_{31} = 2$, $d_{03} = d_{30} = d_{12} = d_{21} = 3$, $d_{11} = d_{22} = d_{33} = 4$, $d_{23} = d_{32} = 5$.

Thus, the average hop distance for a 4×4 MSN is

$$\frac{2 \times 1 + 4 \times 2 + 2 \times 3 + 2 \times 4 + 2 \times 5}{16-1} = 2.93.$$

4.8. Derive the average hop distance and diameter for a p -dimensional binary hypercube.

Answer:

The hop distance between two nodes is given by the number of difference over all the bit positions in each node's address.

Without loss of generality, we can select node 0.

Compared to node 0, $\binom{p}{1}$ nodes differ by one bit position, $\binom{p}{2}$ nodes differ by two bit position, etc.

Thus the average hop distance is given by

$$\frac{\sum_{i=1}^p i \cdot \binom{p}{i}}{\sum_{j=1}^p \binom{p}{j}} = \frac{p \cdot 2^{p-1}}{2^p - 1}$$

4.9. Derive the average hop distance of ShuffleNet [Eqn. (4.1)].

Answer:

The number of nodes that are h hops away is given by

$$n_h = \begin{cases} p^h & h = 1, 2, \dots, k-1 \\ p^k - p^{h-k} & h = k, k+1, k+2, \dots, 2k-1 \end{cases} \quad (4.1)$$

Thus, the average hop distance is given by

$$\begin{aligned} \bar{h} &= \sum_{h=1}^{2k-1} \frac{h n_h}{k p^k - 1} \\ &= \frac{1}{k p^k - 1} \left[\sum_{h=1}^{h=k-1} h p^h + \sum_{h=k}^{2k-1} h (p^k - p^{h-k}) \right] \\ &= \frac{1}{k p^k - 1} \left[\sum_{h=1}^{h=k-1} h p^h + \sum_{h=k}^{2k-1} h p^k - \sum_{h=1}^{k-1} h p^h - \sum_{h=0}^{k-1} k p^h \right] \\ &= \frac{1}{k p^k - 1} \left[p^k \sum_{h=k}^{2k-1} h - k \sum_{h=0}^{k-1} p^h \right] \end{aligned}$$

Now, we employ the following two identities to simplify the above equation:

$$\sum_{h=m}^{h=n} h = \frac{n(n+1)}{2} - \frac{m(m-1)}{2}$$

and,

$$\sum_{h=0}^{h=k-1} p^h = \frac{p^k - 1}{p - 1}$$

Thus, we have,

$$\begin{aligned} \bar{h} &= \frac{1}{k p^k - 1} \left[p^k \left(\frac{2k(2k-1)}{2} - \frac{k(k-1)}{2} \right) - k \frac{p^k - 1}{p - 1} \right] \\ &= \frac{k p^k (p-1)(3k-1) - 2k(p^k - 1)}{2(p-1)(k p^k - 1)} \end{aligned}$$

4.10. Suppose we wish to build a multihop network in which each node has a nodal degree of two, and the diameter of the network is three. We consider only ShuffleNet and Manhattan Street Network. Give the possible logical topologies and compare (i) the maximum number of nodes that can be supported, and (ii) the average hop distance.

Answer:

Possible topologies: (2,2) ShuffleNet, 2×4 MSN, and 3×3 MSN.

- (i) 2×4 MSN and (2,2) ShuffleNet support 8 nodes while the 3×3 MSN supports 9 nodes.
- (ii) The average hop distance is 2 for each of the topologies.

4.11. A de Buijn graph topology is chosen by a major internet network service provider. The network must contain at least one million nodes.

- (a) If a binary ($\Delta = 2$) de Bruijn graph is used, what is the minimum number of bits required to represent the label of a node?
- (b) How many self-loops does the graph contain?
- (c) Find a bound for \bar{h} .
- (d) A routing algorithm finds the next node to send a packet to so that this next node is on a shortest path from node a to b . The computer used to implement the algorithm can perform operations on machine words of size 32 bits in 1 nanosecond. Operations include bit shift, bitwise and, bitwise or, add, subtract, equality test, and inequality test. Outline, or code, a fast routing algorithm or program that runs on node a where $a = (a_1, a_2, \dots, a_D)$, and outputs the bit which is appended to a , after shifting, i.e., output is a_{D+1} , where $(a_2, \dots, a_D, a_{D+1})$ is the label of the next node in the shortest path toward $b = (b_1, b_2, \dots, b_D)$. Attempt to make use of low-level (bit) parallelism in your code, possibly packing a and b as unsigned integers, into machine words, as much as possible. Explain how the bits are stored in the words, including the order.
- (e) Assuming that branches also take 1 nanosecond, find a bound on the running time of your routine. Do not consider compiler optimization.

Answer:

- (a) 20, since $2^{20} > 1,000,000$.
- (b) Only two, the all zero and the all one nodes.
- (c) The formula,

$$\bar{h} \leq D \frac{\Delta^D}{\Delta^D - 1} - \frac{1}{\Delta - 1}$$

applies. This expression evaluated with $D = 20$, and $\Delta = 2$ evaluates to slightly over 20, but we know that all distances are not greater than 20, so our upper bound on \bar{h} is 20.

(d) The following C code will implement the string matching algorithm outlined in the book. It uses machine words for node addresses, which are stored as unsigned integers, a native type to C, with a_1 corresponding to the leftmost (most significant) bit in the integer representation. Since $D = 20$, all bits of a node label fit in one machine word.

```
int next_bit(unsigned a, unsigned b) {
    register unsigned z = ~0; /* all ones, 1ns */
    register k = 0;          /* 1ns */
    while (a!=(b&z)) {        /* bitwise shift and
                               word compare, 2ns */
        z = z << 1;          /* bitwise left shift, 1ns */
        a = a << 1;          /* bitwise left shift, 1ns */
        k = k + 1;          /* increment, 1ns */
    }                        /* 1ns branch */
}
```

```

    if (k==0) return -1; /* match code, 1ns. */
    return (b>>(k-1))&1; /* extract bit # k (from
                           right), 3ns */
}

```

(e) Runtime is bound by $5\text{ns} + 6\text{ns} \cdot D$ which is $(5 + 6 \cdot 20)\text{ns} = 125\text{ns}$.

4.12. In the following topologies, the presence of an undirected link is understood to be bidirectional, i.e., it may be replaced by two links going in opposite directions. Compute the diameter and average hop distance for the following logical topologies (graphs). Also, compute the total number of wavelengths (channels) required to implement these logical multihop topologies with a passive-star coupler as the physical topology.

- (a) a 10-node unidirectional ring
- (b) a 10-node bidirectional ring
- (c) a 10-node complete graph
- (d) the (10-node) Peterson graph (see Fig. 4.12 below)

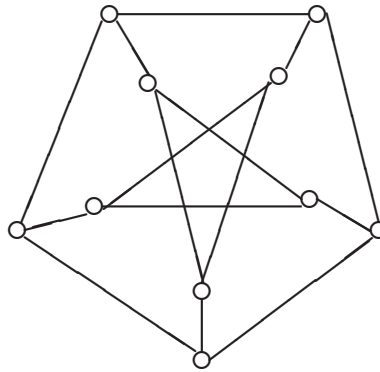


Figure 4.12 The (10-node) Peterson graph.

Answer:

Each graph is totally symmetric, so one may compute the average hop distance from an arbitrary node. The number of required wavelength channels is just the number of directed edges in the graphs.

(a) $D = 9$, $(1+2+3+4+5+6+7+8+9)/9 = 5.00$, $W = 10$.

(b) $D = 5$, $(1+2+3+4+5+4+3+2+1)/9 = 2.77$, $W = 20$.

(c) $D = 1$, $(1+1+1+1+1+1+1+1+1)/9 = 1.00$, $W = 90$.

(d) $D = 2$, $(1+1+1+2+2+2+2+2+2)/9 = 1.67$, $W = 30$.

If it is stated that pair (i, j) may use the same wavelength channel as pair (j, i) , then the W values for b through c may be halved.

4.13. Consider an 8-node (2,3) de Bruijn graph. Determine the total number of paths from node 0 to node 5 that have at most 8 hops.

Answer:

Using the signal flow graph method, we have the following equations:

$$\begin{aligned}
 X_0(t) &= DX_0(t-1) + DX_4(t-1) \\
 X_1(t) &= DX_0(t-1) + DX_4(t-1) \\
 X_2(t) &= DX_1(t-1) + DX_5(t-1) \\
 X_3(t) &= DX_1(t-1) + DX_5(t-1) \\
 X_4(t) &= DX_2(t-1) + DX_6(t-1) \\
 X_5(t) &= DX_2(t-1) + DX_6(t-1) \\
 X_6(t) &= DX_3(t-1) + DX_7(t-1) \\
 X_7(t) &= DX_3(t-1) + DX_7(t-1)
 \end{aligned}$$

Solving for X_5 in terms of X_0 and calculating the transfer function yields:

$$T(D) = D^3 + D^4 + 2D^5 + 4D^6 + 7D^7 + 13D^8 + \dots$$

for a total of 28 paths with at most 8 hops.

- 4.14. Find the transfer function between points A and B in the following graph. How many paths of distance 10 hops are there from A to B ? How many paths of distance 15 hops are there from A to B ?

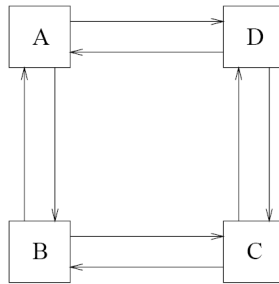


Figure 4.13 A bidirectional ring network.

Answer:

We first create the modified graph as follows:

1. Add auxiliary node a' , and an edge from a' to a .
2. Assign a weight of 1 to the edge from a' to a , and a weight of D to every other edge in the graph.
3. Remove all outgoing links from b . This way, no path will visit Node b multiple times.

Now, we write down the node equations. We get:

$$\begin{aligned}
 a &= a' + Dd \\
 b &= Da + Dc \\
 c &= Dd \\
 d &= Da + Dc
 \end{aligned}$$

Solving for b , we get:

$$b = \frac{Da'}{1 - 2D^2}$$

Thus the transfer function between Nodes a and b is given by:

$$T = \left(\frac{Da'}{1 - 2D^2} \right) \frac{1}{a'} = \frac{D}{1 - 2D^2}$$

Using Taylor series expansion, we get:

$$T = D + 2D^3 + 4D^5 + \dots = \sum_{l=0}^{\infty} 2^l D^{2l+1}$$

Thus the coefficient of D^{10} is 0, and there are no paths of length 10 between a and b .

The coefficient of D^{15} is $2^7 = 128$, thus there are 128 paths of length 15 between a and b .

- 4.15.** Draw a two-dimensional radix-3 hypercube. Is the resulting graph isomorphic to a 3×3 torus? Will a two-dimensional radix-4 hypercube be isomorphic to a 4×4 torus? What is the number of edges in a d -dimensional radix- r hypercube?

Answer:

This is isomorphic to a 3×3 torus. A 2 dimensional radix 4 hypercube is not isomorphic to a 4×4 torus. This can be easily seen because a 2 dimension radix 4 hypercube will have $[(4-1)2] \times 4^2/2 = 3 \times 16 = 48$ edges, while a 4×4 torus will only have 16 edges (which is shown in Fig. 4.14).

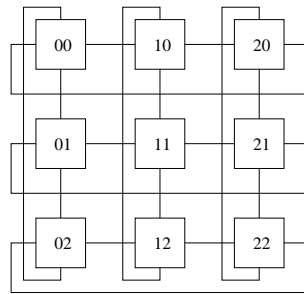


Figure 4.14 A two-dimensional radix-3 hypercube.

- 4.16.** Consider a multidimensional torus, with all links being bidirectional, and with all dimensions (M) being of the same size, i.e., an $N \times N \times \dots \times N$ structure, of size N^M . Now consider the generalized hypercube with bidirectional links. Is it possible for a graph (logical topology) to be both? Does one class contain the other? In other words, can you find two graphs such that one is a generalized hypercube, but not a multidimensional torus, and the other is a torus, but not a generalized hypercube? Give a simple, one-sentence explanation of the difference the essential difference between these two structures.

Answer:

It is possible for a graph to fit both descriptions, a simple case would be a square, but any binary hypercube is multidimensional torus. The two classes are distinct. As an example of a torus that is not a generalized hypercube, simply consider a ring of size five. This is clearly a torus, but it cannot be a generalized hypercube, since five is a prime number, and therefore it would have to be a one dimensional hypercube of radix five, but this is the same as a complete graph of size five. The complete graph of size five is then also an example of a generalized hypercube that is not a torus, by similar reasoning as above. The difference between the structures is that within a dimension, the torus forms a ring, whereas the hypercube forms a complete subgraph (clique).

4.17. Draw a generalized hypercube with 12 nodes. Explain how channel-sharing can be used in this hypercube network.

Answer:

The 12-node hypercube graph is shown in Fig. 4.15.

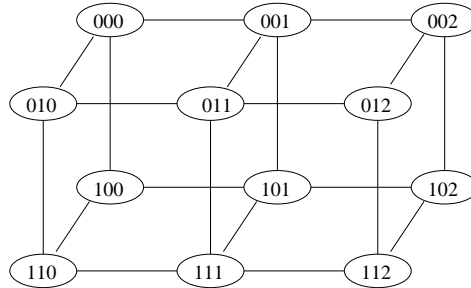


Figure 4.15 A generalized hypercube with 12 nodes.

Channel sharing: (Each node needs 3 pairs of transceivers.)

1. In dimension 1, there are 4 channels, say C1, C2, C3, and C4.

Nodes (000), (001), (002) share C1.

Nodes (010), (011), (012) share C2.

Nodes (100), (101), (102) share C3.

Nodes (110), (111), (112) share C4.

2. In dimension 2, there are 6 channels, say C1, C2, C3, C4, C5, and C6.

Nodes (000), (010) share C1.

Nodes (001), (011) share C2.

etc.

3. In dimension 3, there are 6 channels.

Note: For $N = 12$, we can have multiple hypercube structures. Let $N = 2 \times 6$, then we get a 2-dimensional hypercube. Using a above-mentioned channel sharing scheme in that hypercube, only 8 channels are required.

4.18. One of the disadvantages of a binary hypercube is that the degree of any node is unbounded (also true for generalized hypercubes). That is, for a hypercube of size $2^N = M$, the degree of a node is $N = \log_2 M$ for a binary hypercube. One solution to keep the nodal degree in check so that the structure can be scalable is to replace each node by a ring of N nodes. Each of the new nodes can then handle one of the connections required by the hypercube node. Such a graph is called a *cube-connected cycle*. It is also important to mention that the cycle must be ordered, i.e., hypercube nodes are connected when their labels differ by only one bit. The cycle must occur in bit order, so that connected cycles (hypercube nodes) are connected by corresponding cycle nodes. More precisely, if hypercube nodes a and b are connected, then their labels differ in one bit position, say the j^{th} position. Thus, cycle node a_j and cycle node b_j must be connected. Also, for all cycles a , cycle nodes a_j and $a_{j+1 \bmod N}$ are connected.

When routing within the cube-connected cycle, a pair of nodes within a cycle must follow the cycle to form a shortest path. Outside of a cycle, routing can be accomplished without being much worse than the hypercube distance. Explain how. In other words, describe an efficient routing algorithm. Also, find a bound on the diameter of the graph.

Hint: Consider the routing algorithm of a hypercube.

Answer:

A hypercube's routing algorithm essentially flips the bits, one at a time to follow a path toward its destination. For the cube-connected cycle, the same idea is followed. Each node in the cycle corresponds to a bit position within the hypercube's node label. Thus, to generate a path in the cube-connected cycle, consider the cycle's label, as a hypercube node label. The source cycle has a label, and the destination cycle has a label. Cycle through the bits of the source label, flipping them when they are different from the destination label. Each advance to the next bit position corresponds to an advance in the cycle, each flip corresponds to hopping over to a neighboring cycle. Once the correct cycle is reached, a simple shortest path can be taken (at most half of N). This routing algorithm then yields N hops for looking at each bit position in the label, at most N hops for the hypercube distance, and $\lfloor \frac{N}{2} \rfloor$ for the last cycle. Thus, our diameter bound is $\lfloor 2.5N \rfloor$.

4.19. Consider a six-dimensional binary hypercube.

- (a) How would you represent the root of this hypercube?
- (b) Consider node (010110_2) . List the members of the 3-cube to which this node belongs.
- (c) Consider node (101011_2) . Which node is its "partner" in the 6-cube? 4-cube?

Answer:

Root of hypercube: 000000_2

Members of 3-cube: $010000, 010001, 010010, 010011, 010100, 010101, 010110, 010111$.

Partner in 6-cube differs in the 6th bit: 001011_2 .

4-cube partner: 100011_2 .

4.20. Given three nodes and traffic matrix A , use the Min-Max flow-based heuristic to find a near-optimal node placement on a linear dual bus.

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 5 & 0 & 7 \\ 4 & 3 & 0 \end{bmatrix}$$

Answer:

In the min-max algorithm, for each new added node, find the appropriate side to place it such that the maximum link flow is minimized.

Step 1: Find the minimum-flow link, say m_{13} , then choose Node 1 and Node 3 to begin the construction.

Step 2: Compute the flow on each link for the situation in which Node 2 is placed next to Node 1, and the situation in which Node 2 is placed next to Node 3. We find that Node 2 should be placed next to Node 3. The final ordering is 1-3-2.

4.21. List the advantages of channel-sharing.

Answer:

Reduces the hop distance.

The number of channels doesn't need to grow as fast as the number of links in the logical topology.

Facilitates multicasting.

4.22. Discuss some of the salient features of GEMNET.

Answer:

- (i) Represents a family of network structures.
- (ii) Can incorporate any number of nodes.
- (iii) deBruijn graphs and ShuffleNets are special cases.
- (iv) Scalable.
- (v) Simple routing.

4.23. Show that a $(1, 2^N, 2)$ GEMNET is equivalent (having a one-to-one correspondence between all nodes and links) to a $(2, N)$ de Bruijn graph.

Answer:

Since $K = 1$, $\hat{c} = c = 0$, and one can remove the column from consideration. So we also remove the row from consideration and we refer to label a , with $0 \leq a < 2^N$, as label r was used in the text. Then, $\hat{a} = 2a \bmod 2^N$, and out-links to node a are nodes $2a \bmod 2^N$ and $2a \bmod 2^N + 1$.

Next consider the de Bruijn Graph in which each node is labeled with $b = (b_1, b_2, \dots, b_N)$. Now let each of the b_i (for $1 \leq i \leq N$) be the bits in the binary representation of b' (an integer representing b) with b_N corresponding to the lowest bit value, ie the ones place. Then the out-links of b' are just $2b' \bmod 2^N$ and $2b' \bmod 2^N + 1$. This is just the above one column formula for GEMNET. Since both graphs have equivalent labels and the procedure for determining links is equivalent, the graphs themselves must also be equivalent (the technical term is isomorphic).

4.24. Show that a $(1, \Delta^N, \Delta)$ GEMNET is equivalent (having a one-to-one correspondence between all nodes and links) to a (Δ, N) de Bruijn graph.

Answer:

This argument is an extension to the previous answer. Basically, one must notice that $\hat{a} = \Delta a \bmod \Delta^N$, and that node a , where $0 \leq a < \Delta^N$, has out-links $\Delta a \bmod \Delta^N, \Delta a \bmod \Delta^N + 1, \Delta a \bmod \Delta^N + 2, \dots, \Delta a \bmod \Delta^N + \Delta - 1$. Of course, this also is just the scheme for finding out-links in GEMNET. The only difference is that we are dealing with digits in base Δ instead of base 2.

4.25. Show that a (K, P^K, P) GEMNET is equivalent (having a one-to-one correspondence between all nodes and links) to a ShuffleNet.

Answer:

Both networks (graphs) are connected arranged in $P \times P^K$ rectangles, and both have out-degree P . Also, in both networks, each node is defined to be connected to nodes in the next column in a wrap-around fashion. The only thing we must prove is that for equal enumerations of the nodes, the formula for connecting the nodes is the same. ShuffleNet connects a node in row i to nodes $(i \bmod P^{K-1}) * P, (i \bmod P^{K-1}) * P + 1, \dots, (i \bmod P^{K-1}) * P + P - 1$ in the next column whereas the GEMNET connects a node in row r to nodes $rP \bmod P^K, (rP + 1) \bmod P^K, \dots, (rP + P - 1) \bmod P^K$. But, $(rP + j) \bmod P^K = (r \bmod P^{K-1}) * P + j$, for $0 \leq j < P$, since $((rP + j) \bmod P^K) \bmod P = j$ and $\lfloor ((rP + j) \bmod P^K) / P \rfloor = r \bmod P^{K-1}$ so the two networks really have the same links as well as nodes.

4.26. Given nine nodes, find a GEMNET topology for which $K > 1$ and $M > 1$.

Answer:

A $(3, 3, 2)$ GEMNET is one possible solution.

4.27. Under what conditions do we obtain the maximum and minimum hop distances in a GEMNET?

Answer:

When we cover distinctly new edges every time we cycle back to the same column we started from, then we have minimum hop distance.

When the clusters of nodes in each subsequent hop overlaps perfectly with the cluster of nodes in the previous hop, then we have maximum hop distance.

4.28. Explain Eqn. (4.2).

Answer:

$\delta = [(K + c_d) - c_s] \bmod K$ is the column distance. Thus, the number of hops h , has to be of the form $(\delta + jK)$, where j is an integer, and K is the number of columns. j should be large enough so that r_d is within the reachable nodes from c_s .

Note that if P^h is large enough, the equation will be satisfied.

$(r_s \cdot p^h)$ is the row number of the node at the end of the set of nodes that can be reached in h hops from r_s . If $(r_s \cdot P^h) > M$, then we have a wrap around, and thus the generalized form is

$$(M + r_d - (r_s \cdot P^h) \bmod M) \bmod M$$

Now, there are two cases we have to consider:

Case 1: $P^h < M$. In this case, we have not reached all the nodes as yet, but since the row distance given by $[M + r_d - (r_s \cdot P^h) \bmod M] \bmod M$ is less than what we can reach, we can find a route from the source to the destination.

Case 2: $P^h > M$. In this case, since we can reach all the nodes in a row, the problem is solved trivially.

4.29. Find the average hop distance for a (2,5,2) GEMNET (see Fig. 4.6). Assume shortest path routing. Show how a packet is routed from node 0 to node 9.

Answer:

Node (0,0) reaches 2 nodes in one hop, 3 nodes in two hops, 3 nodes in three hops, and 1 node in four hops. Thus, the average hop distance for Node (0,0) is $21/9$. By symmetry, Nodes (1,0), (0,4), and (1,4) have the same average hop distance.

Node (0,1) reaches 2 nodes in one hop, 3 nodes in two hops, 3 nodes in three hops, and 1 node in four hops. Thus, the average hop distance for Node (0,1) is $21/9$. By symmetry, Nodes (1,1), (0,3), and (1,3) have the same average hop distance.

Node (0,2) reaches 2 nodes in one hop, 4 nodes in two hops, and 3 nodes in three hops. The average hop distance for Node (0,2) is then $19/9$. By symmetry, Node (1,2) has the same average hop distance.

For the entire network, the average hop distance is:

$$\frac{8 \cdot \frac{21}{9} + 2 \cdot \frac{19}{9}}{10} = 2.29$$

Routing from Node 0 to Node 9:

$$R = (M + r_d - (r_s P^h) \bmod M) \bmod M < P^h \Rightarrow 4 < 2^h$$

Smallest $h = 3$, and $R = 4$.

Route code $R = 100^2$.

Route: 0-3-4-9.

- 4.30. Consider a (3,7,2) GEMNET. Find the route code R from source node (0,4) to destination node (2,1).

Answer:

We have:

$$R = \left[7 + 1 - (4 \cdot 2^h) \bmod 7 \right] \bmod 7 < 2^h$$

where h is an integer of the form

$$h = (2 + j \cdot 3).$$

The smallest h which satisfies these expressions is $h = 5$, thus $R = 6$, or represented in five binary digits, $R = (00110)$.

- 4.31. Find the number of shortest paths in a (3,27,3) GEMNET between nodes $s = (1, 25)$ and $d = (0, 10)$.

Answer:

$$\delta = (0 + 3) - 1 = 2$$

Thus hop distance is of the form $h = 3j + 2$.

Now $r_s = 25$, $r_d = 10$.

