2 Interaction of Charged Particles with Strong Electromagnetic Wave in Dielectric Media. Induced Nonlinear Cherenkov Process

What can we expect from particle–strong wave interaction in a medium essentially different from that of a vacuum?

It is well known that in a medium with the refractive index $n(\omega) > 1$ *(dielectric media) the Cherenkov effect takes place — charged particle moving with a velocity* $\mathbf{v} = \mathrm{const}$ *radiates spontaneously transverse EM wave of frequency* ω *at the angle* θ *satisfying the condition of coherency* $\cos\theta = c/\mathrm{v}n(\omega)$. *This means that in the presence of an external plane EM wave of the same frequency* ω *propagating at this angle with respect to the particle motion the spontaneous Cherenkov radiation of the particle will acquire induced character and the inverse process of Cherenkov absorption from the incident wave by the particle is possible as well. This is the general character of arbitrary type spontaneous radiation process in corresponding induced one. However, in contrast to the noncoherent process (e.g., bremsstrahlung), if the spontaneous process is of coherent nature, such as the Cherenkov process, for the satisfaction of the condition of coherency the external wave should be weak enough to not change considerably the particle initial velocity* $\mathbf{v}$ *and violate the mentioned condition of coherency of the spontaneous process. Consequently, this explanation of formation of induced process with the charged particles (induced free–free transitions in quantum terminology) corresponds to the linear theory.*

The behavior of induced Cherenkov process in the strong EM wave field is quite different from the mentioned one. The existence of the threshold value of the particle velocity for the spontaneous Cherenkov radiation ($\mathrm{v} > c/n(\omega)$) *stipulates for the threshold value of the wave intensity essentially changing the character of the dynamics of the particle–wave interaction in a medium and, consequently, the character of electromagnetic processes in dielectriclike media, proceeding in the presence of strong radiation fields. As we will see later, the peculiarities which arise at the nonlinear interaction of charged particles with strong EM waves are the general features of coherent processes like the Cherenkov one.*

To reveal the nonlinear behavior and principal peculiarities of a particle–strong wave interaction in a medium, this chapter will present the nonlinear classical theory of induced Cherenkov process.

2.1 Particle Classical Motion in the Field of Strong Plane EM Wave in a Medium

A plane quasi-monochromatic EM wave in a medium may be described by the vector potential $\mathbf{A}(t, \mathbf{r}) = \mathbf{A}(t - n_0 \nu_0 \mathbf{r}/c)$, where $n_0 \equiv n(\omega_0)$ is the refractive index of the medium at the carrier frequency of the wave (actually laser radiation). For the electric and magnetic fields we will have respectively

$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(t - n_0 \nu_0 \mathbf{r}/c); \quad \mathbf{H}(t, \mathbf{r}) = \mathbf{H}(t - n_0 \nu_0 \mathbf{r}/c); \quad \mathbf{H} = n_0 \left[\nu_0 \mathbf{E}\right]. \tag{2.1}$$

Hereafter we will assume that the frequency ω_0 is far from the main resonance transitions between the atomic levels of the medium to prohibit the wave absorption and nonlinear optical effects in the medium and consequently $n_0 = \sqrt{\varepsilon_0 \mu_0} = \text{const}$ will correspond to the linear refractive index of the medium (ε_0 and μ_0 are the dielectric and magnetic permittivities of the medium, respectively).

Without loss of generality we will direct vector ν_0 along the OX axis of a Cartesian coordinate system: $\nu_0 = \{1, 0, 0\}$ and the relativistic classical equations of motion of a charged particle in the field (2.1) will be written in the form

$$\frac{dp_x}{dt} = n_0 \frac{e}{c} \left[\mathrm{v}_y E_y(\tau) + \mathrm{v}_z E_z(\tau)\right], \tag{2.2}$$

$$\frac{dp_y}{dt} = e \left(1 - n_0 \frac{\mathrm{v}_x}{c}\right) E_y(\tau); \quad \frac{dp_z}{dt} = e \left(1 - n_0 \frac{\mathrm{v}_x}{c}\right) E_z(\tau), \tag{2.3}$$

where $\tau = t - n_0 x/c$ is the retarding wave coordinate of the quasi-monochromatic plane EM wave in a medium.

The integration of Eqs. (2.2) and (2.3) is carried out as was done for Eqs. (1.3) and (1.4) and with Eq. (1.9) one can obtain the particle transversal momentum

$$p_y = p_{0y} - \frac{e}{c} A_y(\tau); \quad p_z = p_{0z} - \frac{e}{c} A_z(\tau) \tag{2.4}$$

and integral of motion

$$K \equiv \mathcal{E} - \frac{c}{n_0} p_x = \text{const}, \tag{2.5}$$

which together with the relation $\mathcal{E}^2 = \mathbf{p}^2 c^2 + m^2 c^4$ determine the energy of the particle in the field of strong quasi-monochromatic plane EM wave in a medium:

$$\mathcal{E} = \frac{\mathcal{E}_0}{n_0^2 - 1} \left\{ n_0^2 \left(1 - \frac{\mathrm{v}_{0x}}{c n_0}\right) \mp \left[\left(1 - n_0 \frac{\mathrm{v}_{0x}}{c}\right)^2 \right.\right.$$

$$-\frac{\left(n_0^2-1\right)}{\mathcal{E}_0^2}\left(e^2\mathbf{A}^2\left(\tau\right)-2ec\mathbf{p}_0\mathbf{A}\left(\tau\right)\right)\Bigg]^{1/2}\Bigg\}. \qquad (2.6)$$

Here $\mathbf{p}_0 = \{p_{0x}, p_{0y}, p_{0z}\}$, $\mathcal{E}_0$, and v_{0x} are the particle initial momentum, energy, and longitudinal velocity, respectively, at $\tau = -\infty$ ($\mathbf{A}(\tau)\mid_{\tau=-\infty} = 0$ according to unique definition of the vector potential of the wave (1.7)).

Equation (2.6) describes the energy exchange between the charged particle and plane transverse EM wave of arbitrary intensity in a medium in the general case. However, besides the formula of the energy for the description of the particle nonlinear dynamics in this process we will need the formula for the longitudinal velocity of the particle in the field — a major characteristic of the induced Cherenkov process. The latter can be defined from the relation $\mathrm{v}_x = c^2 p_x/\mathcal{E}$ within the expression for the longitudinal momentum of the particle p_x, which is determined by the integral of motion (2.5) and Eq. (2.6). Then for the longitudinal velocity of the particle we will have

$$\mathrm{v}_x = cn_0\frac{1-\mathrm{v}_{0x}