Thus,

$$R = [27 + 10 - (25 \cdot 3^h) \bmod 27] \bmod 27 < 3^h$$

For $j = 1$, we have $h = 5$, $3^5 > 27$, thus $j = 1$ works.

Hence,

$$R = [37 - (25 \cdot 3^5) \bmod 27] \bmod 27 = 10 = (101)_3.$$

Thus, the number of shortest paths is:

$$Y = \lceil (3^5 - 10)/27 \rceil = 9$$

- 4.32. Consider a lightly but uniformly loaded (K, M, P) GEMNET. Suppose one link fails. How will hop distance be affected? If shortest path routing is being used, how many connections will have to be rerouted?

Answer:

In GEMNET, failure of different links will have different effects, i.e., all links are not equivalent. Thus, we will look at an “approximate” study. R is the routing code which can be represented as an h digit base- P number. If a link fails, all connections through that link will have to be rerouted. Some of these rerouted connections will experience an increase in hop distance by K , some will not. Let us first calculate the approximate number of rerouted connections. Since the in degree of all nodes is P , the number of connections passing through a column can potentially use any of the PM links. Thus, assuming that the load is evenly distributed over all these links, the number of connections that need rerouting is $1/PM$.

From each node, P^h nodes can be reached in h hops. There are M nodes per column, and thus, the number of nodes “missed” while finding the shortest path is approximately:

$$MK - \sum_{i=0}^{\log_p M} p^i = KM - \frac{MP - 1}{P - 1}$$

The “missed” nodes will have multiple paths from the source, and thus when a link fails, the hop distance for these routes will not change. Thus, the number of paths for which the route changes is approximately given by $\frac{MP-1}{P-1}$.

- 4.33.** Compute the diameter and average hop distance for a 3×3 GEMNET. Compute the base route code from node (1,0) to node (1,2). Find all shortest paths from node (1,0) to node (1,2).

Answer:

Diameter = 4.

$\bar{h} = 2.25$.

Route code from Node 1 to Node 7: $(010)_2$.

From Node 1 to Node 7, there are two shortest paths: $(010)_2$ and $(101)_2$.

- 4.34.** We wish to build a 10-node network with a GEMNET topology, trying to achieve the best performance. Come up with a GEMNET topology and explain your choice. Assume each node has a degree of two.

Answer:

For a network with 10 nodes, there exist three possible GEMNET topologies: (1,10,2), (2,5,2), and (5,2,2). The (2,5,2) GEMNET is the preferred topology because it results in the lowest average hop distance ($\bar{h} = 2.33$), and there are multiple shortest paths, allowing traffic to be better balanced. The average link load is also the lowest for the (2,5,2) GEMNET.

- 4.35.** Draw all possible GEMNETs having six nodes and a nodal degree of two. Which of these will have the shortest average hop distance? The longest?

Answer:

The possible GEMNET topologies are: (1,6,2), (6,1,2), (2,3,2), and (3,2,2). The (2,3,2) and (3,2,2) GEMNETs have the shortest average hop distance with $\bar{h} = 1.8$.

- 4.36.** If we are trying to minimize the maximum link load in the network, which GEMNET design is better: one with a higher number of columns or one with a higher number of rows?

Answer:

Two conflicting factors compete when we try to minimize the maximum link load. One is that, as a GEMNET is widened (by increasing K), a larger number of multiple shortest paths exist, allowing traffic to be better balanced, thereby decreasing load on the most congested link. On the other hand, as a GEMNET is widened, its average hop distance increases which will proportionally increase the average link load. Thus the optimal point lies somewhere between the two extremes.

- 4.37.** A (4,4,2) GEMNET is to be upgraded into a (4,5,2) GEMNET by adding a row of nodes. What is the number of retunings required?

Answer:

The number of retunings needed:

$$\sum_{i=2}^p \left\lceil \frac{N}{p} \right\rceil (i-1) = \frac{N}{2}(2-1) = 8.$$

- 4.38.** Consider a network of six nodes. Construct a SC_GEMNET logical topology for each of the following cases:

(a) Number of channels = 6

(b) Number of channels = 3

(c) Number of channels = 2

Assume a $FT - FR$ system in each case.

Answer:

i) The only possible logical topology is a ring.

ii) In this case there are several GEMNET possibilities: (1,6,2), (2,3,2), (3,2,2), and (6,1,2).

iii) In this case the possible GEMNET topologies are: (1,6,3), (2,3,3), (3,2,3), and (6,1,3).

4.39. Construct a (2,4,2) SC_GEMNET with $w = 3$, and show a possible TDM frame structure.

Answer:

Number of nodes = 8.

Since $8/3$ is not an integer we can either use $w = 2$ or have unequal sharing.

With $w = 2$:

λ_0	0	1	2	3
λ_1	4	5	6	7

With unequal sharing:

λ_0	0	3	6
λ_1	1	4	7
λ_2	2	5	2

4.40. Find all possible channel-sharing configurations for a network with 10 nodes. Consider the situation in which w is an exact divisor of 10.

Answer:

There are four possible patterns:

1) $N = 10, w = 1, P = 10, K = 1$ (1,10,10) SC-GEMNET

2) $N = 10, w = 2, P = 5, K = 2$ (2,5,5) SC-GEMNET

3) $N = 10, w = 5, P = 2, K = 5$ (5,2,2) SC-GEMNET

4) $N = 10, w = 10, P = 1, K = 10$ ring topology

Topology	\bar{h}	Stable load range	Average packet delay
(1,10,10) SC-GEMNET	1	$\lambda \leq 0.1$	7.88
(2,5,5) SC-GEMNET	1.44	$\lambda \leq 0.139$	7.49
(5,2,2) SC-GEMNET	2.78	$\lambda \leq 0.18$	10.32
(10,1,1) ring	5	$\lambda \leq 0.2$	20

$\lambda_{max} = 0.2$ packets/slot/node.

$w_{max} = 10$.

$w^* = 2$ for $\lambda = 0.15$.

- 4.41. Consider a (3,4,2) SC_GEMNET with unicast traffic. Packets arrive at each node according to a Poisson process with rate $\lambda = 2 \cdot 10^5$ pkt/s. The distance between each node and the star coupler is 10 km. There are six wavelengths used in the system. Slot length is equal to 1 μ s. Find the average packet delay. (Approximate the delay on each wavelength channel as a M/M/1 queueing system.)

Answer:

$$R = 100 \text{ slots.}$$

$$\lambda = 0.2 \text{ pkts/slot.}$$

$$\bar{h} = \frac{27}{11}$$

$$\lambda' = 0.2 \cdot 12 \cdot \frac{27}{11} \cdot \frac{1}{6} = \frac{10.8}{11} \text{ pkts/frame.}$$

$$T = \frac{27}{11} \left(1 + \frac{2 \cdot \frac{10.8}{11}}{2(\frac{0.2}{11})} + 1 + 100 \right) = 382.9 \text{ slots.}$$

- 4.42. Consider a 12-node SC_GEMNET with $w = 3$. The source node is node 0, and it has a multicast group consisting of nodes 2, 4, 6, and 10. Provide an efficient design of this SC_GEMNET. Justify your design. Compare your design with the nonshared case.

Answer:

$$\bar{h}_m = 0.75$$

$$\bar{g}_m = 1.75$$

In the non-shared case:

$$\bar{h}_m = 2.25$$

$$\bar{g}_m = 5.5$$

Optical Access Networks

5.1. Let us consider the costs of deploying a point-to-point network, a curb-switched network, and a passive optical network (PON) shown in Fig. 5.2. Ten homes in a residential network wish to be connected to a central office (CO) via fiber, so as to receive a shared bandwidth connection of 1 Gbps.

Assume the following estimated costs (numbers are for example only):

- (a) Cost of fiber installation: \$200/meter
- (b) Cost of curb switch: \$15000 + \$500 × number of ports
- (c) Cost of passive optical splitter: \$750 + \$100 × number of ports
- (d) Cost of a transceiver (either at CO/OLT or at ONU): \$750
- (e) Distance of homes from CO: 5 km
- (f) Possible location of curb switch/splitter: 50 meters away from each home

Compare the costs of network deployments using each of the three approaches. Please try to justify the importance of a PON.

Answer:

Point-to-point network

Total Cost = Cost of 10 fibers + Cost of 20 transceivers

$$\begin{aligned}\text{Total Cost} &= \$10 * 200 * 5000 + \$20 * 750 \\ \Rightarrow \text{Total Cost} &= \$10,015,000\end{aligned}$$

Curb-switched network

Total Cost = Cost of 1 fiber of length 5 km + Cost of 10 fibers of length 50 meters + Cost of curb switch of 10 output ports + Cost of 11 transceivers

$$\begin{aligned}\Rightarrow \text{Total Cost} &= \$200 * 5000 + \$10 * 50 * 200 + \$15000 + \$500 * 10 + \$750 * 11 \\ \Rightarrow \text{Total Cost} &= \$1,128,250\end{aligned}$$

Passive optical network

Total Cost = Cost of 1 fiber of length 5 km + Cost of 10 fibers of length 50 meters + Cost of passive splitter + Cost of 11 transceivers

$$\Rightarrow \text{Total Cost} = \$200 * 5000 + \$10 * 50 * 200 + \$750 + \$100 * 10 + \$750 * 11$$

$$\Rightarrow \text{Total Cost} = \$1,110,000$$

Thus, out of the three topologies, the passive optical network has the lowest cost of deployment. The Curb-switched network requires active power at the curb and therefore maintenance, which the passive optical network does not require.

Note: variations in assumptions may lead to other solutions also.

5.2. Assume that the PON solution was chosen after analyzing the data in Problem 5.1. Several years after the deployment, the homes wish to upgrade their connection from a 1-Gbps shared line to a 10-Gbps shared line. Let us consider the following possibilities for upgrading the connection:

- (a) *Rate upgrade:* Upgrade all the transceivers from 1 Gbps to 10 Gbps.
- (b) *Spatial upgrade:* Deploy nine additional fibers, so that each ONU can now have one fiber dedicated to it, thus giving it a bandwidth of 1 Gbps, and making the cumulative bandwidth 10 Gbps.
- (c) *WDM upgrade:* Use WDM to support 10 channels of 1 Gbps each on the same fiber. A passive AWG routes each channel to a different ONU.

In addition to the costs in Problem 5.1, assume the following additional cost estimates:

- (a) Cost of AWG router: $\$4000 + \$300 \times \text{number of ports}$
- (b) Cost of a 10-Gbps transceiver: \$2000
- (c) Cost of multiplexing 10 channels at OLT for WDM: \$1000

Compare the costs for each of the three types of upgrades. Analyze the advantages and disadvantages of each approach.

Answer:

Rate upgrade

Cost of upgrade = Upgrade of 11 10-Gig transceivers

$$\Rightarrow \text{Cost} = \$2000 * 11$$

$$\Rightarrow \text{Cost} = \$20000$$

Spatial Upgrade

Cost of upgrade = Cost of 9 additional fibers + Cost of 9 1-Gig transceivers (at OLT)

$$\Rightarrow \text{Cost} = \$5000 * 200 * 9 + \$750 * 9$$

$$\Rightarrow \text{Cost} = \$9,006,750$$

WDM Upgrade

Cost of upgrade = Cost of AWG + Cost of 9 1-Gig transceivers

$$\Rightarrow \text{Cost} = \$4000 + \$300 * 10 + \$750 * 9$$

$$\Rightarrow \text{Cost} = \$13750$$

The cost of upgrading by using a WDM-PON is much less while rate-upgrade has the additional advantage that the maximum line rate is 10 Gig. However incremental upgrade is difficult in rate-upgrade because all the end-user transceivers will have to be changed in one step.

5.3. Consider the IPACT protocol with the *limited service scheme*, and a maximum scheduling timeslot W_{MAX} of 2500 bytes. Draw a timing diagram for a 4-ONU EPON system, similar to the timing diagram is in Fig. 5.12.

Assume the following sequence of requests as shown in Table 5.1. Assume RTTs are constant but different for different ONUs and assume a 5- μ s guard timeslot between data transmissions of different ONUs. The polling order starts with ONU 1.

Table 5.1 Sequence of requests from ONUs.

ONU No.	Request 1	Request 2	Request 3	RTT
1	1800	1000	3000	100
2	1500	1200	4500	200
3	3000	3000	1000	250
4	2700	2500	2500	400

Answer:

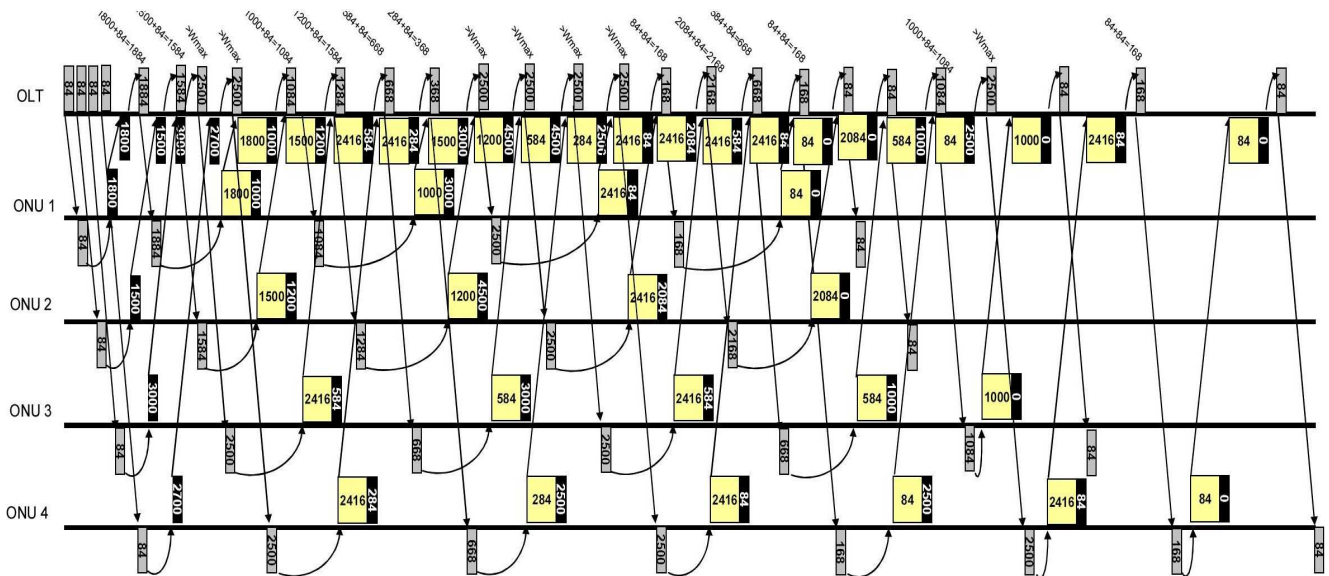


Figure 5.27 Solution for Problem 5.3.

5.4. Assume that, in an EPON, the downstream delay is equal to the upstream delay. When a REPORT message arrives from ONU_i , the OLT's local time is 10. The timestamp on the REPORT message is 2. What is the current local time at ONU_i ? Assume negligible processing time at the OLT.

Answer:

$$RTT = T_{local} - T_{timestamp}$$

$$\Rightarrow RTT = 10 - 2 = 8$$

The current local time at the ONU is:

$$\begin{aligned}
& T_{local} - 0.5 * RTT \\
\Rightarrow T_{current} &= 10 - 0.5 * 8 \\
\Rightarrow T_{current} &= 6
\end{aligned}$$

5.5. Consider IPACT with *limited service scheme*. Consider a maximum scheduling timeslot W_{MAX} of 5000 bytes. The number of ONUs in the EPON is 16. Assume a guard time interval of 1 μs . Compute:

- (a) the maximum cycle time,
- (b) average throughput for each ONU, when all ONUs are active,
- (c) maximum throughput when only one ONU is active, while all the other ONUs are idle,
- (d) average throughput for active ONUs, when 8 ONUs are active, and the remaining 8 are idle.

Answer:

(a) Consider an EPON with the line rate of 1 Gbps:

$$\begin{aligned}
\text{In the limited service scheme } T_{MAX} &= \sum_{i=1}^{i=16} T_{guard} + W_{MAX}/R \\
\Rightarrow T_{MAX} &= 16 * (1 + \frac{(5000+84)*8}{1000}) \\
\Rightarrow T_{MAX} &= 666us
\end{aligned}$$

Note that the 84 bytes above correspond to the length of the REPORT message that each ONU must send.

(b) Each ONU may send 5000 bytes in each cycle time:

Therefore average throughput for each ONU =

$$\begin{aligned}
& \frac{5000*8}{666*10^{-6}} \\
&= 60.06Mbps
\end{aligned}$$

(c) Refer to Figure 5.12:

One ONU is active while all the other 15 ONUs are idle. Even when an ONU is idle, it must be allocated 84 bytes by the GATE message so that it may report the size of the queue.

Assuming that the active ONU will have backlogged queues at all time. Therefore the length of the maximum time cycle is:

$$\begin{aligned}
T_{MAX} &= 16 + \frac{(5000+84)*8}{1000} + 15 * \frac{84*8}{1000} \\
\Rightarrow T_{MAX} &= 66.752us
\end{aligned}$$

Therefore the throughput for the active ONU is:

$$\begin{aligned}
& \frac{5000*8}{66.752*10^{-6}} \\
&= 600Mbps
\end{aligned}$$

The utilization in IPACT with only one active ONU is very poor compared to the line rate. This is because of the overhead of polling each ONU for queue sizes. Subsequent protocols have addressed this issue of improving the utilization for IPACT.

(d) Work on the approach illustrated in problem (c) with 8 active ONUs and 8 inactive ONUs.

5.6. Once a window is granted to an ONU, there are two approaches for transmitting packets in this window. The first approach is to transmit packets in order. This means that, if the first packet in the queue is smaller than the remaining timeslot, this packet will not be transmitted, which is also known as *head-of-line blocking*. Another approach is to use techniques such as bin-packing algorithms to ensure that as many packets may be transmitted in the allocated timeslot as possible. Contrast and compare these two approaches. Write an algorithmic description for the second approach.

Answer:

Head-of-line queuing is simpler to implement as the router will look at only the first packet in the queue before sending it out to the output buffer. Bin-packing (for discrete sized packets) is an NP-hard problem, but several approximation algorithms are available for it. Implementing a bin-packing algorithm therefore may lead to better channel utilization, but is harder to design in hardware. Therefore, most queuing practices currently are based on the head-of-line queuing approach.

- 5.7. In a PON, the downstream traffic is point-to-multipoint, whereas the upstream traffic is point-to-point. Discuss the implications of such an architecture from a network security point of view.

Answer:

In downstream direction, PONs are susceptible to eavesdropping because all ONUs receive the downstream traffic, and therefore an ONU can eavesdrop traffic destined to other users if the traffic is not encrypted. Another security problem is theft-of-service, when an ONU registers itself in the network stealing the identity of another ONU in the network which is temporarily turned off. This issue may be resolved using a public-private key encryption while registering the ONU in the network. Denial-of-service attacks are also possible in which an ONU does not respect the transmission window in the GATE message and turns its laser ON when some other ONU has been allocated a transmission window.

- 5.8. Draw a transition state diagram for discovery handling, GATE message handling, and REPORT message handling, as defined in the MPCP protocol for an ONU.

Answer:

Please refer to Fig. 5.28 for the solution.

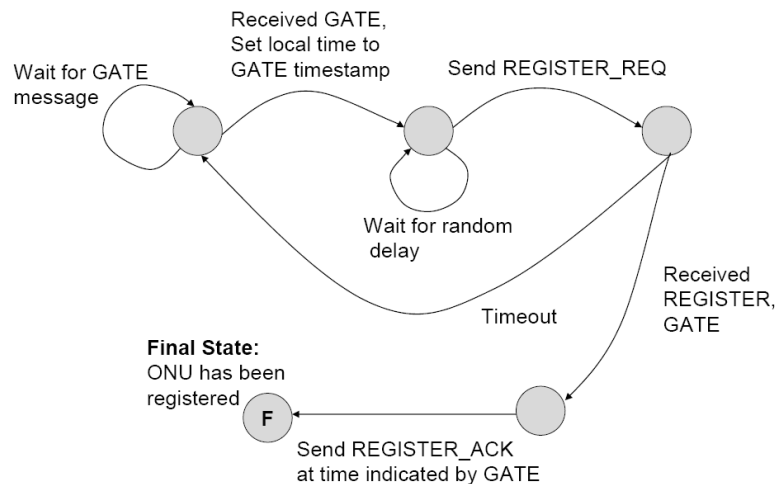


Figure 5.28 State Diagram for ONU discovery.

- 5.9. Consider the PON network architecture. We would like to modify the PON architecture to build a $(1 + 1)$ protected PON, so that if any component, namely, the OLT or ONU transceivers, fiber, or splitter fails, the PON network is not disrupted at all. Draw a suitable architecture for such a $(1 + 1)$ protected PON.

Answer:

Please refer to Fig. 5.29 for the solution.

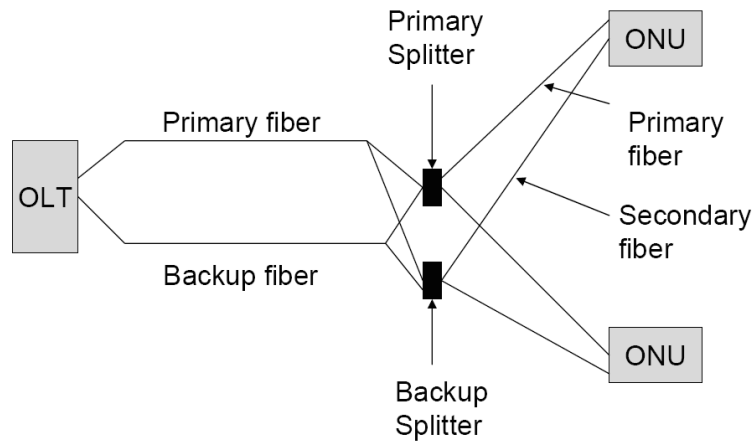


Figure 5.29 (1+1) protected PON.

- 5.10.** Consider a PON which has 32 ONUs. Let the transmission power of the OLT transceiver be 0.01 mW, and the receiver be able to detect signals of at least 0.0001 mW for a reasonable bit-error rate. Assume that the splitter is 10% lossy. The optical fiber has an attenuation of 0.2 dB/km. What is the maximum possible distance of an ONU from the OLT. What is the RTT corresponding to this distance.

Answer:

Maximum power loss allowed in this EPON system=

$$10\log_{10} \frac{0.01}{10^{-4}} = 20dB$$

Power loss at the splitter (Splitting signal into 32 output channels and 10% inherent power loss)
=

$$10\log_{10} \frac{0.9}{32} = -15.5dB$$

Therefore, the remaining power loss that we may have over the fiber = 4.5 dB

Therefore. the maximum length of fiber that we may have =

$$4.5/0.2 = 22.5km$$

RTT corresponding to this distance = $2 * 22.5 * 5\mu s/km$
= 225 ms.

- 5.11.** Consider the *near-far problem* in an EPON with four different ONUs. The distances of different ONUs from the OLT are: 1 km for ONU_1 , 5 km for ONU_2 , 10 km for ONU_3 , and 20 km for ONU_4 .

Assume that each ONU's transmission power is 0.01 mW. Also assume that the splitter is lossless, and the attenuation in the fiber is 0.2 dB/km.

Let the adjustment time for the OLT's transceiver to determine the 0-1 threshold for a signal to be linearly dependent on the difference between the power of the current signal and the previous signal, and let it be defined by the following equation:

$$t_{0-1} = 500 \times [P_{new}[\mu W] - P_{old}[\mu W]] \quad ns$$

What is the minimum value of the guard time interval, in order to allow for the adjustment time of the OLT's transceiver?

Answer:

Nearest ONU = 1km

Farthest ONU = 20km

Power of transmitted signal received from nearest ONU =

$$0.01 * 10^{\frac{-0.2}{10}} = 9.5 \mu W$$

Power of transmitted signal received from farthest ONU =

$$0.01 * 10^{\frac{-0.2*20}{10}} = 4 \mu W$$

Therefore, the maximum 0-1 threshold adjusting time for the signal =

$$500 * 5.5 = 2.75 \mu s$$

Therefore, the minimum value of guard time = 2.75 μs .

5.12. What is the *light-load penalty*. Discuss the possible solutions for avoiding the light-load penalty.

Answer:

Discussed in Section 5.3.5 (in Page 253) of textbook.

5.13. Identify whether the following statements are *true* or *false*. Justify your answer.

- (a) The Ethernet PON is based on the CSMA/CD protocol.
- (b) Fixed timeslot assignment (synchronous TDMA) is an efficient scheme in EPON.
- (c) Dynamic slot assignment in EPON is based on a distributed (decentralized) approach.
- (d) The GFP-PON uses ATM cells for the MAC frames.

Answer:

- (a) False, IEEE 802.3ah does not use CSMA/CD, it is based on MPCP.
- (b) False, synchronous TDMA leads to very poor utilization.
- (c) False, dynamic slot assignment in EPON is centralized at the OLT.
- (d) False, the GFP-PON uses GFP frames which may adapt traffic from the higher layers over SONET/SDH.

Optical Metro Networks

- 6.1. For interconnected double rings, what switching strategy should we adopt for small, medium, and large traffic-grooming ratio? Explain why.

Answer:

We compared several ways to decompose the inter-ring traffic for strategy 4 and tried to find the best operating point. For example, we place only very regulated traffic on the logical ring but all remaining traffic on the local rings. We were able to achieve very low cost on the logical ring but this increased the local ring cost too much so that the overall network cost increased. To conclude, if the grooming ratio is small or very large, switch all the inter-ring traffic optically. For networks with medium grooming ratio, consider using TDM-level switching. Thus, in order to design an OXC that has TDM-switching functionality and is intended only for interconnecting rings, the grooming ratio should be between 8 and 64.

- 6.2. Consider an optical metro ring networks consisting of five nodes, where node 0 is a hub node. OC-48 rings can be supported, which can groom four OC-12 connections. Traffic originates only from node 0 and is transmitted to the other nodes based on the following 1×5 traffic vector:

$$(0 \ 2 \ 3 \ 3 \ 4)$$

The entry (i, j) in the vector represents the number of OC-12 connections of traffic demand from node i to node j . There are three wavelengths available. Design the rings so that the minimum number of ADMs is employed in the network.

Answer:

The configured structure is similar to Fig. 6.11(c). But we put node #0 at the interconnected position. We then put node #1 and node #4 at the left ring, and put node #1 and #3 at the right ring. Under the constraint of 3 available wavelengths, the total number of ADM's is 12 without any wavelength converter.

- 6.3. For the network topology solution obtained in Problem 6.2, draw the node architecture of node 3. Indicate which switches are in the cross state and which are in the bar state.

Answer:

Based on the structure, there is a wavelength which carries a channel from node #0 to node #4 through node #1. On this OC-12 channel, the switch at node #1 needs to be set to the bar state. All other switches have to be set to cross state for locally dropping.

- 6.4. Consider the network in Fig. 6.32. The network is an unidirectional ring with two wavelengths (λ_1 and λ_2) and two timeslots on each wavelength channel (C_1 and C_2). Assume that the traffic matrix $T = [t_{ij}]$ is 3×3 .
- Write the objective function for node 1.
 - Write the traffic-load constraints for the connection between node 1 and node 2 (assume $t_{12} = 3$).
 - Write the channel-capacity constraints for the physical link $L_{12}^{C_1 \lambda_1}$.
 - Write the transmitter and receiver constraints for node 1. What is the grooming ratio?

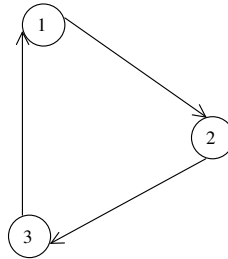


Figure 6.32 3-node unidirectional network.

Answer:

Please refer to Formulation 6.2.

- 6.5. The bidirectional ring network in Fig. 6.33 has uniform traffic request, with grooming ratio 3. Each request is one unit of sub-channel capacity. Find a feasible solution to put all the connection requests on two wavelengths such that the minimum number of ADMs are used. Verify the minimum ADMs used with the tabular result given by the ILP solver using the simulated-annealing-based traffic-grooming algorithm.

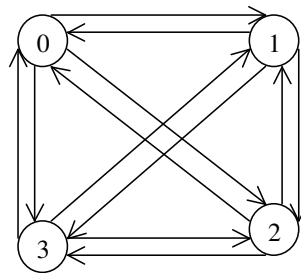


Figure 6.33 4-node bidirectional network.

Answer:

Based on Table 6.3, under the constraint of grooming ratio 3 and 4 nodes, the minimum number of ADMs is 7 at simulated annealing algorithm in an unidirectional ring with uniform traffic. When the bidirectional network could be regarded as two unidirectional networks, each wavelength is assigned to one of two unidirectional networks. At the same direction, we could group these requests into a wavelength by using a logic ring on the network. For each link, it has at most 3 logic rings allowed. Under the solution, we have to employ 12 ADMs at least.

- 6.6. Consider the bidirectional single-point-connected double-ring network shown in Fig. 6.34. Given

the following traffic matrix

$$A = \begin{pmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 1 & 0 & 4 & 3 & 2 & 1 & 7 \\ 3 & 8 & 0 & 5 & 7 & 9 & 3 \\ 5 & 7 & 11 & 0 & 1 & 4 & 6 \\ 7 & 6 & 3 & 8 & 0 & 12 & 2 \\ 9 & 5 & 8 & 2 & 13 & 0 & 9 \\ 11 & 4 & 14 & 7 & 6 & 5 & 0 \end{pmatrix}$$

Show:

- (a) the decomposed traffic matrix on each ring for strategy 1 (DXC), and
- (b) one reasonable decomposition of the traffic matrix on each ring for strategy 2 (OXC).

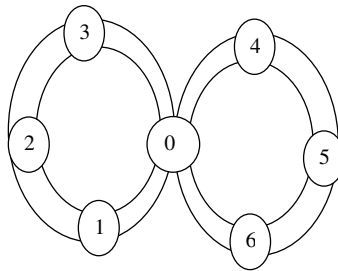


Figure 6.34 Single-point-connected double-ring network.

Answer:

In Formulation 6.1, please replace t_{ij} with t'_{ij} to get the decomposed traffic matrix. For the strategies 1 and 2, please refer to Section 6.4.2.

6.7. Given the following traffic matrix for an unidirectional ring:

$$A = \begin{pmatrix} 0 & 1 & 3 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Assume $c = 1$ and transmission in the clockwise direction only. Give a virtual connection assignment. Calculate the number of ADMs and the number of wavelengths required.

Answer:

Assume that nodes #0, #1, #2 and #3 are aligned on the ring orderly. Because of $c = 1$, we need 5 wavelengths and 8 ADMs at least.

6.8. Describe the relation between grooming ratio and the required ADMs of strategy 1 (DXC) compared to the strategies 2 and 3 (OXC) under uniform traffic. Explain why this is so.

Answer:

Please refer to Section 6.4.3.

6.9. For the mathematical problem formulation for traffic grooming in a single-hub multihop networks, extend it to the bidirectional formulation for uniform traffic, and analyze its complexity in terms of big- O notation.

Answer:

Please refer to Formulation 6.2.

- 6.10.** Same question as Problem 6.9, for the mathematical problem formulation for traffic grooming in a single-hop unidirectional ring, extend it to the bidirectional formulation for uniform traffic, and analyze its complexity in terms of big- O notation.

Answer:

Please refer to Formulation 6.1.

Routing and Wavelength Assignment

- 7.1.** In the linear programming formulation in Section 7.2.1, we minimize F_{\max} . What is F_{\max} ? Why should we minimize F_{\max} ?

Answer:

Let λ_{sd} denote the traffic (in terms of a lightpath) from any source s to any destination d . We consider at most one lightpath from any source to any destination; hence $\lambda_{sd} = 1$ if there is a lightpath from s to d ; otherwise $\lambda_{sd} = 0$. Let F_{ij}^{sd} denote the traffic (in terms of number of lightpaths) that is flowing from source s to destination d on link ij . Then F_{\max} is defined as:

$$F_{\max} \geq \sum_{s,d} F_{ij}^{sd} \quad \forall \quad ij \quad (7.1)$$

Minimizing F_{\max} will minimize the number of lightpaths passing through each link, which is the objective of the linear programming formulation in Section 7.2.1.

- 7.2.** Let G be a graph with $V(G) = v_1, v_2, \dots, v_n$, where $\deg(v_i) \geq \deg(v_{i+1})$ for $i = 1, 2, \dots, n-1$. Prove that $\chi(G) \leq \max_{1 \leq i \leq n} \min(i, 1 + \deg(v_i))$.

Answer:

As proved in Section 7.2.4, some particular sequential vertex coloring will yield a $\chi(G)$ coloring, which we called optimal sequential vertex coloring.

Other sequence yields coloring $\geq \chi(G)$.

Assuming the k th vertex has $k = \deg(v_k)$, $\min(i, 1 + \deg(v_i)) = i$ for $i \leq k$; and $\deg(v_i) + 1$ for $i > k$. Then, add vertex according to the order of $\langle v_1, v_2, \dots, v_n \rangle$. For each vertex $i \leq k$, assign a new color for it. There will be k colors when vertex k is added. For each vertex $i > k$, we can always find a color for it from $k - (1 + \deg(v_i))$ colors. Keep adding vertex until all vertices are added, which will yield k coloring $\geq \chi(G)$.

From above procedure, it is obvious that $\chi(G) \leq k = \max_{1 \leq i \leq n} \min(i, 1 + \deg(v_i))$.

- 7.3.** Given a graph $G = (V, E)$. Define

$$k = \max_{1 \leq i \leq n} 1 + \deg_{\langle v_1, v_2, \dots, v_n \rangle}(v_i) \quad (7.2)$$

where $v_1, v_2, \dots, v_n \in V$ and $\langle v_1, v_2, \dots, v_n \rangle$ is a vertex ordering. Define *smallest-last* (SL) vertex ordering $\langle v_1, v_2, \dots, v_n \rangle$ to be the vertex ordering such that $\deg(v_{i+1}) \leq \deg v_i$, for $1 \leq i \leq n$. Show that, over all the $n!$ possible vertex orderings, the SL vertex ordering minimizes the value of k .

Answer:

Open for readers.

- 7.4.** We know that the static lightpath establishment (SLE) problem is NP-complete. Show a simple transformation that transforms a SLE problem into a graph coloring problem.

Answer:

Assigning wavelength colors to different lightpaths, so as to minimize the number of wavelengths (colors) used under the wavelength-continuity constraint, reduces to the graph coloring problem, as stated below.

- (a) Construct a graph $G(V, E)$, so that each lightpath in the system is represented by a node in graph G . There is an undirected edge between two nodes in graph G if the corresponding lightpaths pass through a common physical fiber link.
- (b) Color the nodes of the graph G such that no two adjacent nodes have the same color.

This problem has been shown to be NP-complete.

- 7.5.** Consider the network shown in Fig. 7.1. Let the connection requests be as follows:

B-H, A-E, B-D, D-F, B-F, C-E, C-H, A-G, A-C

Set up lightpaths to satisfy the above connection requests using at most three wavelengths per link. Assume no wavelength conversion.

Answer:

λ_1 :

A \rightarrow 1 \rightarrow 5 \rightarrow 8 \rightarrow 9 \rightarrow E; B \rightarrow 7 \rightarrow 10 \rightarrow D; C \rightarrow 7 \rightarrow 8 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow H;

λ_2 :

A \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow G; B \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 4 \rightarrow F; C \rightarrow 7 \rightarrow 10 \rightarrow 9 \rightarrow E;

λ_3 :

A \rightarrow 1 \rightarrow 5 \rightarrow 8 \rightarrow 7 \rightarrow C; B \rightarrow 6 \rightarrow 1 \rightarrow 2 \rightarrow H; D \rightarrow 10 \rightarrow 9 \rightarrow 8 \rightarrow 4 \rightarrow F.

- 7.6.** Consider the network in Fig. 7.1, and the following lightpaths:

- (a) C-7-8-9-E
- (b) A-1-5-8-9-E
- (c) H-2-1-5-8-7-C
- (d) B-6-7-8-9-E
- (e) A-1-6-7-10-D
- (f) G-3-2-1-6-B
- (g) H-2-3-4-F

Color the lightpaths using the minimum number of wavelengths.

Answer:

4 colors are used. Multiple wavelength assignments are possible. We only give one possible solution.

color 1: (a),(e);
 color 2: (b),(f);
 color 3: (d),(g);
 color 4: (c).

7.7. Consider the NSFNET physical topology shown in Fig. 9.1. Remove the nodes WA, CO, NE, and GA. Suppose the connection requests are as shown in Fig. 7.14. What is the minimum number of wavelengths needed to satisfy all the connection requests?

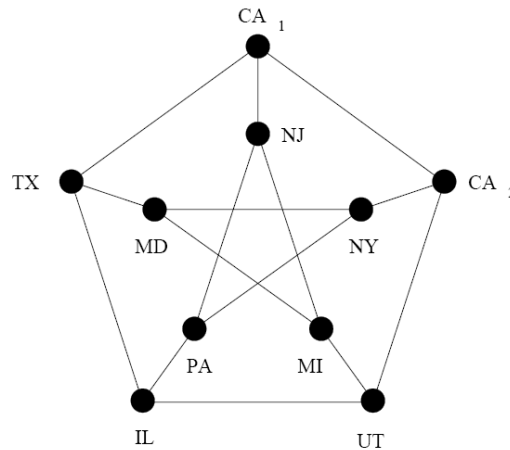


Figure 7.14 Connection requests.

Answer:

3 wavelengths are needed. There may be several possible solutions. Here one solution is listed.

λ_1 :

CA2→UT→MI→NJ;
 CA2→CA1→TX;
 TX→MD→NY→PA→IL;
 MI→MD→NY;

λ_2 :

CA2→CA1;
 TX→MD;
 UT→MI→NJ→PA→IL;
 PA→NY→MD→MI→NJ;

λ_3 :

CA1→CA2;
 UT→MI; MI→NJ;
 PA→NY;
 PA→NJ; NY→MD;
 CA1→TX→MD→NY.

7.8. Compare the characteristics of various routing schemes.

Answer:

Fixed Routing

Definition: Offline; Routing is predefined, e.g, shortest-path, simple;

Advantage: Minimize average hops per connection;

Disadvantage: High blocking probability; More wavelengths needed; Hard to handle link failure.

Fixed-Alternate Routing

Definition: Offline; Multiple link-disjoint routes are predefined;

Advantage: Simple; Some degree of tolerance upon link failure;

Disadvantage: Connection blocking probability is reduced but it can still be high.

Adaptive Routing

Definition: Online; Routing decision is made based on the real-time link status, e.g, congestion of the link;

Advantage: More reduction of blocking probability; More tolerance upon link failure;

Disadvantage: Computational complexity, need simplified version, e.g., only examines the first k links on each path.

Fault-Tolerant Routing

Definition: Offline or Online;

Advantage: Backup path is computed besides the primary path;

Disadvantage: Good for fault tolerance.

7.9. Compare the characteristics of various wavelength-assignment schemes.

Answer:

1. Random Wavelength Assignment: This scheme finds the set of all wavelengths available on the required route, and then chooses one randomly.
2. First-Fit: All wavelengths are numbered. The available wavelength with least number is chosen. Compared to random assignment, First-Fit requires no global information, and the computation cost is lower. It performs well in terms of blocking probability and fairness.
3. Least-Used: It selects the wavelength that is the least used in the network, thereby attempting to balance the load among all the wavelengths. The scheme ends up breaking the long wavelength paths quickly. It performs worse than 1, and requires global information and high computation complexity.
4. Most-Used: It is opposite of 3. It outperforms 3 significantly. The communication overhead, storage, and computation cost are similar to those in 3. It also slightly outperforms 2 by packing connections into fewer wavelengths and conserving the spare capacity of less-used wavelengths.
5. Min-Product: It works in multi-fiber networks to pack wavelengths into fibers, thereby minimizing the number of fibers in the network. It does not perform as well as the multi-fiber version of 2 in which the fibers and wavelengths are ordered.
6. Least-Loaded: It works in multi-fiber networks to select the wavelength that has the largest residual capacity on the most-loaded link along route p . When used in single-fiber networks, it reduces to 2. It outperforms 5 in terms of blocking probability in a multi-fiber networks.
7. Max-Sum: It is applied to multi-fiber and single-fiber cases. It considers all possible paths in the network and attempts to maximize the remaining path capacities after a lightpath is established.
8. Relative Capacity Loss (RCL): It is based on 7 but it chooses wavelength j to minimize the relative capacity loss. Both 7 and 8 may be used for non-uniform traffic by taking a weighted sum over the capacity losses. 8 has been shown to perform better than 7.

9. Distributed Relative Capacity Loss: It is well suited for a distributed adaptive routing environment. Unlike 7 and 8 which need global knowledge of the network state, this scheme maintains information about routing and RCL table, which are updated when a connection request arrives. The schemes from 1 to 9 attempt to minimize the blocking probability. However schemes such as Wavelength Reservation and Protecting Threshold attempt to protect longer paths. When the two schemes are used, the overall blocking probability performance in the network may be higher, but a greater degree of fairness can be achieved, in that blocking probability of longer paths will not be significantly higher.

- 7.10. Consider the addition of a ninth unidirectional lightpath in Fig. 7.2 between the left-most node (as source) and the right-most node (as destination). Let the route of this lightpath follows the lower three links. Draw the corresponding auxiliary graph, and determine the number of wavelengths needed to solve the problem.

Answer:

Number of wavelengths needed = 4.

Please refer to Fig. 7.15.

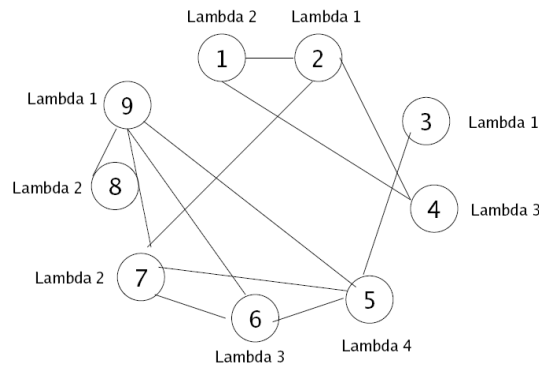


Figure 7.15 Solution to Problem 7.10.

- 7.11. Repeat the above problem where the ninth lightpath is routed over the top three links.

Answer:

Number of wavelengths needed = 4.

Please refer to Fig. 7.16.

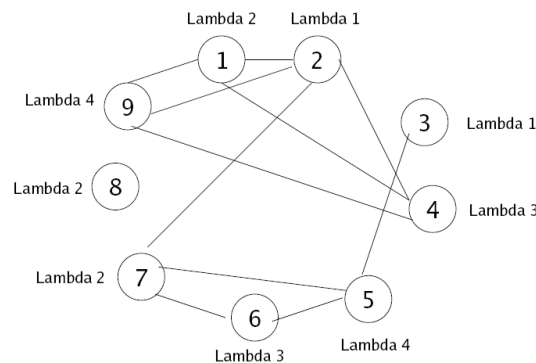


Figure 7.16 Solution to Problem 7.11.

- 7.12.** Choose a different route for the ninth lightpath from the ones chosen in the previous two problem. Repeat the analysis.

Answer:

Number of wavelengths needed = 4.

Please refer to Fig. 7.17.

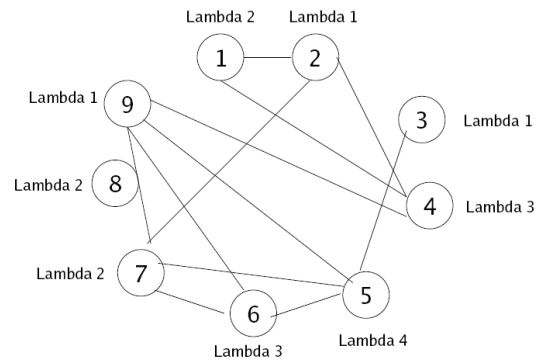


Figure 7.17 Solution to Problem 7.12.

Here, the ninth lightpath is routed over the bottom three links and it shares links with lightpaths 5, 6 and 7.

Elements of Virtual-Topology Design

8.1. What are the advantages of embedding a virtual topology on a physical topology?

Answer:

The virtual topology, which is operated as a packet-switched network and which consists of a set of “lightpaths,” is set up to exploit the relative strengths of both optics and electronics viz., packets of information are carried by the virtual topology “as far as possible” in the optical domain using optical circuit switching, but packet forwarding from lightpath to lightpath is performed via electronic packet switching, whenever required. The collection of lightpaths, i.e., the virtual topology, is also referred to as a “Lambda Grid”, or just Grid, for short.

8.2. What is the wavelength-continuity constraint? Which of the constraint equations relate to the wavelength-continuity constraint?

Answer:

If a lightpath is constrained to using single wavelength without wavelength converter, that is called the wavelength-continuity constraint.

Equation 8.13 is the wavelength-continuity constraint, which requires that a lightpath be of one color only.

8.3. Consider the NSFNET physical topology shown in Fig. 8.1. Remove nodes WA, CO, NE, and GA.

- (a) Draw the new physical topology.
- (b) Set up lightpaths on the new topology so that the resulting virtual topology is the Petersen graph. In your virtual topology, what is the maximum number of wavelengths used on any link in the network?
- (c) Show the details of the UT (Utah) switch (similar to the one in Fig. 8.4.
- (d) Assume a uniform traffic matrix, i.e., equal amount traffic between any two nodes. Assume that packets are routed via the shortest path. Calculate the average packet hop distance when packets are routed over the physical topology and when packets are routed over the virtual topology.

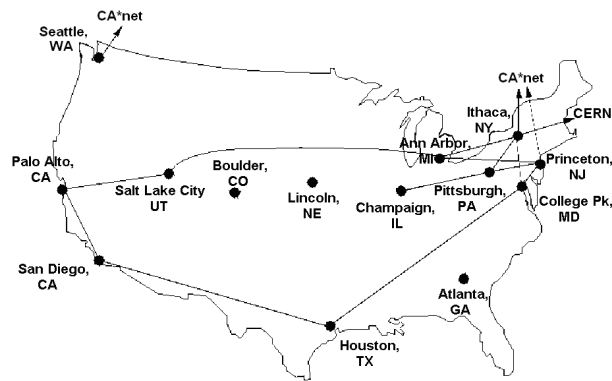


Figure 8.12 Revised NSF-T1 physical topology.

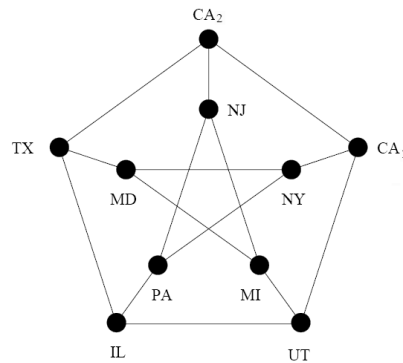


Figure 8.13 Petersen graph.

Answer:

a) See Fig. 8.12. b) See Fig. 8.13. In Petersen graph, the maximum number of wavelengths is 2 on any link. c) Please refer to Fig. 8.4. d) For the virtual topology, the average hop distance is 1.5. For the case of the physical topology, it is calculated as follows:

Average hop distance from UT to other nodes = $\frac{1+2+3+1+2+2+3+4+3}{9} = 2.33$. Similarly,

Average hop distance from MI to other nodes = 2

Average hop distance from NY to other nodes = 2.44

Average hop distance from CA1 to other nodes = 2.66

Average hop distance from CA2 to other nodes = 2.77

Average hop distance from TX to other nodes = 2.55

Average hop distance from MD to other nodes = 2.22

Average hop distance from NJ to other nodes = 1.88

Average hop distance from PA to other nodes = 2.33

Average hop distance from IL to other nodes = 3.2

Therefore, average hop distance =

$$\frac{2.33+2+2.44+2.66+2.77+2.55+2.22+1.88+2.33+3.2}{10} = 2.71$$

- 8.4. Give a set of lightpaths which embeds a 4×4 Manhattan Street Network with bidirectional links onto the NSFNET physical topology shown in Fig. 8.2. How many wavelengths are required for this embedding?

Answer: The embedded Manhattan street network is shown in Fig.8.14. The required number

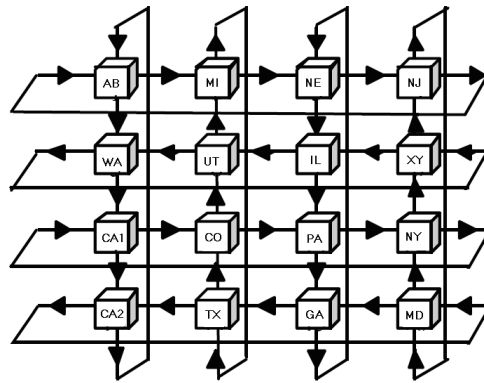


Figure 8.14 4×4 embedded Manhattan street network.

of wavelengths should be enough to support all lightpaths on the virtual network. But the virtual network is rooted from its physical network. With the constraint of Equation 8.14, we need at least 4 wavelengths for this embedding with bidirectional links (see Fig. 8.15).

- 8.5.** For the physical NSFNET topology shown in Fig. 8.2, find a logical ring embedding which uses only one wavelength.

Answer:

Please refer to the solution above.

- 8.6.** The embedding shown in Fig. 8.6 assumes that all of the local laser-filter pairs at a node operate on different wavelengths. What are the advantages of this design? What are the disadvantages?

Answer:

Please refer to the example in Section 8.2.3.

- 8.7.** In Fig. 8.10, at low loads, the average delay in a network employing full WDM is more than the average delay in a network with no WDM. Why?

Answer: This is because, in the full WDM case, the shortest path along the physical topology

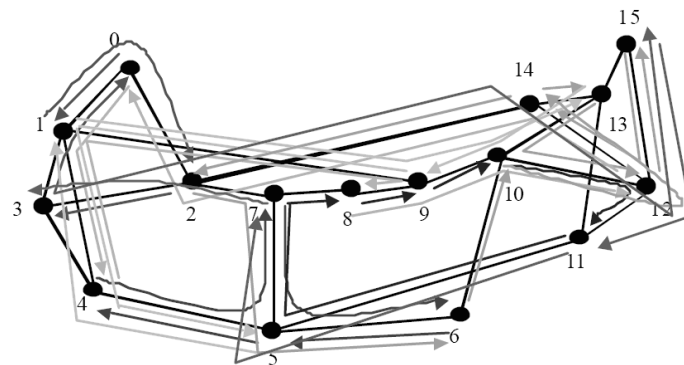


Figure 8.15 An example of an embedded Manhattan street network on NSFNET physical network.

cannot always be chosen because of the virtual-topology embedding, so that some packets may have to travel longer distances, in general. However, the scaleup in the virtual topology is much

more than that for the other two schemes; so the addition of switches at intermediate nodes to perform wavelength routing provides a significant improvement in throughput for the network.

Advanced Topics in Virtual-Topology Optimization

9.1. In this chapter, average packet hop distance is used as the objective function. Why?

Answer:

The average packet hop distance is defined as the number of lightpaths that a packet has to traverse on average, and is a function of the virtual topology. We simplify the objective function to minimize the average packet hop distance (which is inversely proportional to the network throughput under balanced network flows and which is a linear objective function). By changing the objective function to minimizing the hop distance and by relaxing the wavelength-continuity constraints (i.e., assuming wavelength converters at all nodes), we demonstrate that the entire optical network design problem can be linearized and hence, it can be solved optimally.

9.2. Heuristics **MaxSingleHop** and **MaxMultiHop** assign lightpaths based on traffic flows. **MaxSingleHop** adjusts the traffic matrix after assigning a lightpath, while **MaxMultiHop** does not. Why?

Answer:

In a packet-switched network, the traffic carried by a link may include forwarded traffic as well as traffic originating from that node. Intuitively, it seems that any lightpath-establishment heuristic which accounts for the forwarded traffic that the lightpath will carry should provide better performance than a heuristic which only tries to maximize the single-hop traffic. This intuition led to the derivation of the current heuristic. The heuristic starts with the physical topology as the initial virtual topology, and attempts to add more lightpaths one by one.

9.3. For this exercise, consider the physical topology shown in Fig. 9.14 and the traffic matrix shown below. Assume two channels per link, where each channel has a capacity of five units. Use the **MaxSingleHop** heuristic to set up lightpaths. Assume that each node has sufficient number of transceivers. The traffic matrix is:

$$T = \begin{bmatrix} 0 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

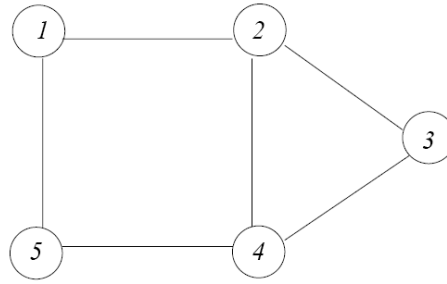


Figure 9.14 Physical network topology.

Answer:

Each link has two channels, each with 5 units of capacity.

Based on these conditions, the set of lightpaths are as follows:

- 1→2→3: 5;
- 1→2: 4; 3→4: 2;
- 3→4→5: 3;
- 3→2→4→5→1: 1.

- 9.4.** Draw the virtual topology of the network shown in Problem 9.3. Calculate V_{ij} , λ_{ij}^{sd} , and p_{mn}^{ij} for this network.

Answer:

The virtual topology is straightforward gotten based on the set of lightpaths above. Accordingly, the V_{ij} , λ_{ij}^{sd} , and p_{mn}^{ij} can be calculated based on the virtual topology.

- 9.5.** Derive Eqn. (9.1). Show that the inequality holds for the network shown in Problem 9.3. When will the equality hold?

Answer:

When the hops are minimized, the equality is held.

For the case in Problem 9.3, the inequality holds because the number of hops is not minimized.

- 9.6.** Consider the network shown Fig. 9.15 with two transmitters and two receivers per node, two wavelengths, capacity of each wavelength equal to 10 units, and the following traffic matrix:

$$\Lambda = \begin{bmatrix} 0 & 2 & 2 & 1 & 4 & 3 \\ 1 & 0 & 4 & 4 & 3 & 2 \\ 3 & 2 & 0 & 6 & 2 & 1 \\ 1 & 1 & 2 & 0 & 1 & 7 \\ 3 & 5 & 2 & 1 & 0 & 1 \\ 2 & 4 & 5 & 3 & 2 & 0 \end{bmatrix}$$

Determine a set of lightpaths using the **MaxSingleHop** heuristic.

Answer: The lightpaths with their routes and wavelength-assignment are shown in Table 9.1. In this solution traffic grooming is also considered.

- 9.7.** For the network in Problem 9.6, run the **MaxMultiHop** heuristic to determine a set of lightpaths.

Answer:

Same as above.

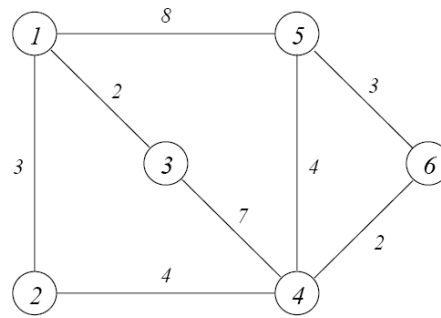


Figure 9.15 Physical network topology.

- 9.8.** Assume that the network equipment cost budget is \$1,000,000. Using Table 9.1, find the network configurations (i.e., number of transceivers and wavelengths) that can be supported. Which network configuration maximizes the total network throughput?

Answer:

If the network equipment cost budget is bounded to \$1,000,000, there are 6 kinds of configurations based on Table 9.1.

Using 4 transceivers and 6 wavelengths maximizes the total network throughput.

- 9.9.** Assume the Petersen graph as the physical topology. Assume that the network can support 10 wavelengths. Now assume that we embed a complete graph on 10 nodes as the virtual topology. (Use shortest-path routing to embed the virtual topology). Using the cost model described in Section 9.4, compute the cost to build the network. Which type of component has the highest networkwide cost?

Answer:

This architecture, the famous ‘Petersen Graph’ is shown in Fig. 4.11 in PP. 218. The average hop count is 1.5, but the worst case is only 2. We use the average hop count to compute the cost.

In the example, $M = 15$ links, $N = 10$ nodes, $W = 10$ wavelengths, $H_P = 1.5$ average hops. The approximate number of transceivers per node is:

$$T_i = R_i \approx \frac{MW}{NH_P} = 10 \quad (9.1)$$

Then, according to Equation 9.29, the cost = 2,305,000 \$.

The wavelength converters represent the highest portion of the cost, with around 1,300,000 \$.

Table 9.1 Lightpath provisioning using MaxSingleHop.

Demand	Route	Wavelength
4→6	4→6	4→6 (λ_1)
3→4	3→4	3→4 (λ_1)
5→2	5→1→2	5→1 (λ_1), 1→2 (λ_1)
6→3	6→5→1→3	6→5 (λ_1), 5→1 (λ_1), 1→3 (λ_1)
1→5	1→5	1→5 (λ_2)
2→3	2→1→3	2→1 (λ_1), 1→3 (λ_1)
2→4	2→4	2→4 (λ_1)
6→2	6→4→2	6→4 (λ_2), 4→2 (λ_2)
1→6	1→5→6	1→5 (λ_2), 5→6 (λ_2)
2→5	2→4→5	2→4 (λ_1), 4→5 (λ_1)
3→1	3→1	3→1 (λ_2)
5→1	5→1	5→1 (λ_2)
6→4	6→4	6→4 (λ_2)
1→2	1→2	1→2 (λ_2)
1→3	1→3	1→3 (λ_2)
2→6	2→4→6	2→4 (λ_1), 4→6 (λ_1)
3→2	3→1→2	3→1 (λ_2), 1→2 (λ_2)
3→5	3→4→5	3→4 (λ_1), 4→5 (λ_1)
4→3	4→3	4→3 (λ_2)
5→3	5→4→3	5→4 (λ_2), 4→3 (λ_2)
6→1	6→4→2→1	6→4 (λ_2), 4→2 (λ_2), 2→1 (λ_2)
6→5	6→5	6→5 (λ_1)
1→4	1→3→4	1→3 (λ_1), 3→4 (λ_1)
2→1	2→1	2→1 (λ_1)
4→1	4→3→1	4→3 (λ_2), 3→1 (λ_2)
4→2	4→2	4→2 (λ_2)
4→5	4→5	4→5 (λ_1)
5→4	5→4	5→4 (λ_2)
5→6	5→6	5→6 (λ_1)

Chapter 10

Wavelength Conversion

- 10.1.** What are the different methods by which we can increase the capacity of a WDM optical network? Also, mention the changes in the network that will be required to implement the method.

Answer:

Increasing the capacity of a WDM optical network can be achieved using one of the following ways:

- 1) Increasing the number of channels in the fiber;
- 2) Increasing number of fibers;
- 3) Increasing the throughput per channel by employing wavelength conversion in the network;
- 4) Increasing throughput by using traffic grooming.

- 10.2.** Suppose we have an optical network N_1 with one fiber between adjacent nodes in the physical topology and *four* wavelengths per fiber. Network N_1 does not allow wavelength conversion. Now consider another network N_2 with *four* fibers between adjacent nodes in the physical topology and one wavelength per fiber. Let N_3 be a network similar to N_1 , but with full wavelength conversion. Assume that connection requests are set up dynamically. Let p_1 , p_2 , and p_3 be the average blocking probabilities of networks N_1 , N_2 , and N_3 , respectively. What can we say about how p_1 , p_2 , and p_3 compare with one another?

Answer:

N_1 : Network with one fiber 4 wavelengths/fiber in fiber and no wavelength conversion;

N_2 : Network with 4 fibers and 1 wavelength/fiber and no wavelength conversion;

N_3 : Network with one fiber 4 wavelengths/fiber in fiber and with wavelength conversion.

If the four links in N_2 contain the same wavelengths like in N_1 , i.e. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, then we expect p_1 and p_2 to be equivalent because the same wavelengths exist between each two nodes. Otherwise, N_2 can achieve lower blocking probability, i.e. p_2 can be less than p_1 . For network N_3 , and because of using wavelength conversion, we expect lower blocking probability than in N_1 and N_2 , i.e. p_3 is less than p_1 and p_2 .

- 10.3.** For a network with *two* wavelengths, show an example of dynamic connection setup requests which can be satisfied with wavelength conversion and cannot be satisfied otherwise.

Answer:

The network in Fig. 10.13 shows an example where we can provision a connection only when wavelength conversion is used. In this figure, connection 1-4 will not succeed in the case of wavelength continuity constraint because the network does not have a single wavelength on both links (1,2) and (2,4). Otherwise, if wavelength conversion is available, then we can set up the connection by using λ_2 on link (1,2) and λ_1 on link (2,4)

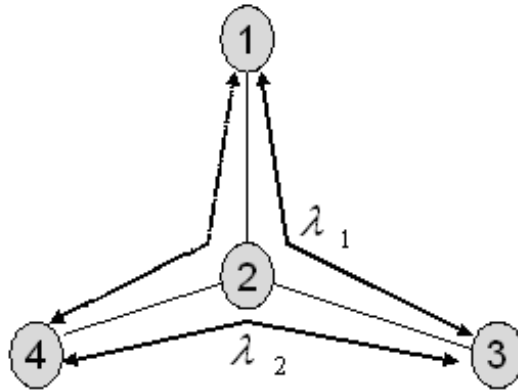


Figure 10.13 An example network.

- 10.4.** Given an optical network with the facility of recoloring existing lightpaths, show that such a network may block a connection request which could have been satisfied if wavelength conversion was allowed.

Answer:

If the network is given same as one in Problem 10.8. Connection requests are give as follows (same as the set (a) in Problem 10.8):

- λ_1 from input 1 to output 1
- λ_2 from input 1 to output 1
- λ_3 from input 1 to output 2
- λ_1 from input 2 to output 1
- λ_2 from input 2 to output 1
- λ_3 from input 2 to output 2
- λ_4 from input 2 to output 2

The connections with λ_1 and λ_2 from input 2 to output 1 could be blocked without wavelength converters. But the set of connections can be satisfied when wavelength are applied, please refer to Problem 10.8.

- 10.5.** Explain why employing multiple fibers between nodes is better (i.e., results in lower blocking probabilities) than increasing the number of wavelengths?

Answer:

Using single fiber, all wavelengths must have different colors. Using multiple fibers, colors of wavelengths on different fibers can be similar or different. Hence, the network has more flexibility in using the wavelengths, which normally reduces blocking probability.

- 10.6.** Let N_1 and N_2 be two networks with the same physical topology and number of wavelengths per fiber. Wavelength conversion is not allowed in network N_1 while it is allowed in network N_2 . Assume that we use the *least-congested path* routing scheme to satisfy dynamic connection requests. Let the blocking probabilities for a sequence of connection requests S be p_1 and p_2 ,

for the networks N_1 and N_2 , respectively. Is $p_1 > p_2$ for all S ? If yes, prove it. If not, show an example of a network topology and sequence of connections S , such that $p_2 > p_1$.

Answer:

Please refer to Section 10.4.1, and Equations 10.1 and 10.3.

- 10.7.** Figure 10.11 shows a plot of the percent gain from using full conversion vs. the network load. Explain the local maximum in the plot.

Answer:

At light load, there is not much need for wavelength conversion, this is because few connections find routes to their destinations. As the load increases, the need for wavelength conversion increases, and we can find the load at which the benefit from wavelength conversion is maximum. In Fig. 10.11 for the NSFNET network topology, this maximum occurs at a load of around 25 Erlangs.

- 10.8.** Consider a node with two input fibers and two output fibers. There are four wavelengths that can be used in the system. For each of the following sets of connections, determine which node architecture – share-per-node, share-per-link, and dedicated converters – can support the connections.

- (a) λ_1 from input 1 to output 1
 λ_2 from input 1 to output 1
 λ_3 from input 1 to output 2
 λ_1 from input 2 to output 1
 λ_2 from input 2 to output 1
 λ_3 from input 2 to output 2
 λ_4 from input 2 to output 2
- (b) λ_1 from input 1 to output 2
 λ_2 from input 1 to output 2
 λ_3 from input 1 to output 2
 λ_1 from input 2 to output 1
 λ_2 from input 2 to output 2
 λ_3 from input 2 to output 1
 λ_4 from input 2 to output 1

Answer:

Wavelength converters are mainly used to avoid the output contention.

For the set (a), conversion requirements are from both inputs. So share-per-node architecture can support the set of connections by only directing the wavelength which require conversion to a converter bank.

For the set (b), conversion requirements are occurred on a particular link. So share-per-link architecture can support the set of connections.

- 10.9.** Consider a path consisting of six links. Each link supports up to four wavelengths, and average link utilization is 0.5. Calculate the blocking probability with and without wavelength conversion. What is the gain for a blocking probability of 0.8?

Answer:

In the case of wavelength conversion, $P'_b = 0.375$ according to Equation 10.2.

In the case without wavelength conversion, $P_b = 0.815$ according to Equation 10.4.

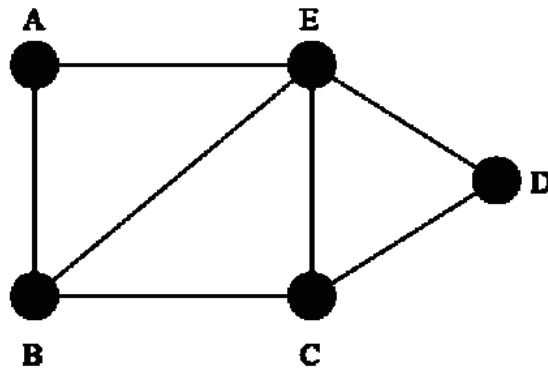


Figure 10.14 Network with uniform loading.

The gain is given based on Equation 10.5.

For $P_b = 0.8$, $G = 1.244$.

- 10.10.** Given a path consisting of five links, suppose full wavelength conversion is allowed between the second and third links, but not at any other location along the path. There are five wavelengths in the system, and the average link utilization is 0.6. What is the blocking probability?

Answer:

The node between the second and third links is only equipped with the full wavelength conversion. In fact, the path could be regarded as two links in the wavelength conversion case, i.e., $H = 2$. According to Equation 10.2, $P'_b = 0.15$.

- 10.11.** Calculate the pass-through traffic for each node in the network shown in Fig. 10.14). Assume uniform loading.

Answer:

Assuming symmetric routing, i.e., path from point A to B is the same as path from points B to A . To calculate the pass-through traffic, we must decide the fixed routing among all source destination pairs plus assuming the symmetry in routing. In this problem, if we assume a minimum hop distance routing and assuming traffic demand among all node pairs is one wavelength, then the overall pass-through in the network is 6 wavelengths.

Chapter 11

Survivable WDM Networks

11.1. (Dedicated-Path Protection) Consider the network topology in Fig. 11.20.

- (a) Find the shortest primary path from source node 3 to destination node 21.
- (b) Find the shortest backup path from node 3 to node 21 so that this path is link disjoint to the path found in part (a). Calculate the total cost of this two-step approach (sum of primary and backup paths) for dedicated-path protection between nodes 3 and 21. Is the backup path found by you node disjoint to the primary path?
- (c) We now describe a *one-step* algorithm for finding the primary and the backup paths concurrently for dedicated-path protection:
 - Reverse the links of the primary path found in part (a). We will refer to this path as *reverse* path.
 - Find another shortest path from the source node to the destination node on the updated network with the above *reverse* path.
 - Remove the links on this newly found shortest route which use links on the *reverse* path.
 - Reverse the links of the *reverse* path.

Use the above one-step algorithm to find two link-disjoint paths between nodes 3 and 21. Compare the cost of the one-step algorithm with the two-step approach. Is the new backup path found by you node disjoint to its primary?

Answer:

Assume cost of link $(3, 5) = 1000$.

- (a) The primary path is $(3, 7, 9, 12, 16, 21)$. The cost is 5000.
- (b) The backup path is $(3, 2, 6, 9, 10, 13, 17, 22, 21)$. The cost is 6950. Total cost = $5000 + 6950 = 11950$. They are not node disjoint (node 9 is a common node).
- (c) Using one-step algorithm, the primary path is $(3, 2, 6, 11, 12, 16, 21)$, the cost is 5050. The backup path is $(3, 7, 9, 10, 13, 17, 22, 21)$, the cost is 6600. Total cost = $5050 + 6600 = 11650$, less than two-step algorithm. The new backup path is node disjoint to the primary.

11.2. (Shared-Path Protection) Consider the same network topology in the previous problem.

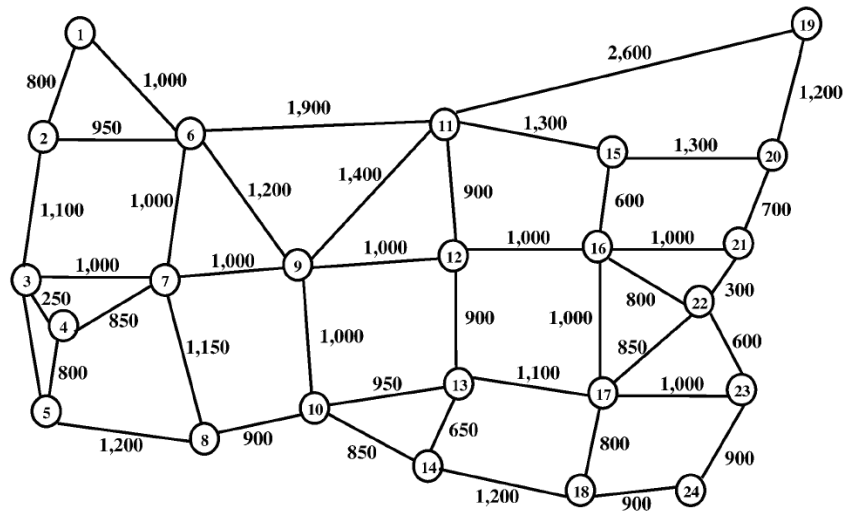


Figure 11.20 A sample 24-node network topology.

- We now wish to establish another connection from node 5 to node 15. Use the one-step algorithm to find the primary and backup paths between nodes 5 and 15. What is the total cost of establishing the two connections (3-21 and 5-15) with dedicated-path protection? Is this cost optimal for the two connections?
- On the network found in Problem 11.1, find the primary path for the connection from node 5 to node 15. Also find its backup while allowing it to share links with the backup path of the connection between nodes 3 and 21. What is the cost of establishing the two connections (3-21 and 5-15) with shared-path protection? In a WDM network, when can two connections not share their backup paths?

Answer:

- Set up a connection from node 5 to node 15 using one-step algorithm. The primary path is (5, 8, 10, 14, 18, 17, 16, 15). The cost is 8550. The backup path is (5, 4, 7, 6, 9, 11, 15). The cost is 9550. Total cost = 8550 + 9550 = 18100. Total cost of two connections is 11650 + 18100 = 29750. The cost is not optimal.
- The primary path is (5, 8, 10, 14, 18, 17, 16, 15). The cost is 8550. The backup path is (5, 3, 7, 9, 10, 13, 17, 22, 21, 20, 15). The new cost is 2000 + 900 + 2300 = 5200. (Note: we only count those links that are not on the backup path of connection (3, 21).) Total cost = 11650 + 8550 + 5200 = 25400.

In a WDM network, two connections can not share their backup paths only when the primary paths of the two connections have common link.

- 11.3.** (Dedicated-Link Protection) Consider the same network topology in Problem 11.1. Suppose now we wish to separately protect every link along the primary path between nodes 3 and 21. Show the backup routes around each link for dedicated-link protection. What is the total cost of this dedicated-link-protection method?

Answer:

Assumption: we first route connection (3, 21).

The backup route for each link along the connection (3, 21) is:

Link(i, j)	Protection-Route	Cost
(3, 7)	(3, 4, 7)	1100
(7, 9)	(7, 8, 10, 9)	4050
(9, 12)	(9, 11, 12)	2300
(12, 16)	(12, 13, 17, 16)	6000
(16, 21)	(16, 22, 21)	1500

Total cost is 14950.

The backup route for each link along the connection (5, 15) is:

Link(i, j)	Protection-Route	Cost
(5, 3)	(5, 4, 3)	2050
(3, 2)	(3, 5, 8, 7, 6, 2)	7300
(2, 6)	(2, 1, 6)	1800
(6, 11)	(6, 9, 10, 13, 12, 11)	7950
(11, 15)	(11, 19, 20, 15)	7500

Total cost is 26600.

So, the total cost for dedicated-link protection is $14950 + 26600 = 41550$.

- 11.4.** (Shared-Link Protection) Consider the same network topology in Problems 11.1 and 11.2. If we allow sharing among the backups of the links, we call this method shared-link protection. Show the backup routes for protecting the two primary paths (between nodes 3 and 21 and between nodes 5 and 15) using shared-link protection. Compare the cost of shared-link protection with dedicated-link protection.

Answer:

Assumption: we assume the resource can be shared when protecting two links in the same connection.

The backup route for each link along the connection (3, 21) is:

Link(i, j)	Protection-Route	Cost
(3, 7)	(3, 4, 7)	1100
(7, 9)	(7, 6, 9)	2200
(9, 12)	(9, 11, 12)	2300
(12, 16)	(12, 9, 10, 13, 17, 16)	5050
(16, 21)	(16, 22, 21)	1500

Total cost is 12150.

The backup route for each link along the connection (5, 15) is:

Link(i, j)	Protection-Route	Cost
(5, 3)	(5, 4, 3)	2050
(3, 2)	(3, 4, 7, 6, 2)	950
(2, 6)	(2, 3, 4, 7, 6)	200
(6, 11)	(6, 9, 11)	0
(11, 15)	(11, 12, 9, 10, 13, 17, 16, 15)	600

Total cost is 3800.

So, the total cost for shared-link protection is $12150 + 3800 = 15950$. Hence, we see the cost for dedicated-link protection is greatly larger than the cost of shared-link protection.

- 11.5.** (Path restoration) Survivability means how quickly and efficiently one can restore a link failure. A faster online solution can be proposed.

Compute a path by using “cache”: Each node’s cache contains information of ‘n’ other neighborhood nodes. The value of ‘n’ depends on the type of protection we need. Let L be the number of backup paths traversing the node. So, the higher the value of L, the higher the value of ‘n’, and vice versa.

According to the constraints, please state a LP problem to minimize the restoration time when some primary path fails and the backup path traverses the node.

Answer:

Given: The network topology or graph, $G=(V,E)$, where V denotes the nodes and E denotes the links in the network; Input: Number of backup paths for a particular connection from the source node ‘s’ to the destination node ‘d’, denoted as L; Constraints: $n \leq V$ and $L \leq binomial(n, 2)$; Minimize: ϵ , which is the restoration time

- 11.6.** Suppose we have $N : 1$ protection (N primary paths protected by 1 shared backup) in Fig. 11.21, and the availability of each path is P . Calculate the possibility that $N : 1$ backup is insufficient (number of failed paths ≥ 2).

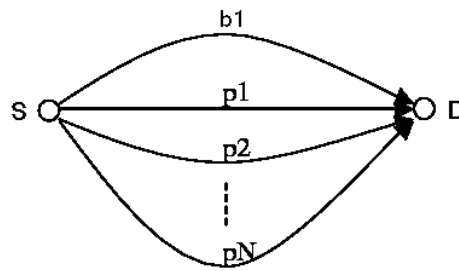


Figure 11.21 $N:1$ backup.

Answer:

$$1 - P^N - N(1 - P)P^{N-1}.$$

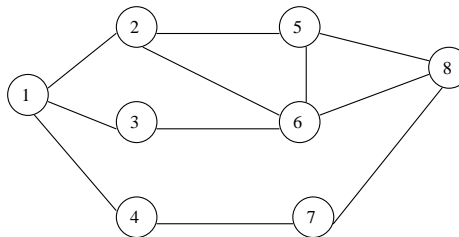


Figure 11.22 Restoration.

- 11.7.** Consider the network topology in Fig. 11.22. A lightpath is set up between nodes 1 and 8 along the link $1 \rightarrow 2 \rightarrow 5 \rightarrow 8$. If link $2 \rightarrow 5$ goes down, which of the following restoration strategies will lead to a path with minimum number of hops? State the number of hops in each case.

- a) Path restoration
- b) Sub-path restoration
- c) Link restoration

Answer:

- (a) Path restoration: (1, 3, 6, 8), 3 hops.
- (b) Sub-path restoration: (1, 3, 6, 5, 8), 4 hops.
- (c) Link restoration: (1, 2, 6, 5, 8), 4 hops.

Path restoration leads to a path with minimum number of hops.

11.8. Given the NSF network in Fig. 8.2,

- a) find two lightpaths that can provide protection for the lightpath from CA1 to NY.
- b) calculate recovery time for a failure on link $UT \rightarrow MI$ (link distances are shown in the diagram in km).

Answer:

- a) Assume that dedicated path protection is applied and path routing considers number of hops as cost function.

Therefore, the primary path: $CA1 \rightarrow UT \rightarrow MI \rightarrow NY$.

The 1st backup path: $CA1 \rightarrow WA \rightarrow IL \rightarrow PA \rightarrow NY$.

The 2nd backup path: $CA1 \rightarrow CA2 \rightarrow TX \rightarrow MD \rightarrow NY$

- b) In dedicated path protection, no OXC configuration is necessary when the failure occurs. If link $UT \rightarrow MI$ in the primary path fails, UT will notify CA1 of the failure. Then CA1 will switch traffic to the 1st backup path:

Restoration time = failure detection time
 + failure notification time
 + propagation delay
 + switching time

Assume that: failure detection time: $100 \mu s$; message processing time at a node: $10 \mu s$; failure notification time + propagation delay : $5 \mu s/km$.

Therefore,

$$\begin{aligned} \text{restoration time} &= 100 \mu s + 10 \mu s \times 6 \\ &+ (120 + 75 + 300 + 60 + 75) \text{ km} \times 5 \mu s/km \\ &= 3310 \mu s = 33.1 \text{ ms.} \end{aligned}$$

11.9. In Fig. 11.23, a primary path between s and d is protected by N backup paths, which is known as $1 : N$ protection. Assume that the availability of path $P1$ is A_{P1} , and availabilities of paths $b1, b2, \dots, bN$ are $A_{b1}, A_{b2}, \dots, A_{bN}$, respectively. Derive an expression for a connection's availability from s to d .

Answer:

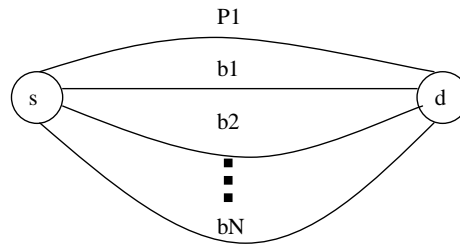


Figure 11.23 1:N backup.

$$\begin{aligned}
 P_{s,d} &= A_{p1} + (1 - A_{p1}) \times A_{b1} + \\
 &\quad (1 - A_{p1}) \times (1 - A_{b1}) \times A_{b2} + \\
 &\quad \cdots + (1 - A_{p1}) \times (1 - A_{b1}) \times \cdots \times (1 - A_{bN-1}) \times A_{bN} \\
 &= A_{p1} + (1 - A_{p1}) \times \sum_{i=1}^N A_{bi} \left(\prod_{j=1}^{i-1} 1 - A_{bj} \right)
 \end{aligned}$$

11.10. Consider the bottleneck-cut identification problem for protection. A “cut” is defined as a set of links such that, if all of the links in this set fail, then the network will become partitioned into two disconnected fragments, e.g., links $1 \rightarrow 2$ and $3 \rightarrow 4$ in Fig. 11.24. A “bottleneck cut” is the cut which disrupts the largest amount of traffic. Given the following traffic matrix, find the bottleneck cut in Fig. 11.24.

$$\begin{bmatrix}
 0 & 0.78 & 0.04 & 0.23 & 0.52 & 0.88 \\
 0.81 & 0 & 0.27 & 9.8 & 0.24 & 0.43 \\
 0.9 & 0.54 & 0 & 0.18 & 0.98 & 0.28 \\
 0.16 & 0.68 & 0.74 & 0 & 0.01 & 0.89 \\
 7.16 & 0.1 & 0.89 & 0.71 & 0 & 0.61 \\
 3.56 & 0.41 & 0.34 & 0.57 & 1.8 & 0
 \end{bmatrix}$$

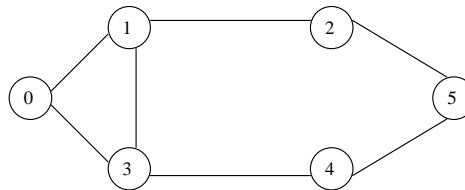


Figure 11.24 Bottleneck cut.

Answer:

(0, 3), (1, 3), (4, 5).

11.11. Consider the network topology in Fig. 11.25, where all links have the same cost. Connection requests arrive between nodes $1 \rightarrow 5$ and nodes $5 \rightarrow 4$. Design a protection scheme for these

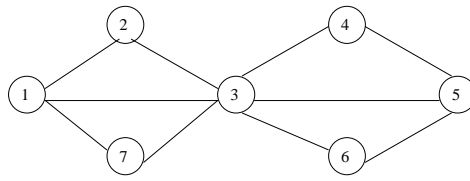


Figure 11.25 A sample network topology.

two connection requests, while minimizing the total number of wavelengths used (for primary as well as backup).

Answer:

The primary path for connection (1, 5) is (1, 3, 5).

The backup path for connection (1, 5) is (1, 2, 3, 4, 5).

The primary path for connection (5, 4) is (5, 4).

The backup path for connection (5, 4) is (5, 3, 4).

So, the total number of wavelengths = $2 + 4 + 1 + 2 - 1 = 8$.

Chapter 12

Light-Tree: Optical Multicasting

12.1. Consider the NSFNET backbone topology in Fig. 12.1. We need to establish a multicast session from source node *CA1* to destination nodes *MI*, *NY* and *NJ*. Find the minimum Steiner Tree to set up the multicast session. What is the total cost of your light-tree?

Now, consider, the weight of the link *UT* – *MI* is increased by 7 to 20 units. Find SMT on the new topology. What is the cost of the new light-tree?

Answer:

- SMT: *CA1* – *UT*, *UT* – *MI*, *MI* – *NY*, *NY* – *MD*, *MD* – *NJ*.
- Total Cost: 25.
- SMT: *CA1* – *UT*, *UT* – *CO*, *CO* – *NE*, *NE* – *IL*, *IL* – *PA*, *PA* – *NY*, *PA* – *NJ*, *NY* – *MI*.
- Total Cost: 31.

12.2. For problem 12.1, please draw the virtual links and show the percentages of traffic from the source node to each of the destination nodes in both the cases.

Answer:

- Virtual links: *CA1* – *MI*(33.33%), *CA1* – *NY*(33.33%), *CA1* – *NJ*(33.33%).
- Virtual links: *CA1* – *NJ*(50%), *CA1* – *NY*(25%), *CA1* – *MI*(25%).

12.3. For the network in Fig. 12.4, show the MWRS configuration at Node *B* using the architectures in Fig. 12.5 (opaque switch) and Figs. 12.6 and 12.7 (transparent switches).

Answer:

Opaque switch in Node *B* takes input only from node *A*, and the input is replicated into three copies. One copy is dropped locally at node *B* itself. For the other 2 copies, one goes to outgoing fiber link for node *C*, and other for node *D*. Same principle applies for transparent switches as well.

12.4. In Fig. 12.4, suppose the cost of link *A*–*B* is increased to 30 units. How will the light-tree now be routed? Draw a new diagram. Show the MWRS configuration of Node *D* using switch architectures in Figs. 12.5 and 12.6.

Answer:

Paths: $F \rightarrow A \rightarrow E \rightarrow D \rightarrow B \rightarrow C$.

MWRS on node D : input from E , signal is replicated into 2 copies, one is locally dropped at D , one goes to outgoing fiber link for node C .

- 12.5.** A transparent switch, with two optical switches, has 12 incoming/outgoing fibers, having 8 wavelengths in each fiber. The sizes of the larger and the smaller optical switches are 98×102 and 20×20 , respectively. Find the splitter-bank size. Assume that the number of local add/drop ports in the larger and smaller switches are 2 and 4, respectively. What is the splitting degree of each splitter?

Now, consider the modified switch having one optical cross-connect. Find the size of the switch. Use the data found before.

Answer:

- Here, $DW + l + B = 102$, solving, $B = 102 - 12 \times 8 - 2 = 4$.
- Here, $Bd + L - l = 20$, solving, $d = \frac{20-4}{4} = 4$.
- Size: $(DW + Bd + L) \times (DW + B + L) = 118 \times 106$.

- 12.6.** In a MILP formulation of multicast traffic with wavelength converters, there are 165 variables. Find the number of nodes in the network if there are 3 multicast sessions. Assume that each fiber link can support 8 wavelengths. What is the upper bound of the number of constraints in the above MILP? How does the number of variables change, assuming no wavelength converter? Also, calculate the number of constraints.

Answer:

- Solve quadratics, $3N + 6N^2 = 165$, $N = 5$.
- Upperbound, 75.
- Variables, 705, Upperbound of Constraints, 600.

- 12.7.** Consider the MILP formulation of multicast traffic with grooming. Find the number of variables and the number of constraints. How does your solution change if you have only 5 splitters at each node, with each having a splitting degree of 3? Use the data found in Problem 12.6.

Answer:

- Variables, 168, Upperbound of Constraints, 75.
- Variables, 720, Upperbound of Constraints, 600.

- 12.8.** For the multicast sessions in Table 12.3, draw the corresponding logical topology. Are the links in the logical topology directed or undirected? Why?

Answer:

Open for readers.

- 12.9.** Consider the network topology in Fig. 12.9. There is only one direct physical bidirectional fiber link between any two nodes. Each link carries one wavelength. We need to establish 5 full-capacity multicast sessions $S_1 = \{\mathbf{E}, A, B, C\}$, $S_2 = \{\mathbf{D}, E, F, C\}$, $S_3 = \{\mathbf{C}, F, A, D\}$, $S_4 = \{\mathbf{A}, E, F\}$ and $S_5 = \{\mathbf{F}, E, D\}$. Solve the routing and wavelength assignment problem using an MILP. What is the total cost?

Answer:

- Route: $E - A, A - B, B - C$, Cost: 10.
- Route: $D - E, E - F, D - C$, Cost: 14.
- Route: $C - B, B - A, A - F, C - D$, Cost: 15.
- Route: $A - E, A - F$, Cost: 6.
- Route: $F - E, E - D$, Cost: 11.
- Total Cost: 56.

12.10. Consider the network topology in Fig. 12.9. There is one direct physical bidirectional fiber link between any two nodes. Each link carries 2 wavelengths. We need to establish 5 full-capacity multicast sessions $S_1 = \{\mathbf{E}, A, B, C\}$, $S_2 = \{\mathbf{D}, E, F, C\}$, $S_3 = \{\mathbf{C}, F, A, D\}$, $S_4 = \{\mathbf{A}, E, F\}$ and $S_5 = \{\mathbf{F}, E, D\}$. Solve the routing and wavelength assignment problem using an MILP. What is the total cost? Assume a no- λ -conversion network. Solve this problem for a full λ -conversion network.

Answer:

Open for readers.

12.11. How would your solutions to Problems 12.9 and 12.10 change if connections S_1 , S_2 and S_4 use 25%, and connections S_3 and S_5 use 50% of channel capacity?

Answer:

Open for readers.

12.12. Consider the network topology in Fig. 12.11. There is only one direct physical bidirectional fiber link between any two nodes. Each link carries 2 wavelengths. We need to establish 6 multicast sessions $S_1 = \{1.0, \mathbf{6}, 1, 5, 4\}$, $S_2 = \{0.25, \mathbf{0}, 6, 10, 2\}$, $S_3 = \{1.0, \mathbf{8}, 6, 7, 5, 14\}$, $S_4 = \{1.0, \mathbf{1}, 6, 8, 5\}$, $S_5 = \{0.5, \mathbf{12}, 4, 5, 8, 9, 10, 0, 2\}$, $S_6 = \{0.5, \mathbf{7}, 6, 5, 1, 14, 13\}$. Solve the routing and wavelength assignment problem using an MILP. What is the total cost? Assume a full λ -conversion network. How many wavelength converters are needed? At which nodes?

Answer:

Open for readers.

12.13. Formulate a dual problem for multicast tree protection without λ continuity.

Answer:

Open for readers.

Chapter 13

Traffic Grooming

- 13.1.** Given the number of flow capacities as follows, how many OC-192 links are needed to carry all the traffic if we use packing/unpacking techniques? (Consider the three cases separately first, and then consider them together.)

4 OC-3

5 OC-12

6 OC-48

Answer:

4 OC-3 = 1 OC-12

4 OC-12 = 1 OC-48

4 OC-48 = 1 OC-192

Thus, we need:

2 OC-12, 3 OC-48 and 1 OC-192 links. Since $2 \times 12 + 3 \times 48 < 192$, then 2 OC-192 links would be enough to carry all the traffic. So, we need 2 OC-192 links.

- 13.2.** Construct an example of traffic grooming in a ring topology, showing that minimizing the number of ADMs may result in non-shortest-path routing.

Answer:

Consider a 7-node bidirectional ring shown in Fig. 13.24. Consider bidirectional traffic demands between nodes 2-3, 3-4 and 2-4. First, assuming that all demands are routed on their shortest path, we use wavelength λ_1 for routing 2-3 and 3-4, and λ_2 for provisioning 2-4 on its shortest path 2-3-4. In this (shortest-path) case, we need 5 ADMs and 2 wavelengths. Second, consider that we provision 2-3 and 3-4 on λ_1 , and now use the longer path 4-5-6-7-1-2 and λ_1 for routing 2-4. In this case, we need less ADMs (3), and only one wavelength.

This example shows that shortest path is not necessarily optimal in minimizing the number of ADMs (or used wavelengths).

An example (on a 9-node ring) for showing that shortest-path routing does not guarantee minimum number of ADMs and that minimum number of ADMs does not necessarily mean minimum number of wavelengths (which can be used as solution for this problem and for problem 13.4) appears in [MoLi01].

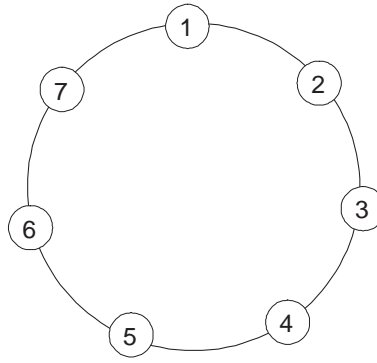


Figure 13.24 A 7-node bidirectional ring.

13.3. Consider the topology shown in Fig. 13.25. Each link has two wavelengths of capacity OC-96 each. Determine the link utilization after satisfying the connection requests in the following traffic matrix. The unity traffic demand is considered to be OC-12.

$$A = \begin{bmatrix} 0 & 2 & 1 & 1 & 2 \\ 2 & 0 & 3 & 2 & 2 \\ 1 & 3 & 0 & 3 & 2 \\ 1 & 2 & 3 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix}$$

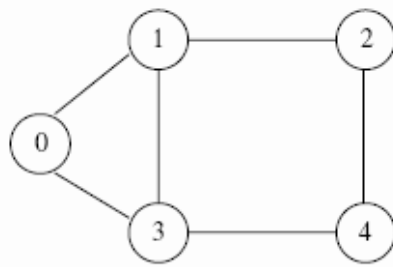


Figure 13.25 Graph representing network topology.

Answer:

2 wavelengths of OC-96 = OC-192.

- Link 0-1 - supports connection 0-1 (OC-24), connection 0-2 (OC-12), 0-4 (OC-24) - in total $60/192 = 31.25\%$ utilization of the 2 wavelengths.
- Link 0-3 - supports the connection 0-3 (OC-12) in total $12/192 = 6.25\%$ utilization of the 2 wavelengths.
- Link 3-1 - supports the connection 3-1 (OC-24), connection 3-2 (OC-36), connection 3-4 (OC-24). The utilization is: 43.75%
- Link 1-2 - supports the connection 0-2 (OC-12), connection 0-4 (OC-24), connection 3-2 (OC-36), connection 3-4 (OC-24), connection 1-2 (OC-36), connection 1-4 (OC-24) The utilization of this link is: 81.25%
- Link 2-4 - supports the connections 0-4 (OC-24), connection 1-4 (OC-24), connection 3-4 (OC-24), and connection 2-4 (OC-24). So, the utilization of this link is: 50%

Link 3-4 is not utilized (there are traffic grooming policies which try to use existing lightpaths, hence avoiding the use of new transceiver resources, even if the chosen route may be longer).

- 13.4.** Construct a traffic-grooming example to show that, for a ring topology, minimizing the number of ADMs may need more wavelengths than the minimal.

Answer:

Consider the 7-node bidirectional ring in Fig. 13.24 and traffic demands between nodes 2-3, 3-4, 2-4, 5-6, 6-7, 5-7 and 7-1. A first solution would use wavelength λ_1 for 2-3, 3-4, 5-6, 6-7, and 7-1, and λ_2 for 2-4 and 5-7 (routed on shortest path). So, this solution requires 2 wavelengths and 11 ADMs. A second solution is to route 2-3, 3-4, and also 4-2 (routed on the long path 4-5-6-7-1-2) on λ_1 , 5-6, 6-7 and 7-5 (routed on its long path) on λ_2 , and 7-1 on λ_3 . This solution uses more wavelengths (i.e., 3), but less ADMs (i.e., 8). So, the solution with smaller number of ADMs (second solution) uses up more wavelengths.

- 13.5.** Consider the network in Fig. 13.26. The corresponding link utilization matrix can be seen below (with 0.3 signifying 30% utilization of a wavelength):

$$\Lambda = \begin{bmatrix} 0.0 & 0.3 & 0.2 & 0.4 \\ 0.3 & 0.0 & 0.1 & 0.6 \\ 0.4 & 0.1 & 0.0 & 0.2 \\ 0.5 & 0.4 & 0.6 & 0.0 \end{bmatrix}$$

- (1) How many wavelengths are saved by using traffic grooming? Draw the connections with and without grooming.
- (2) What is the average utilization for each wavelength in the non-grooming mode? What is the utilization in traffic grooming mode? Which one is better?

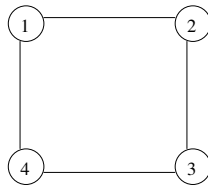


Figure 13.26 The optical network.

Answer:

1. Without using traffic grooming, the connections can be seen in the figure below.

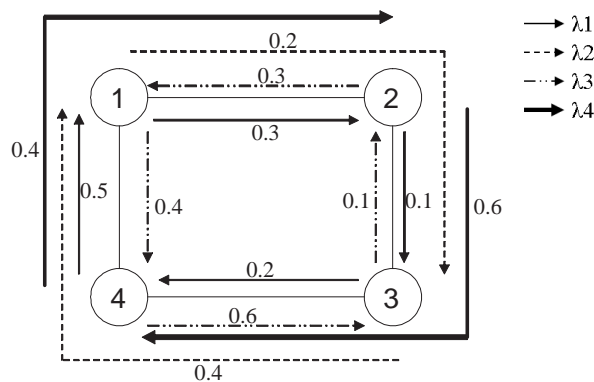


Figure 13.27 The connections without traffic grooming.

As can be seen in Fig. 13.27, 4 wavelengths (counting both fiber directions) are used in the case when we do not employ traffic grooming.

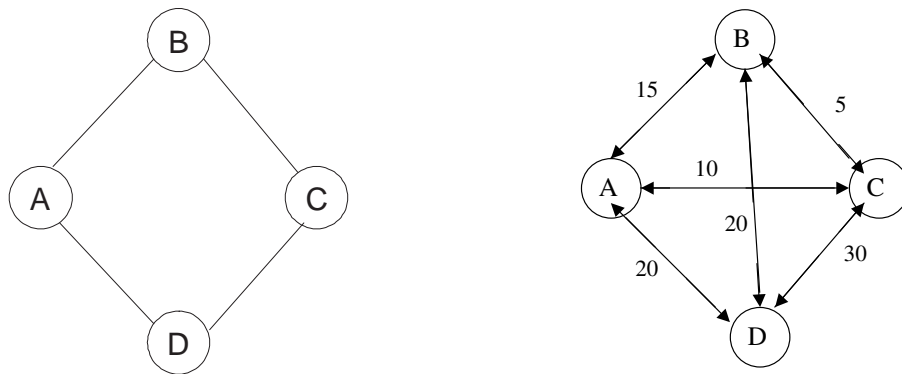


Figure 13.28 Network topology and traffic demands.

Considering traffic grooming (and opaque switches), on the directed link 1-2, the connections 1-2, 1-3, 4-2 can now be packed on the same wavelength λ_1 , using in total 90% of the wavelength. Similarly, on link 2-3, the connections 2-3, 1-3, and 2-4 can also be groomed on a single wavelength (filling up 90% of its capacity). In contrast, on the directed links 3-4 and 4-1, even with grooming, we still require two wavelengths to accommodate the traffic.

However, for all the four network links in the opposite direction, we only require one wavelength. So, overall, by using traffic grooming we only need 3 wavelengths per link.

Therefore, by packing the traffic intelligently on the wavelengths, savings are possible over approaches that do not consider traffic grooming.

2. To compute the average utilization of every wavelength in the non-grooming mode, we have to compute the average utilization of each wavelength on all the edges.

- For λ_1 : $(0.3+0.1+0.2+0.5)/4=0.275$
- For λ_2 : $(0.2+0.2+0.4+0.4)/4=0.3$
- For λ_3 : $(0.3+0.1+0.6+0.4)/4=0.35$
- For λ_4 : $(0.4+0.4+0.6+0.6)/4=0.5$

So, these are the utilizations in the non-grooming mode.

The utilizations in the traffic grooming mode, for our repartition of traffic onto wavelengths, are:

- For λ_1 : $(0.9+0.9+1+1)/4=3.8/4=0.95$
- For λ_2 : $(0+0+0.2+0.3)/4=0.125$
- For λ_3 : $(0.3+0.1+0.6+0.4)/4=0.35$

13.6. Consider Fig. 13.28 which shows a network topology and the traffic demands between each pair of nodes. The capacity of each link is OC-48.

- (1) Without traffic grooming, how many lightpaths are required, and how many wavelengths need to be allocated? Show all the lightpaths and their carried traffic.
- (2) With traffic grooming, how many lightpaths are required, and how many wavelengths need to be allocated? Show all the lightpaths and their carried traffic.

Answer:

1. Without using traffic grooming, we need to establish 6 lightpaths (the same as the number of connections), and to allocate 3 wavelengths per link. A total of 8 wavelength-links are used.

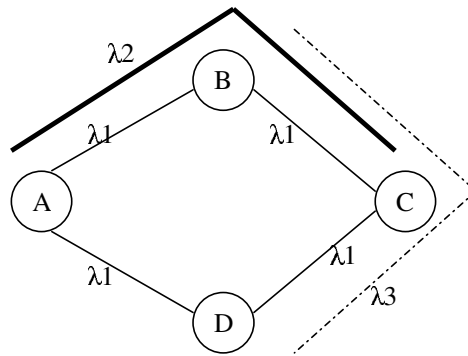


Figure 13.29 The wavelength assignment without traffic grooming.

The wavelength assignment can be seen in the figure below:

So, for the case where we do not employ traffic grooming, we use the following wavelengths (the pair shows the nodes of the traffic demands, followed by the wavelength):

- A-B - λ_1
- A-C - λ_2 (through B)
- A-D - λ_1
- B-C - λ_1
- B-D - λ_3 (through C)
- C-D - λ_1

2. With traffic grooming (considering that nodes have opaque switches which perform optical-electronic-optical conversion), we need to allocate 1 wavelength per link and to establish 4 lightpaths. A total of only 4 wavelength-links are used, as explained next.

Consider the following routing of traffic. A-B is routed directly on link A-B, A-C on path A-B-C, and A-D directly on the same link. B-C is routed on link B-C, B-D is routed on B-A-D, while C-D is routed on link C-D. On each link, all this traffic can be groomed onto one wavelength.

The total traffic carried by link A-B is 45, and by link B-C is 15. The traffic carried by link C-D is 30, and by link A-D is 40.

13.7. Consider the network topology in Fig. 13.30, with capacities shown on links (denoting the maximum “capacity” of the link).

Given the following traffic matrix (the traffic is assumed to be symmetrical in both directions), try to accommodate the traffic optimally in terms of network throughput, by using traffic grooming (with multiple paths). Show the aggregation of traffic into the lightpaths.

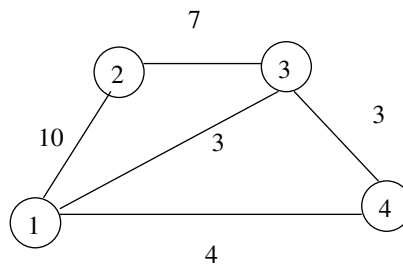


Figure 13.30 The network topology.

The traffic matrix can be seen below:

$$A = \begin{bmatrix} 0 & 1 & 3 & 6 \\ 1 & 0 & 4 & 0 \\ 3 & 4 & 0 & 1 \\ 6 & 0 & 1 & 0 \end{bmatrix}$$

Answer:

We can send 6 units of traffic from node 1 to node 4 through both the paths: 1-4 directly (4 units), and 1-2-3-4 (2 units). In total 6 units.

From node 1 to node 2, we just send directly 1 unit.

From node 1 to node 3, we can send 3 units on the path 1-3.

From node 2 to node 3, send 4 units directly.

From node 3 to node 4, send 1 unit of traffic.

Traffic grooming is a good way of better using the capacities of a link. Also, by allowing traffic grooming with multiple paths, we are able to accommodate more traffic (we could not accommodate the traffic between 1 to 4 without allowing multiple paths), hence achieve better link utilization.

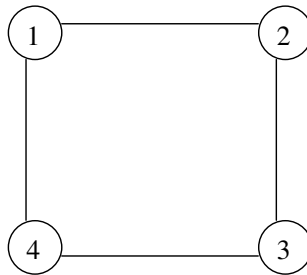


Figure 13.31 A network topology.

13.8. Consider the network topology in Fig. 13.31. The capacity of each wavelength is 10 Gbps. Given the following traffic matrix in Gbps, what is the number of wavelengths needed with traffic grooming and without traffic grooming?

$$A = \begin{bmatrix} 0 & 2 & 3 & 0 \\ 0 & 0 & 3 & 3 \\ 4 & 0 & 0 & 2 \\ 4 & 2 & 0 & 0 \end{bmatrix}$$

Answer:

Without traffic grooming, we need to use 2 bidirectional wavelengths per link.

With traffic grooming, the minimum number of wavelengths we can use is 1.

Let us explain how:

- on the link 1-2 we will carry on the one wavelength the following traffic demands: 1-2, 1-3 and 4-2;
- link 2-3 will carry demands 2-3, 2-4, 1-3;
- link 3-4 will carry demands 3-4, 3-1 and 2-4 on the one wavelength;

- link 4-1 will carry demands 4-1, 3-1 and 4-2.

This way, by employing traffic grooming, we can use only one wavelength.

13.9. Consider the network in Fig. 11.20. The capacity of each wavelength channel is OC-192. Assume that there are no limits on the number of wavelengths on each link and on the number of transceivers at each node. The following THREE lightpaths are already established (including their routes and capacities used):

- (1) From 8 to 12 at OC-96 (8-7-9-12)
- (2) From 12 to 21 at OC-48 (12-16-22-21)
- (3) From 10 to 16 at OC-144 (10-9-12-16)

When a new low-speed connection request arrives, one of the following four schemes can be used to establish it.

Scheme 1: Groom the new connection onto an existing lightpath with sufficient spare capacity.

Scheme 2: Create a new lightpath for the new connection.

Scheme 3: Groom the new connection onto existing lightpaths, thereby establishing a multihop connection.

Scheme 4: Use existing lightpath(s) for some part(s) of its route and set up new lightpaths for the remaining part(s).

We need to establish the following connections. Find the routes – if possible – for each of these connections, indicating the use of existing lightpaths, wherever applicable, and new lightpaths. If a connection cannot be established using a specified scheme, please say so, and justify.

- a) 5 to 20 at OC-48 using Scheme 1
- b) 5 to 20 at OC-48 using Scheme 2
- c) 5 to 20 at OC-48 using Scheme 3
- d) 5 to 20 at OC-48 using Scheme 4
- e) 8 to 21 at OC-48 using Scheme 1
- f) 8 to 21 at OC-48 using Scheme 3
- g) 8 to 21 at OC-144 using Scheme 1
- h) 8 to 21 at OC-144 using Scheme 3
- i) 10 to 21 at OC-48 using Scheme 4

Answer:

a) 5-20 using Scheme 1 is impossible, since there is not a single lightpath going from 5 to 20.

b) 5-20 using Scheme 2.

As there are enough available resources (wavelengths and transceivers) in the network, a solution according to Scheme 2 is to setup a new end-to-end lightpath between 5 and 20, for example 5-8-7-9-12-16-22-21-20.

c) Not possible, since we can not use only existing lightpaths between 5 and 20, as specified in Scheme 3 (there are no existing lightpaths covering the segments 5-8 and 21-20).

d) 5-20 at OC-48 using Scheme 4.

A solution would be: 5-8-7-9-12-16-22-21-20, which means that we set up a new lightpath from 5 to 8, then use first existing lightpath (lightpath 1) from 8-12, then use the second existing lightpath (from 12-21), then create a new lightpath from 21 to 20.

e) We cannot setup a connection between 8 and 21 by using Scheme 1, because there is no single existing lightpath between 8 and 21.

- f) We can accommodate such a connection on route 8-7-9-12-16-22-21. We use the residual capacity on the first two existing lightpaths (lightpath 1 from 8-12 and lightpath 2 from 12-21).
- g) Impossible. Between the three established lightpaths, there is no single lightpath (with sufficient spare capacity) between 8 and 21 (as required in Scheme 1).
- h) Impossible. We would like to accommodate a OC-144 connection onto two existing lightpaths (lightpath 1 and 2); however, lightpath 1 does not have enough remaining capacity to support the new connection.
- i) We can establish such a connection on route 10-9-12-16-22-21. We use lightpath 3 from 10-16, and then establish a new lightpath 16-22-21.

Chapter 14

Advanced Topics in Traffic Grooming

14.1. Consider the network topology in Fig. 14.7, where every link is a bidirectional fiber. Each fiber has four wavelengths, each of capacity $STS - 192$. Use protection-at-lightpath (PAL) scheme to setup the following connections, one at a time.

- $c0(< 1, 10, STS - 3c >)$
- $c1(< 8, 14, STS - 1 >)$
- $c2(< 7, 3, STS - 3c >)$
- $c3(< 7, 15, STS - 12c >)$
- $c4(< 14, 11, STS - 12c >)$
- $c5(< 3, 12, STS - 3c >)$
- $c6(< 18, 19, STS - 48c >)$
- $c7(< 22, 15, STS - 1 >)$
- $c8(< 2, 20, STS - 48c >)$
- $c9(< 18, 17, STS - 3c >)$

Assume that every node has four grooming ports. The connections, once set up, will remain in the network throughout. Specify free/used capacity along the links after all connections are set up. Also specify the number of grooming ports used/available at the nodes after all connections are established.

Answer:

C0: working: $1 \rightarrow 5 \rightarrow 10$; backup: $1 \rightarrow 2 \rightarrow 6 \rightarrow 8 \rightarrow 10$; free capacity: $STS-189$; grooming ports: at 1: $T=3$ and $R=4$; at 10: $T=4$ and $R=3$.

C1: working: $8 \rightarrow 10 \rightarrow 14$; backup: $8 \rightarrow 11 \rightarrow 15 \rightarrow 14$; free capacity: $STS-191$; grooming ports: at 8: $T=3$ and $R=4$; at 14: $T=4$ and $R=3$.

C2: working: $7 \rightarrow 4 \rightarrow 3$; backup: $7 \rightarrow 6 \rightarrow 3$; free capacity: $STS-189$; grooming ports: at 7: $T=3$ and $R=4$; at 3: $T=4$ and $R=3$.

C3: working: $7 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 15$; backup: $7 \rightarrow 6 \rightarrow 8 \rightarrow 11 \rightarrow 15$; free capacity: $STS-180$; grooming ports: at 7: $T=2$ and $R=4$; at 15: $T=4$ and $R=3$.

C4: working: $14 \rightarrow 15 \rightarrow 11$; backup: $14 \rightarrow 10 \rightarrow 11$; free capacity: STS-180; grooming ports: at 14: $T=3$ and $R=3$; at 11: $T=4$ and $R=3$.

C5: working: $3 \rightarrow 6 \rightarrow 8 \rightarrow 11 \rightarrow 12$; backup: $3 \rightarrow 4 \rightarrow 7 \rightarrow 9 \rightarrow 12$; free capacity: STS-189; grooming ports: at 3: $T=3$ and $R=3$; at 12: $T=4$ and $R=3$.

C6: working: $18 \rightarrow 19$; backup: $18 \rightarrow 10 \rightarrow 14 \rightarrow 19$; free capacity: STS-144; grooming ports: at 18: $T=3$ and $R=4$; at 19: $T=4$ and $R=3$.

C7: working: $22 \rightarrow 21 \rightarrow 15$; backup: $22 \rightarrow 16 \rightarrow 15$; free capacity: STS-191; grooming ports: at 22: $T=3$ and $R=4$; at 15: $T=4$ and $R=2$.

C8: working: $2 \rightarrow 6 \rightarrow 8 \rightarrow 11 \rightarrow 15$; backup: $2 \rightarrow 4 \rightarrow 7 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 15$.

Either use separate wavelength for working. If we use same wavelength, make sure, backup paths are on two separate wavelengths, because working paths of C5 and C8 are shared on link $8 \rightarrow 11$. grooming ports: at 2: $T=3$ and $R=4$; at 15: $T=4$ and $R=1$.

C9: working: $18 \rightarrow 19 \rightarrow 20 \rightarrow 21 \rightarrow 16 \rightarrow 17$; backup: $18 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 17$;

same argument as C8 as working path of C6 and C9 share the same link on $18 \rightarrow 19$.

grooming ports: at 18: $T=2$ and $R=4$; at 17: $T=4$ and $R=3$.

- 14.2.** Repeat Problem 14.1, assuming that every connection comes periodically, one after every 10 seconds (i.e., connection-interarrival time = 10 sec.), starting from connection c_0 at $t = 0$, and connection-holding time is 25 seconds.

Answer:

Follow same strategy as Problem 14.1.

- 14.3.** Consider the network topology in Fig. 14.7, where every link is a bidirectional fiber. Each fiber has two wavelengths, each of capacity $STS - 192$. Use mixed protection-at-connection (MPAC) scheme to setup the following connections, one at a time.

- $c_0(< 5, 23, STS - 3c >)$
- $c_1(< 9, 13, STS - 12c >)$
- $c_2(< 20, 8, STS - 3c >)$
- $c_3(< 23, 7, STS - 48c >)$
- $c_4(< 3, 21, STS - 12c >)$

Assume that every node has four grooming ports; and connections, once set up, will remain in the network throughout. Specify free/used capacity along the links after all connections are set up. Also specify the number of grooming ports used/available at the nodes after all connections are established.

Answer:

C0: working: $5 \rightarrow 6 \rightarrow 7 \rightarrow 9 \rightarrow 13 \rightarrow 17 \rightarrow 23$; backup: $5 \rightarrow 8 \rightarrow 11 \rightarrow 12 \rightarrow 16 \rightarrow 22 \rightarrow 23$; free capacity: STS-189 for working, STS-189 for backup; grooming ports: at 5: $T=2$ and $R=4$; at 23: $T=4$ and $R=2$.

C1: working: $9 \rightarrow 13$; backup: $9 \rightarrow 12 \rightarrow 13$; free capacity: STS-177 for working (as $9 \rightarrow 13$ is shared between C0 and C1), STS-180 for backup; grooming ports: at 9: $T=2$ and $R=4$; at 13: $T=4$ and $R=2$.

C2: working: $20 \rightarrow 15 \rightarrow 11 \rightarrow 8$; backup: $20 \rightarrow 19 \rightarrow 14 \rightarrow 10 \rightarrow 8$; free capacity: STS-189 for working and backup; grooming ports: at 20: $T=2$ and $R=4$; at 8: $T=4$ and $R=2$.

C3: working: $23 \rightarrow 17 \rightarrow 13 \rightarrow 9 \rightarrow 7$; backup: $23 \rightarrow 22 \rightarrow 16 \rightarrow 12 \rightarrow 11 \rightarrow 8 \rightarrow 6 \rightarrow 7$; free capacity: STS-144 for working and backup; grooming ports: at 23: $T=2$ and $R=2$; at 7: $T=4$ and $R=2$.

C4: working: $3 \rightarrow 6 \rightarrow 8 \rightarrow 11 \rightarrow 15 \rightarrow 21$; backup: $3 \rightarrow 4 \rightarrow 7 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 21$; free capacity: STS-180 for working, STS-177 for backup as on $7 \rightarrow 9$ link is shared between C0 working and C4 backup; grooming ports: at 3: $T=2$ and $R=4$; at 21: $T=4$ and $R=2$.

- 14.4.** Now, consider that every connection in Problem 14.3 has two additional parameters: connection-arrival time and connection-holding time, i.e., $\langle t_s, t_h \rangle$, in seconds. For the five connections in Problem 14.3, these parameter values are: $\langle 3, 10 \rangle$, $\langle 7, 13 \rangle$, $\langle 21, 7 \rangle$, $\langle 23, 10 \rangle$, and $\langle 24, 4 \rangle$, respectively. How would your solution change? Show all the steps of your solution as time progresses.

Answer:

Follow same strategy as Problem 14.3.

- 14.5.** Repeat Problem 14.4 using separate protection-at-connection (SPAC) scheme.

Answer:

Same solution for Problem 14.3 would apply here, except the number of grooming ports in each node will change.

- 14.6.** Consider the network topology in Fig. 14.7, where every link is a bidirectional fiber. Each fiber has eight wavelengths, each of capacity $STS - 192$. Every node has 16 grooming ports. New connections need to be setup using the following three schemes:

Scheme 1: PAL; **Scheme 2:** MPAC; and **Scheme 3:** SPAC.

The connections are:

- $c0(\langle 11, 2, STS - 3c \rangle)$, use PAL
- $c1(\langle 8, 20, STS - 12c \rangle)$, use PAL
- $c2(\langle 7, 4, STS - 1 \rangle)$, use SPAC
- $c3(\langle 23, 7, STS - 48c \rangle)$, use MPAC
- $c4(\langle 3, 21, STS - 96c \rangle)$, use PAL
- $c5(\langle 5, 20, STS - 96c \rangle)$, use MPAC
- $c6(\langle 10, 21, STS - 3c \rangle)$, use SPAC
- $c7(\langle 13, 17, STS - 48c \rangle)$, use PAL

Assume that the connections, once set up, will remain in the network throughout. Find the primary and backup routes for all connections, if possible. If no route exists, justify why? Specify free/used capacity along the links as before. Also specify the number of grooming ports used/available at the nodes as before.

Answer:

Please refer to Problems 14.1 and 14.3.

- 14.7.** A waveband path can originate from a waveband add port (one of the BA ports), or from an OEO-switch to waveband-switch port (one of the WB ports) (see Fig. 14.4). Similarly, it can terminate at one of the BD ports or BW ports. How many types of waveband paths are there according to how such a path originates and terminates? What constraints does each type of waveband path impose on routing connection requests?

Answer:

There are four types of waveband paths according to how such a path originates and terminates: B-B, B-W, W-B, and W-W, where “B” and “W” indicate the waveband switch and OEO full-grooming switch, respectively. Each type of waveband path imposes different constraints with regard to routing connection requests.

A B-B waveband path can only contain connection requests which share the same source and destination as the waveband path. The start and the end of the waveband path are directly connected to the ingress and egress point, and there is no subwaveband switching along the entire path.

A B-W waveband path can be shared by connections with the same source and different destinations. Since the B-W path terminates at the OEO full-grooming switch, a connection can be switched to another waveband path so that the connection can continue to travel towards its destination (i.e., it allows multi-hopping across waveband paths).

Similarly, a W-B waveband path can be shared by connections with the same destination and different sources.

The W-W waveband path is the most flexible one, because it can be shared by connections with both different sources and destinations.

- 14.8.** It is not desirable to route a connection to a large number of paths from network operation and management point of view. How can you modify the heuristic in Section 14.3.3 so that a connection will be routed on to no more than K path, where K is constant?

Answer:

Suppose the bandwidth requirement of the connection is B. Prior to Step 1, re-define the available capacity of each link as follows:

$$\text{Modified Available Capacity of Link } L = \left\lceil \frac{\text{current available capacity of link } L}{\lceil B/K \rceil} \right\rceil$$

Remove links of modified available capacity 0; execute Step 1 to Step 3 (it might be necessary to adjust the capacity of the final path).

- 14.9.** Based on Figs. 14.12-14.14, please show the network configuration after provisioning a fourth connection (5, 3, STS-12c, t4) using PAL, MPAC, and SPAC.

Answer:

Please see Figs. 14.16, 14.17 and 14.18.

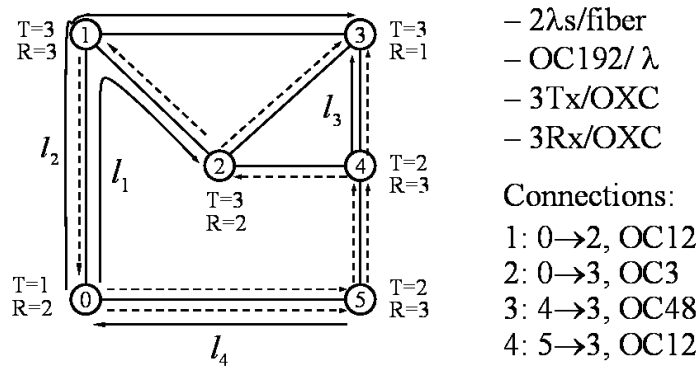


Figure 14.16 Protection-at-lightpath (PAL) level.

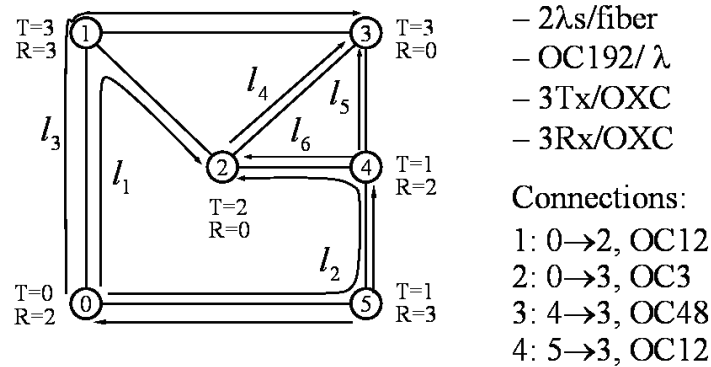


Figure 14.17 Mixed protection-at-connection (MPAC) level.

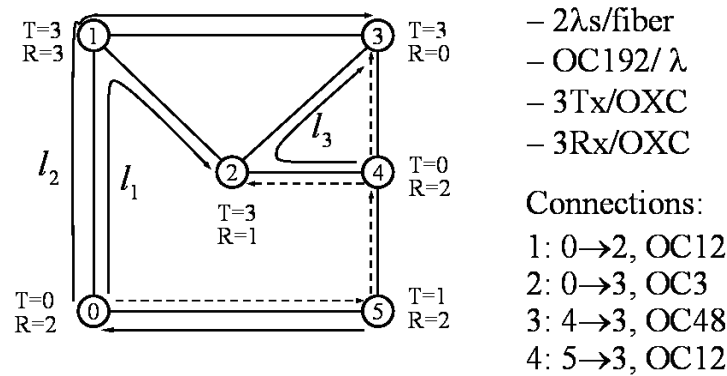


Figure 14.18 Separate protection-at-connection (SPAC) level.

14.10. In Chapter 13, we introduced the generic graph model for traffic grooming. Can this model also handle the hierarchical optical switches shown in Fig. 14.3? Justify your answer. If not, how can we extend the model to cover this architecture? Justify your modification.

Answer:

Open for readers.

14.11. Another hierarchical optical switch architecture might contain a separate wavelength switch between the waveband switch and the OEO grooming switch. Does the architecture proposed in this chapter have wavelength switching capability? What are the advantages and disadvantages of each architecture?

Answer:

Open for readers.

14.12. When a traffic stream is broken into multiple components and routed through different paths towards a destination node using virtual concatenation, the destination node may need an external RAM to compensate for the differential delay between the bifurcated streams when they are reconstructed. What are the factors that determine the amount of RAM needed at the destination node?

Answer:

Two factors determine the amount of RAM needed at the destination node, i.e., a) the length difference between each path, and b) the data-rate supported by the shorter path (paths).

Example: assume a traffic request is carried by two route, route A and B . The route length difference is x micro second (in term in optical signal propagation speed). Assuming the data-rate of the shorter route is Y Gbps, end-node buffer size should be $x \times Y$ kbits.

- 14.13.** Under a static network design case, in which all traffic requests are known and do not change, which virtual concatenation benefits listed in Section 14.3.3 may not apply?

Answer:

Since all traffic requests are static and known in the network design stage, time-slot alignment and continuity constraints could always be met.

Chapter 15

All-Optical Impairment-Aware Routing

- 15.1.** From data-transparency perspective, what are the three network models that can be used for a WDM mesh network? What are their main characteristics?

Answer:

There could be three types of network configurations such as opaque network, translucent network and transparent network. In an opaque network, data signal is received and retransmitted at every intermediate node along a lightpath via OEO conversion. The operating expenses of such a point-to-point system are high due to the large amount of regenerators required at the nodes.

In a translucent network, the data regeneration functionality only employed at some nodes instead of at all nodes in the opaque network. Thus, the regeneration cost could be reduced.

In a transparent network, data signal remains in the optical domain for the entire lightpath. Hence, the transparent network eliminates the expensive OEO conversions, and offers transparency to bit rate, signal format, and protocols. The transparent network also can provide high-speed node-bypass.

- 15.2.** What are the major issues in a transparent network?

Answer:

Because of transparency of the transparent network, noise and signal distortion due to linear and nonlinear effects of optical components accumulate along the lightpath and may cause significant signal degradation. At the destination node, the received signal quality may be so poor that the bit-error rate (BER) can reach an unacceptably high value, and thus the lightpath is not usable.

- 15.3.** What are physical-layer blocking and network-resource blocking?

Answer:

At the destination node, if the received signal quality is so poor that the bit-error rate (BER) can reach an unacceptably high value, the lightpath is not usable. This phenomenon is called physical-layer blocking. If no route or wavelength is available, a connection is blocked due to lack of resources in the network layer. This type of blocking is called network-resource blocking.

- 15.4.** What are PMD blocking and OSNR blocking?

Answer:

If a call needs to be routed farther than the PMD limit of transmission length, it should be blocked. This kind of blocking caused by the PMD impairment is referred to as PMD blocking.

If the accumulated noise degrades the OSNR to below a required threshold, the lightpath should not be used. This form of physical-layer blocking is referred to as OSNR blocking.

15.5. What is transparent transmission length?**Answer:**

It is the geometrical length for which the pulse broadening of a transmitted signal is less than a certain standard requirement such that the PMD effects can be tolerated.

15.6. How could physical-layer devices affect the performance of a RWA algorithm?**Answer:**

A lightpath may traverse through a number of non-ideal transmission devices, such as optical fibers, optical amplifiers, and optical crossconnects (OXC's). Because of transparency of an optical network, noise and signal distortion due to linear and nonlinear effects accumulate along the lightpath and may cause physical-layer blocking. Thus, the lightpath computed by a RWA algorithm may not be usable resulting in high blocking performance.

15.7. Please describe the major characteristics of an impairment-aware RWA.**Answer:**

Compared with traditional impairment-unaware RWA, an impairment-aware RWA takes the physical-layer impairments into consideration while computing and setting up a lightpath. The criterion of resource acceptability is different for call admission. The call-admission criterion of the traditional algorithms depends on resource availability, whereas the criterion of the impairment-aware algorithms depends not only on resource availability but also on the lightpath's signal quality. In the impairment-aware algorithm, only the qualified available lightpaths can be used to set up a call request.

15.8. Besides IAFF and IABP, design another impairment-aware RWA. Specify the algorithm as shown for IAFF and IABP. Qualitatively compare your algorithm with IAFF and IABP.**Answer:**

Open for readers.

15.9. What are the major impairment effects in an optical network with 40-Gbps data rate?**Answer:**

PMD, GVD and fiber nonlinear effects.

15.10. What is the transparent transmission length at 40 Gbps? Use appropriate data from Table 15.1. How does the transparent transmission length change at 20 Gbps? and at 10 Gbps?**Answer:**

For first question, the results are straightforward according to Equation 15.1 and the parameters given in Table 15.1.

For next questions, based on Equation 15.1, we see that the transparent transmission length is inversely proportional to the square-root of data rate. When data rate is decreased, its transparent transmission length is increased accordingly.

- 15.11.** Let us apply the results from the above problem to the network topology in Fig. 15.5. Let us consider *PMD only* and no other impairments. At 40 Gbps, which node pairs can communicate with each other?

Answer:

Refer to the answer above.

- 15.12.** Repeat the above problem for a data rate of 20 Gbps. Now, Node 1 can communicate with which other nodes (assuming no regeneration anywhere in the network)?

Answer:

Same as above.

- 15.13.** Repeat the above problem for Nodes 5, 19, and 24.

Answer:

Same as above.

- 15.14.** Based on your results in the last two problems, determine the nodes at which you will place the minimum number of regenerators so that all nodes can communicate with one another at a data rate of 20 Gbps.

Answer:

Same as above.

Chapter 16

Network Control and Management

- 16.1.** Explain the advantages and disadvantages of the three adaptive routing techniques that are based on global information: the completely centralized algorithm, the link-state approach, and the distance-vector approach.

Answer:

The centralized routing algorithms have the advantage of maintaining the whole network state information, and so these schemes often make the most optimal routing decisions. The decisions are easy to take. It doesn't have to have a high degree of coordination between the nodes.

The main disadvantage of the centralized architecture is the fact that it is very vulnerable to failure, and sometimes it may not be scalable.

The link-state approach is also based on global information. It has the downside that every node in the network must maintain complex network state information, which could waste a lot of resources, and again it can be unscalable. The update messages are communicated through broadcasts, and this can result in significant control overhead. The advantage is again, that the protocols are simple, and the decisions can be very accurate.

The distance-vector approach has the advantage that it only uses very little resources, and that it is very scalable with the network size. Despite all these, it has the disadvantages that the neighbors have to periodically exchange information, that the network converges to a stable state relatively slow, and that the protocols are more complicated, and that, sometimes, the decisions are based on the local optimum, instead of a global optimum.

- 16.2.** Given the topology in Fig. 16.16 (where the numbers on the edges represent the “costs”, say, distance in km), let us assume that we want to reserve a lightpath between the source (s) and the destination (d).

Let us assume that the lightpath of the lowest cost is tried first, and that it is available. Using the data below, compute the times needed to reserve the whole lightpath, using:

- parallel reservation
- hop-by-hop forward reservation.

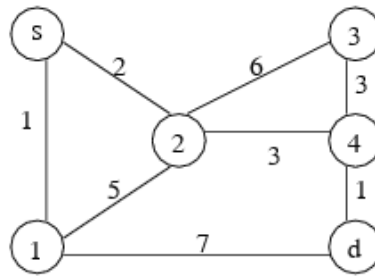


Figure 16.16 Network topology.

Which is better? What are the advantages and disadvantages of the two schemes?
Consider the following times:

- Time to process a message at a node: $T=10$ microseconds;
- Time to configure a node, and setup a crossconnect: $C=100$ microseconds;
- Propagation delay per km of fiber: $P=5$ microseconds;

Answer:

The shortest path that is chosen for our lightpath is the following: s - 4 - 2 - d.

Using parallel reservation the source will send [in parallel] a message to every node, in which it will try to reserve the desired wavelength on each link. In return it will send the acknowledgements in parallel. If all the links send positive ACKs, the lightpath is setup.

Since all the messages are sent in parallel, the total time necessary to reserve the lightpath is the maximum of all the times.

So, from s-4-2-d, the maximum time is the roundtrip between s-d, which is: $2 \times 3 \times T + C + 2 \times P \times (1+3+2)$. There are 3 processing nodes (2 times because of the return of the ACK), one setup time and a roundtrip propagation delay of $2 \times \text{number of km.} \times P$. So, the total time is: $60+100+60=220$ microseconds.

For hop-by-hop forward reservation, the reservation takes place at every node, so we will have a setup time for every node on the path.

The total time to setup the paths is the following.

The time to get to the destination (and reserve all the nodes on the way):

$$3 \times T + 3 \times C + P \times (1+3+2)$$

The time for the acknowledgement of 'lightpath setup' to get back to the source:

$$3 \times T + P \times (1+3+2).$$

So, the total time in this case is: 420 microseconds.

Even if the time is clearly larger in the hop-by-hop reservation, this method can be used without knowing the exact topology of the network, just by knowing the next hop. So, the hop-by-hop reservation mechanism is more scalable.

- 16.3.** (Path protection) Consider the following backbone network in Fig. 11.20 in PP. 557 with a primary and backup path pair between node 3 and 21. Suppose that a failure occurs on link $9 \rightarrow 12$ on the primary path. Calculate the total failure-recovery time. Assume that the propagation delay in the network is 0.005 ms per kilometer.

Answer:

Primary path: $3 \rightarrow 7 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 21$.

Backup path (for example): $3 \rightarrow 4 \rightarrow 5 \rightarrow 8 \rightarrow 10 \rightarrow 13 \rightarrow 17 \rightarrow 22 \rightarrow 21$.

Assuming the connection is 1:1 protected, no OXC configuration time needed. Failure-recovery time = time to notify source and destination. Assuming node 9 notifies node 3, and node 12 notifies node 21 in parallel, failure detection time is negligible and message processing time = 10 microseconds at each node.

$$\begin{aligned}
 \text{Failure recovery time (in ms)} &= (2000) \times 0.005 + 10/1000 \times 2 \\
 &= 10 + 2/100 \\
 &= 10.02 \text{ ms}
 \end{aligned} \tag{16.1}$$

- 16.4.** Given the topology in Fig. 16.17, where we have the path 1-2-3-4 already setup, and assuming that there is a failure on link 2-3, show the recovery procedure for the three known restoration techniques – path restoration, sub-path restoration, and link restoration. (In sub-path restoration, the upstream end-node of the failed link discovers the latter part of the path to the destination.)

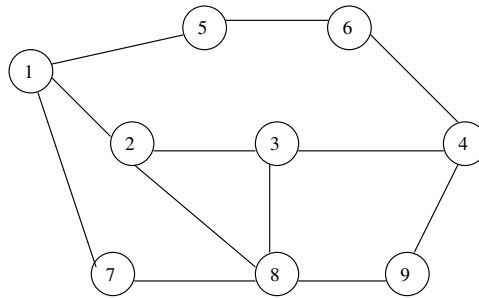


Figure 16.17 Another Network topology.

Answer:

The following are examples of:

- Path restoration: 1-7-8-9
- Subpath restoration: 1-2-8-9-4
- Link restoration: 1-2-8-3-4

- 16.5.** Given a network $G = (V, E)$, cost defined on each link $Cost : E \rightarrow \mathbb{Z}^+$, congestion on each link $C : E \rightarrow \lambda$, where C gives the number of available wavelengths on a link. Given a source s and destination d , design an algorithm to find the least-cost path from s to d with number of available wavelengths on the path $\geq N$, where N is the desired number of available wavelengths. State the time complexity of your algorithm.

Answer:

- 1) Remove all links in the network with number of free available wavelengths ($< N$).
- 2) Run shortest-path algorithm from S to D in the new network. Using Dijkstra's algorithm and a heap for calculating shortest paths, time is $O(E + V \log V)$.

- 16.6.** Consider a network where naive deflection routing is implemented. Nodes maintain a routing table indicating, for each destination, one or more alternate pre-computed outgoing links to reach

that destination, which are preferentially ordered. If resources are unavailable on the preferred link, the next preferred link is chosen for the route. Nodes maintain only local information. Construct an example complete with routing tables at each node and preferred links, where a link failure can lead to routing loops in a 4-node network.

Answer:

Routing Table at Node 4.

Destination	Next-Hop
3	3, 1
1	1,3
2	1, 3

Routing table at node 1.

Destination	Next-Hop
3	4, 2
...	...

When link 4→3 fails, node 4 forwards packets with destination node 3 to node 1, but node 1 uses node 4 as its referred next hop to node 3, and link 1→4 is working, node 1 forwards the packet back to node 4, and the packet loops between node 1 and node 4.

- 16.7.** In Section 16.5.2, stabilizing time was defined. Find the average stabilizing time using the link-state approach for the network in Fig. 16.10. The number on each link represents its length in units of 10 km. The propagation delay of a link with length 10 units, i.e., 100 km, is 500 μ s. Assume time to transmit/switch an LSA is 10 μ s.

Answer:

Stabilizing time as defined in the solution to Problem 16.5 is $\frac{1}{N} \sum_i T_i$. where

$$T_{ij} = \max(H_{ij}R + d_{ij})$$

d_{ij} = propagation delay from node i to j

H_{ij} = number of hops in this path

R = time to transmit/switch an LSA

For node 9, for instance:

For 9→7, time = $1 \times 10 \mu\text{s} + 190 \times 5 \mu\text{s}$.

For 9→10, time = $1 \times 10 \mu\text{s} + 110 \times 5 \mu\text{s}$.

Calculate delay from node 9 to all nodes in the network and take the maximum. This is T_9 . Similarly, find T_1, T_2, \dots, T_{15} . $\frac{1}{15} \sum_i T_i$ = stabilizing time on average in the network.

- 16.8.** Consider the two-step approach to find link-disjoint primary and backup paths. For the network shown in Fig. 16.18 find link-disjoint primary and backup paths using this approach from node A to node F. Assume all links have equal number of available wavelengths, which is equal to the number of free transmitters and receivers at each node. The numbers on links denote the cost.
- Is the two-step approach successful in finding link-disjoint primary and backup paths?
 - If not, find a pair of link-disjoint primary and backup paths from A to F without using the two-step approach.

Answer:

a) Two-step approach will not find a path pair. It chooses the min-cost path in the first step (A→B→C→D→E→F) and no further link disjoint paths can be found.

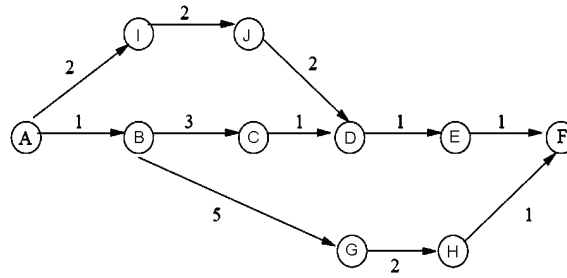


Figure 16.18 Disjoint primary and backup paths.

b) Link-disjoint paths are follows.

Primary path: $A \rightarrow I \rightarrow J \rightarrow D \rightarrow E \rightarrow F$. Backup path: $A \rightarrow B \rightarrow G \rightarrow H \rightarrow F$.

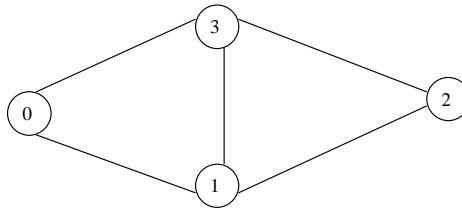


Figure 16.19 A sample network for protection switching.

- 16.9.** Consider the network topology in Fig. 16.19. Assume connection request $0 \rightarrow 2$ arrives. Find a primary and a backup path for the request. Calculate the average protection-switching time in case of a failure (of either of the two links) on the primary path. Message processing time at each node $D = 10 \mu s$, propagation delay on each link $P = 400 \mu s$, time to set cross connection is $C = 100 \mu s$, and time to detect a link failure $F = 100 \mu s$.

Answer:

Primary path: $0 \rightarrow 3 \rightarrow 2$. Backup path: $0 \rightarrow 1 \rightarrow 2$.

If $0 \rightarrow 3$ fails, we assume node 3 informs node 2. Node 0 detects a failure and sends traffic on path $0 \rightarrow 1 \rightarrow 2$. Assuming dedicated protection, no OXC configuration is necessary. So total time = failure detection ($100 \mu s$) + notify nodes ($400 \mu s$) + message processing ($10 \mu s$) = $510 \mu s$.

- 16.10.** Figure 16.20 shows the availability of various links in a network. Availability is the probability that a link is found in the operating state at any point of time. Assuming that the pair of link-disjoint primary and backup paths found in Problem 16.8 is used for a connection C, between nodes A and F, find the availability of connection C. Assume that the only failures are link failures and they are independent of one another.

Answer:

A = availability.

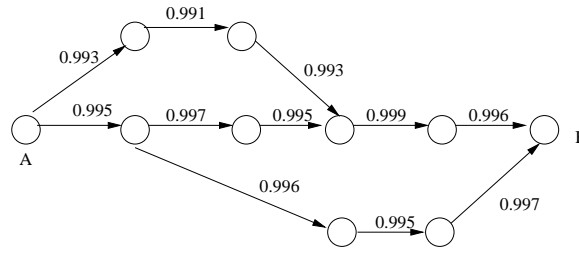


Figure 16.20 Availability of a connection carried over disjoint primary and backup paths.

$$\begin{aligned}
 A_{\text{connection}} &= A_{\text{primary}} + A_{\text{backup}} - A_{\text{primary}} \times A_{\text{backup}} \\
 &= (0.993 \times 0.991 \times 0.993) \times 0.999 \times 0.996 \\
 &\quad + 0.995 \times 0.996 \times 0.995 \times 0.997 - 0.9723 \times 0.983 \\
 &= 0.99953
 \end{aligned}
 \tag{16.2}$$

- 16.11.** Given a network with availability defined on links as in Problem 16.10, design an algorithm to find the most-reliable path (path with maximum availability) from source to destination. State the time complexity of your algorithm.

Answer:

We need to find a path such that A_{path} is maximized.

To find a path with availability greater than same value.

$$A_{e_1} \times A_{e_2} \times \cdots \times A_{e_m} \geq A \tag{16.3}$$

where e_1, e_2, \dots, e_m are links on the path, A_{e_i} is availability of e_i . Taking log on both sides,

$$\log A_{e_1} + \log A_{e_2} + \cdots + \log A_{e_m} \geq \log A \tag{16.4}$$

$\log A_{e_i} \leq 0$, availability is less than 1. The equation above is multiplied by -1.

$$-\log A_{e_1} - \log A_{e_2} - \cdots - \log A_{e_m} \leq -\log A \tag{16.5}$$

This is similar to a shortest path algorithm to find a path with maximum availability.

- 16.12.** What are the different categories of signaling and reservation protocols? State the advantages and disadvantages of each.

Answer:

The different protocols for signaling and resource reservation are:

1) Parallel

Advantage: Faster since nodes process messages in parallel.

Disadvantage: Requires global knowledge, as both path and wavelength must be known ahead of time.

2) Hop-by-hop reservation

Within this category, there are 2 sub-categories:

a) Forward reservation:

Advantages: No global knowledge needed, nodes locally exchange messages to reserve wavelengths.

Disadvantages: Long set-up time.

b) Backward reservation:

Advantage: No over-reservation of resources, no global information required.

Disadvantage: If multiple connections are being set up, even when one wavelength is available in forward direction, it may not be the message traverses in reserve direction.

- 16.13.** Differentiate between the link-state and distributed-routing approaches for connection management.

Answer:

Link-state: 1) updates flooded to all nodes in network; 2) fast set up of routes.

Distance vector: 1) updates sent to neighboring nodes; 2) slower to converge.

- 16.14.** Discuss the advantages and disadvantages of link vs. path protection.

Answer:

Link protection:

Advantage: signaling is fast.

Disadvantage: non-optimal route may be chosen for a connection.

Path protection:

Advantage: optimal end-to-end path is chosen.

Disadvantage: source and destination have to be notified, thus long signaling delay.

- 16.15.** What is the advantage of the Max-Shared-First heuristic over First-Fit and Last-Fit heuristics for wavelength assignment?

Answer:

Max-Shared First has the advantage that it maximizes sharing of wavelength between backup paths.

Chapter 17

Optical Packet Switching (OPS)

- 17.1.** What is meant by the electronic bottleneck in optical networks? What is the main cause of the electronic bottleneck? Is there a remedy for the electronic bottleneck? Justify.

Answer:

Electronic bottleneck refers to the phenomenon that the transportation capacity is limited by the electronic devices in an optical network. This issue is raised from the growing mismatch between the electronic operating frequencies of processors (currently 1-2 GHz) and the optical line rates today, namely, 10 Gbps and expected to exceed 80Gbps, even 160 Gbps per wavelength channel in the future. Currently, due to the premature of optical logical devices and optical random access memory (RAM), electronic devices must be adopted to process optical signaling in the optical networks. In order to avoid the electronic bottleneck, the current effort is to develop the optical switching.

- 17.2.** Please compare the characteristics of the various contention-resolution schemes in OPS networks? How can you efficiently solve the contention problem in OPS networks?

Answer:

Basically, there are three kinds of solutions in space, time and wavelength domains, respectively.

In space domain, deflection routing works quite well in small networks with high connectivity, i.e., if the nodes have several neighbors. Additionally, deflection routing can be only used in networks with low load. If average traffic load is high, deflected packets will only decrease the efficiency of the network. Deflection routing can be improved by accepting only certain ports. In general, deflection routing is an alternative where no buffer are implemented.

Switching in time domain in optical systems is problematic due to the lack of suitable optical RAM. An optical buffer consists of delay lines. When a packet is guided to a delay line, it is not possible to take it back before it has reached the other end of the line. Another problem of optical buffers is the power loss signal suffers when guided through a delay line. To compensate the power loss, either amplifiers or signal regenerators have to be used. There are two classes of optical delay line buffers: recirculating type and feed-forward type. In practice, fiber delay line could be avoided as much as possible.

Optical networks have an additional dimension, the wavelength. If contention occurs in a system using wavelength conversion, one packet is passed through and the other is guided to the same

output port but with a different wavelength. This solution is optimal in the respect that neither packet is delayed. This approach is more suitable for circuit switching, because a need for fast tunable wavelength converters restricts the use in optical packet switching networks.

The most effective combination of these is to use space deflection, buffering and wavelength conversion together. If cheap solution is needed, then minimum optical buffering with space deflection is the solution

- 17.3.** In an OPS network, fiber delay lines (FDLs) have been proposed to resolve contentions. Please compare contention solutions achieved by recirculating FDLs and feed-forward FDLs.

Answer:

The recirculating type offers the possibility to use same delay line several times, i.e., to recirculate packet. The problem with the recirculating type is the power loss that leads to the need for amplification and therefore also to noise. This limits the maximum buffering time or leads to the need for regeneration between the buffering times.

In the feed-forward type buffers the time packet is buffered is determined totally beforehand; a delay line cannot be used several times. Of course, if there are several delay line groups serially connected, the delay can be determined in several pieces, but the length of the delay lines determines the maximum delay anyway. An example of a cascaded delay line system implemented is optical synchronizer in KEOPS project. This way more accurate delays can be implemented without several recirculations. On the other hand, extra components (optical gate for each delay line) are needed.

- 17.4.** What is the difference between asynchronous and synchronous networks? Why does contention resolution perform better in a synchronous network than in an asynchronous network? For what price is this improved performance obtained?

Answer:

For the difference between asynchronous and synchronous networks, please refer to Section 17.2 in Pages from 798 to 803. In a synchronous network, we can predict packet arrivals based on the alignment of slotted time and fixed packet length. So it is helpful to plan and reserve the network resources for the coming packets. In an asynchronous network, the packet lengths are varied and their arrivals are random. This phenomenon could result in more contention in the scenario due to its non-predictability. That is why the contention resolution could be improved in a synchronous network than in an asynchronous network. But the synchronous network needs the fast clock recovery and bit-level synchronization for the alignment of slotted time.

- 17.5.** Clarify the differences between a slotted network and an unslotted network. It is important to realize synchronization of optical packet in slotted networks. Please design a synchronization scheme for an OPS network.

Answer:

Please refer to the synchronization scheme in KEOPS project that is shown in Fig. 17.27.

- 17.6.** Why can one set up a guard time between optical packets? Clarify the meaning and role of jitter in an optical network?

Answer:

Between the header and the payload there is a gap, the guard time, which ensures that jitter will not cause any problems and the payload can be transmitted to the destination undamaged. There is a guard time before and after the payload. The duration of the guard time varies, and consists of two parts. The first part is fixed time, which is always waited. Additionally, there is a second part, which is proportional to the time slot duration. The optimal payload duration

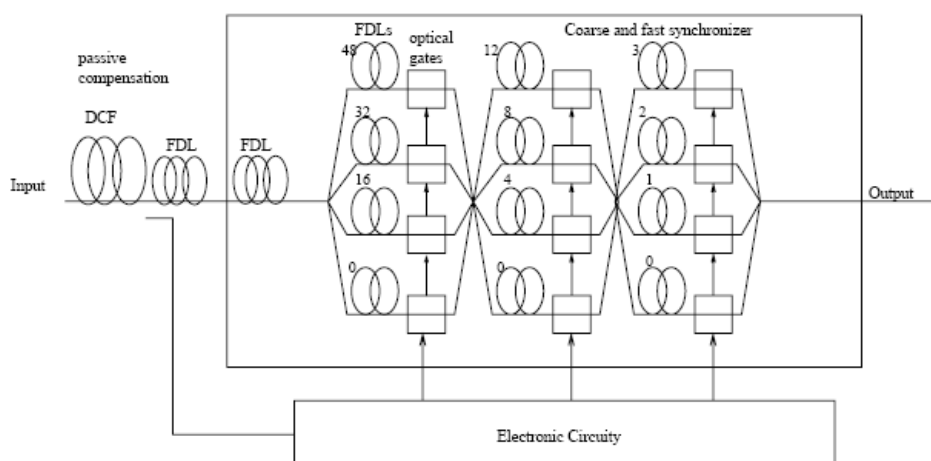


Figure 17.27 Synchronization scheme in KEOPS.

analyzed for ATM traffic is $2.726 \mu s$. The value is based on the transmission efficiency and, on the other hand, the fact that the length of the delay line buffers used for buffering the packets is limited.

The guard time should be enough margin for allowed uncertainties of optical signal jitter. Jitter is different from shift. When the signal will be distorted unidirectionally, the phenomenon is called as shift that is caused by the long-term affection such as temperature. Jitter refers to the signal aberrancy within the short-term time. Jitter is most from the unstable operation of devices. We can not eliminate the jitter from a system because there is no way to keep the devices be absolutely stable. That is why we need to provide a guard time to tolerate the limited jitter.

- 17.7. Please design a format of an optical packet, which may reduce the overhead in an OPS network with a comparative performance.

Answer:

Overhead is mainly from the process of packet head. In order to reduce the overhead, the packet head should be simplified as much as possible. But the routing and signaling information are embedded in the packet head. So overhead could be minimized when the routing and signaling protocols would be intelligent. Here is a revised version of packet format in KEOPS (see Fig. 17.28).

In the packet format, the structure of the header is the following. 2 bytes are dedicated to routing labels and signaling, and 1 bytes are reserved for identification of network resources. 0.5 byte flags the position of the payload relative to the header. 0.5 bytes were reserved to header synchronization pattern. Total length of the header is therefore 4 bytes.

- 17.8. Which would be practical for contention resolution in an OPS network: hybrid contention resolution or optical domain resolution?

Answer:

Hybrid contention resolution would be preferential when the optical hardware is not mature at the moment. In the hybrid resolution, all measures could be applied together in electronic and optical domains. For example, when electronic buffers are added into each switching node in case that optical payload could be overflowed in an intermediate node.

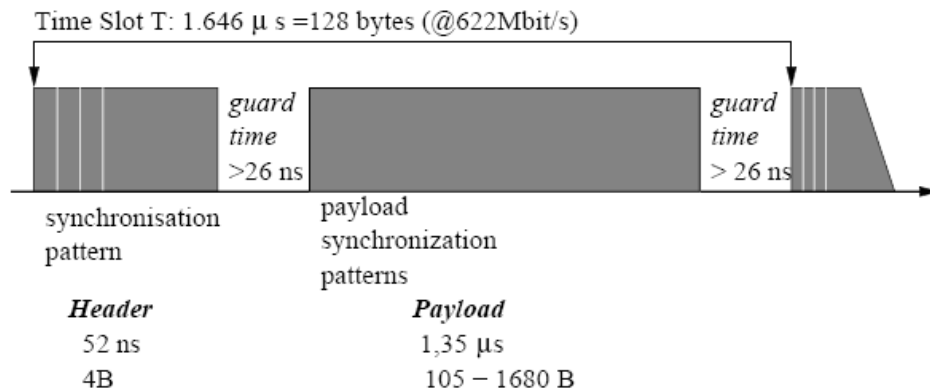


Figure 17.28 Packet format.

17.9. What factors should be used to classify different priority services in an OPS network? Please design a reasonable scheme.

Answer:

For differentiated services, latency, throughput, and bandwidth should be considered. For a specific priority service, some special requirements could be applied, such as live-on video.

An OPS architecture is shown in Fig. 17.29. In the scheme, the high-priority services are switched without extra delay, and the low-priority services are recirculated into the degenerate FDLs if they meet contentions.

17.10. Please design a contention resolution scheme by using FDLs, which would be helpful to reduce the possibilities of a second contention of optical packets in an OPS network.

Answer:

Such an architecture is shown in Fig. 17.30. First, FDLs are used for the input queues. These queues reduce the first contention. After the first queues, a packet could meet the contention at output ports again. In order to avoid the second contention, these packets could be re-queued or re-buffered. So recirculated FDLs could be used to deal with the second contention.

17.11. Please design a scheme for clock recovery in an OPS network.

Answer:

There is an example that is shown in Fig. 17.31.

17.12. Why is it difficult to realize a practical optical logic device? When an all-optical network is built, how can it be made compatible with existing electronic telecommunication networks, or should we totally discard the outdated telecommunication networks? Please imagine what a future optical network will be like if optical logic devices would become mature in the future.

Answer:

There are two main reasons that limit the development of optical logic devices. One is lacking of fast and deep optical random access memory. The other one is infant of integrated optical control contrasted with the matured silicon industry in the electronic field.

When the all-optical network is built, the existing electronic telecommunication networks could be redundancy in terms of technologies. But it could be operated as the upper layer to be integrated with the all-optical network. When the optical logic devices will be applied, the optical switching technologies could catch up with the development of optical transmission technologies.

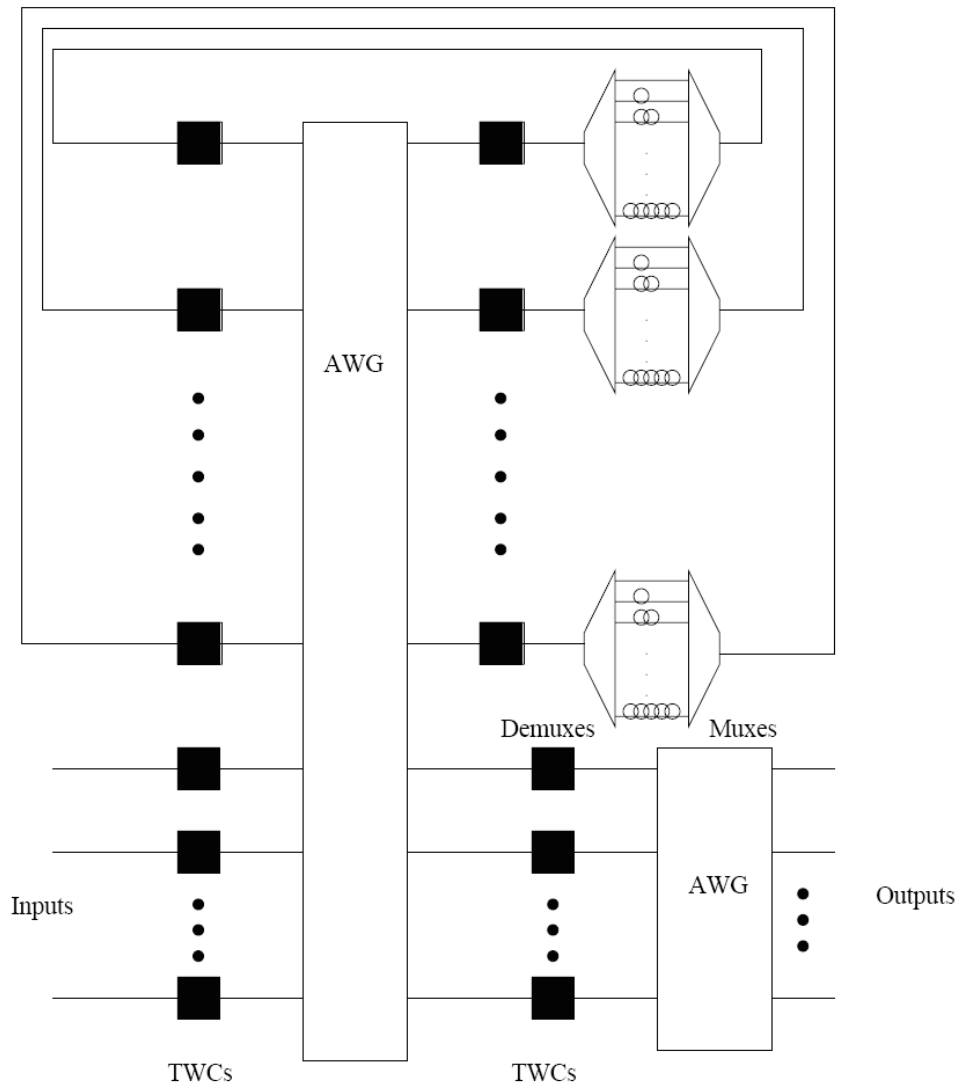


Figure 17.29 Switching architecture for differentiated services.

17.13. Consider an optical buffer in an optical packet switch. The buffer consists of four delay lines. An incoming packet can be switched to any one of the delay lines, or it can be dropped. Packet lengths are fixed to one time slot. Consider two cases. In the degenerate case, the delay lines provide delays of 1, 2, 3, and 4 slots. In the non-degenerate case, the delay lines provide delays of 1, 2, 3, and 6 slots.

- Show for each case the buffer contents in each time slot if 3 packets, A, B, and C arrive in the first time slot, and 3 packets, D, E, and F arrive in the second time slot.
- What are the advantages and disadvantages of degenerate versus non-degenerate buffers?
- Compare the situation in which 4 packets arrive every other time slot to the situation in which 2 packets arrive every time slot. For each case, attempt to design a buffer (degenerate or non-degenerate) that can accommodate all packets over a period of 4 time slots while using the minimum number of delay lines. How many delay lines would you need in each case if void filling was not allowed?

Answer:

- We suppose that four delay lines are parallel to form the buffer.

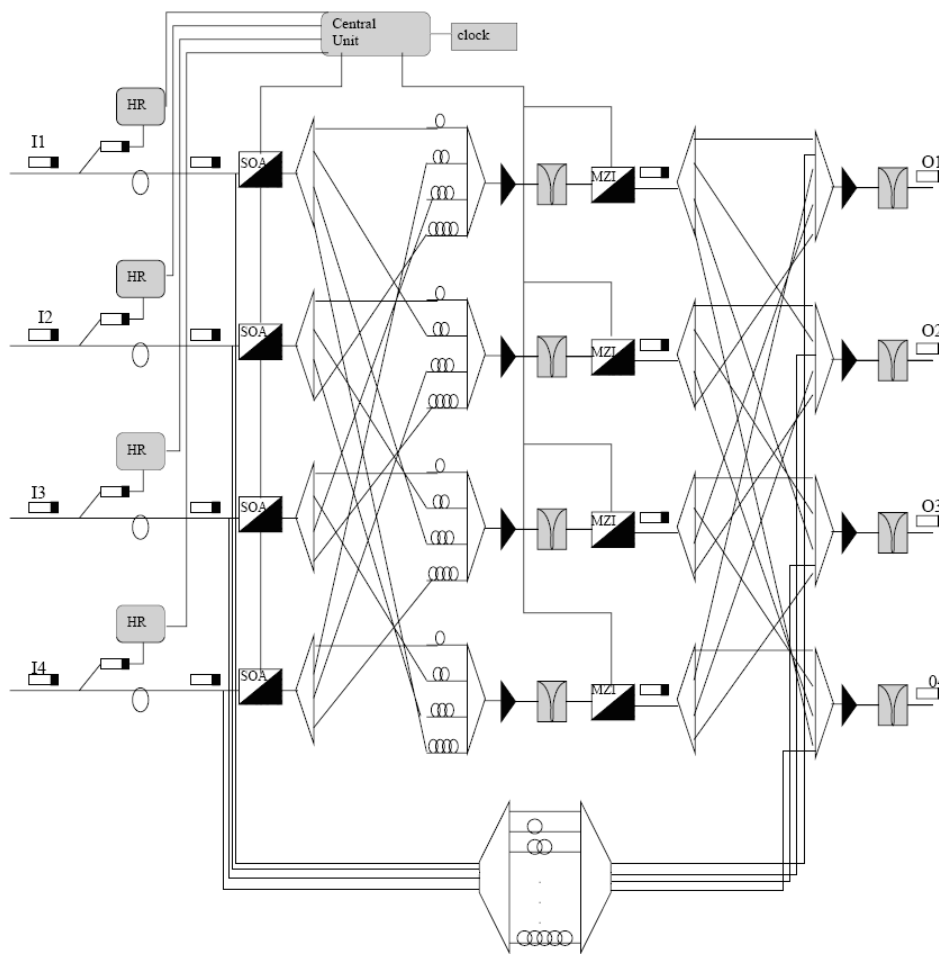
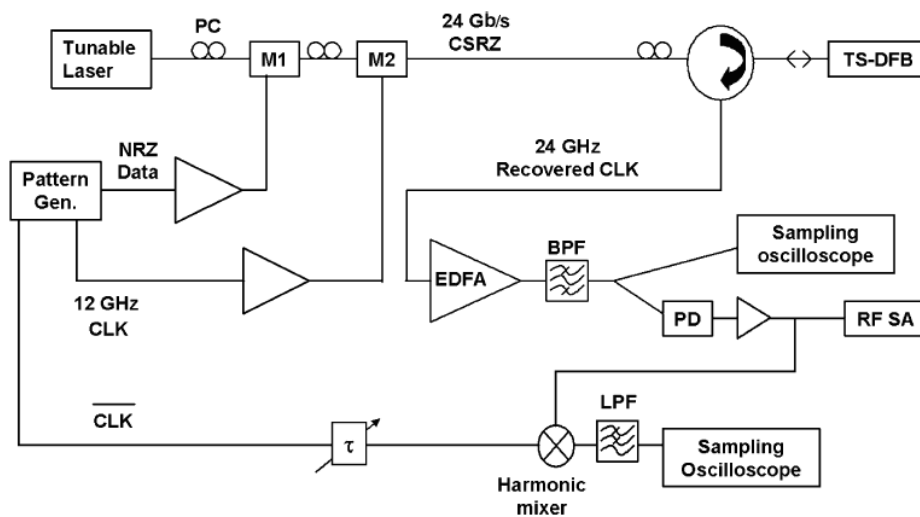


Figure 17.30 Switching architecture.



(M1, M2: modulators; PC: polarization controller; RF SA: RF spectrum analyzer; PD: photodetector; τ : delay).

Figure 17.31 All-optical clock recovery (Source: Inwoong Kim and etc. “Dynamics of All-Optical Clock Recovery Using Two-Section Index- and Gain-Coupled DFB Lasers,” IEEE Journal of Lightwave Technology, vol. 23, no. 4, pp. 1704–1712, April 2005.)

Time slot 1

A
Bo
Coo
oooo

Where o is empty slot in delay line The effective queue is: oCBA

Time slot 2

o
oB
DCo
Eooo

B and C move forward by one slot, and D and E enter the delay lines. The effective queue is: EDCB

Time slot 3

o
oo
oDC
oEoo

Effective queue is: oEDC

For non-degenerate: Slot 1

A
Bo
Coo
oooooo

Slot 2

o
oB
DCo
Eooooo

Slot 3

o
oo
oDC
oEoooo

(b) Degenerate: in-order delivery of packets, holds fewer packets resulting in higher packet loss
Non-degenerate: can store greater number of packets, may have longer delays and out-of-order packets

(c) 4 packets every other time slot, 4 delay lines:

A	o	E	o
Bo	oB	Fo	oF
Coooo	oCooo	GoCoo	oGoCo
Dooooo	oDoooo	HoDooo	oHoDoo

queue: DCooBA oDCooB HGDCFE oHGDCF

2 packets every time slot, 2 delay lines:

A	C	E	G
Boooo	DBooo	FDBoo	HFDBo

queue: BoooA DBooC FDBoE HFDBG

If no void filling is allowed, the first case would require 6 delay lines, and the second case would require 5 delay lines.

Chapter 18

Optical Burst Switching (OBS)

- 18.1.** Why is OBS considered to be a hybrid architecture? Please point out each component of OBS, how it can be implemented, and which domain the corresponding function can be realized in (electronic or optical)?

Answer:

A hybrid architecture refers such an architecture with both electronic and optical components. OBS is designed to transfer traffic data in high-speed and high-capacity. But optical random access memory is still infant, electronic component is adopted to process transmission signaling. Thus, traffic data could be kept in optical domain to achieve the high-speed and high-capacity transmission while processing the transmission signaling in electronic component. Optical to Electronic to Optical converter must be applied for processing the signaling in OBS network.

- 18.2.** What are the differences between OPS and OBS? Considering current limits of optical hardware technologies, which approach (OPS or OBS) seem more promising for realization today?

Answer:

Please refer to Table 18.1 in Page 846 for the comparison between OPS and OBS. Because the optical logic and optical memory devices are premature in application level, optical logic signaling must be converted into electronic signal for processing. Rather than OPS, logic functions are separated from its relational switching functions in OBS. Please refer to Figure 1.2 in Page 9, OBS just adopts the system “B” that is cost-effective, flexible, and scalable. So, OBS is more promising for realization than OPS in terms of the limits of optical hardware technologies.

- 18.3.** If optical logical devices and optical random access memory would become mature in the future, will OBS become outdated? Please state and justify your views of OBS technology’s future promise.

Answer:

Originally, OBS was proposed for the tradeoff between optical hardware limits and optical transportation technologies mature. More importantly, the concept of its burst switching is matched with the fundamental character of self-similar traffic today. Since the self-similar feature is more evident with the traffic explosively increasing, the burst switching is most significant. Currently, OBS is the hybrid architecture. Thus, the development of optical logical devices and optical

random access memory would powerfully advance OBS into all-optical architecture rather than outdated.

18.4. Compare JET and TL. In what aspects does TL surpass JET? Why?

Answer:

Please refer to Table 18.2 in Page 853. Compared with JET, there is no prediction with TL that is a refined reservation of resources. The precision of reservation could be improved by 2 - 3 orders magnitude in TL that is most helpful to avoid contentions of reservations.

18.5. As the volume of data traffic continues to grow, why is it essential to consider the nature of traffic in optical switching? If not, what will happen to optical switching?

Answer:

Today, data traffic demonstrates the self-similar feature that is quite different from the conventional voice traffic. Self-similarity in data traffic has been seen to pose an immediate threat to the current network at fundamental, where challenges have been encountered on various aspects of network design, such as network traffic engineering, performance evaluation, resource planning, and operational procedures established by the former studies over the past decades. Thus, it is more than essential to consider the fundamental nature of traffic patterns in such a circumstance when next-generation optical switching technology is devised.

18.6. What issues should be discussed in Optical Self-Similar Cluster Switching (OSCS)? Please justify your analysis.

Answer:

The special brightness in OSCS is to take full advantages of the unfavorable self-similar feature of data traffic rather than overcoming it. Thus, the source of network performance deterioration is released. OSCS extends the more possibilities of optical switching applications in the future.

18.7. Which stage is key in OSCS? Why? Can you determine any other solution for the key stage?

Answer:

In OSCS, it is most important how to filter out the self-similar clusters from data traffic. We could define two thresholds to filter out the clusters from data traffic, i.e., upper threshold and lower threshold. Lower threshold could define the minimal length of clusters with the constrain of switching efficiency. Upper threshold is for the network resources assignment. If the upper threshold is infinite, it could lead to more contention of resources assignment. The last part of the question is open for readers.

18.8. In an OBS network, nodes are classified into two types: edge nodes and intermediate (or core) nodes. What is required for intermediate nodes in order to guarantee that an OBS network can be implemented without any extra delay? Since edge nodes are important for OBS networks, what functions should be realized in edge nodes? Please design an edge node to implement its basic functions.

Answer:

In order to achieve an OBS network without any extra delay, the core nodes should be transparent. Please refer to Fig. 18.8 in Page 859 that is such an architecture. In OBS, the signaling and routing functions are separated from switching function with a certain offset time. For implementing the network resource reservations, the signaling and routing information must be initialized in the edge nodes. The last part of the question is up to readers.

- 18.9. In an OBS network, what are the two kinds of signaling protocols used? Which kind does TL belong to? Please point out the differences between two kinds of signaling protocols.

Answer:

There are two kinds of signaling protocols, i.e., one-way and two-way. TL is the one-way signaling protocol. The big difference between them is whether there is an acknowledge mechanism or not. One way is more efficient and easy-operation without an acknowledge mechanism. But, the switching performance could be improved in a two-way signaling protocol because of the acknowledge mechanism.

- 18.10. What is the function of a routing protocol? What is the first consideration behind developing OBS-specific routing algorithms?

Answer:

A routing protocol implements its routing algorithms, maintains the information about topology and network resources and distributes non-local resources along the routing path. In an OBS network, signaling protocol is run independently from the transportation of traffic data. The situation should be considered when we develop OBS-specific routing algorithms. Otherwise, the routing path assigned by a routing algorithm could be conflicted with its signaling protocol.

- 18.11. Is it possible to bind signaling and routing protocols together without any other extra protocols? Please try to compare TS-LSP with GMPLS.

Answer:

Yes, TS-LSP is one of these efforts. The fundamental goal of TS-LSP is to bind signaling and routing together while increasing the network efficiency and improving network performance. At the same time, the overhead is more saved by ignoring the link management protocol that is the important function in GMPLS.

- 18.12. How will the network operations, management, measurement, and control plane architectures exist across administrative domains in an OBS network? Is it possible to realize a seamless integration between OBS and ASON?

Answer:

In ASON networks, there are three planes, i.e., transport plane, control plane and management plane. Management plane acts to tune up the relations between transport plane and control plane. Three planes are independent each other in terms of logic functions. But they could share the same physical elements. In OBS networks, control functions are separated from transport functions, in which they may or may not share a same physical elements. Furthermore, all related logic functions in OBS networks are implemented in administrative domains. In fact, there are only two planes in OBS networks, i.e., switching plane and control plane. Regarding the control-plane structure, OBS is quite different from ASON. Thus, it is not possible to integrate OBS with ASON due to total differences of control structures.

- 18.13. Is OBS an asynchronous or a synchronous network? Burst contention is a major challenge towards realization of OBS networks. How can one solve the possible contentions in an OBS network? Please discuss the issues from the views of contention and switching mechanisms.

Answer:

OBS is an asynchronous network. In the asynchronous network, burst arrives and enters are random. This could lead to more contention of bursts. Each burst has more traffic data. It could drop off more traffic data if one burst would be lost. That is why the contention solution is most important in OBS network. There are three kinds of ways to solve the issue, i.e., wavelength, space, and time dimensions. One or two of them could be implemented.

- 18.14.** Consider a GRID Network. What are the relationships between such a network and an OBS network?

Answer:

GRID network is designed for transferring large files between users, such as, such as e-science, e-business, e-health, and e-government. In order to implement these GRID network applications, some issues must be addressed first, such as throughput, priority, security, latency, cost, control and management, QoS and storage capacity. OBS seems to be a better choice to deal with these issues above. That is the reason that OBS provides huge capacity and relatively low latency, controls and allocates bandwidth dynamically. OBS is also more resilient to a wide variety of traffic characteristics and distributions. Optical GRID network infrastructure could be one of OBS applications in the future.

- 18.15.** About optical switching development, which is better: traffic-oriented switching or user-oriented switching?

Answer:

It is a open question for readers. To some extent, traffic is associated with users. Traffic-oriented switching focuses on the traffic patterns in each switching node. User-oriented switching is to meet the requirements of users. In fact, we need to determine how to assign network resources before designing a switching architecture.

- 18.16.** How long is the required offset between a burst and its header in an OBS network using the JET (just enough time) protocol? How is the offset adjusted for bursts of different priorities?

Answer:

The minimal offset must be no less than the sum of signaling process time in each node of routing path. For the differentiated services, the length of offset is assigned according to the priorities of traffic. Usually, network resources will be prioritized to the high-priority traffic. So, the length of offset is increased accordingly based on the priorities.

- 18.17.** In the last two chapters' references, various versions of optical switching are proposed. Please give your concerns about these versions, i.e., WR, OPS, OBS, Optical flow switching, Optical Self-similar Cluster Switching, Optical label switching, Optical code switching, and so on.

Answer:

In terms of switching granularities, WR is the largest one, and OPS is the smallest one of all. All other switching mechanisms are between WR and OPS. From the point of technologies mature view, WR is most favorable. For the traffic pattern, self-similar cluster switching is a better choice. Regarding to the network security, optical label switching and optical code switching are better than other ones. About the switching capacities, optical code switching and OPS are most valued.



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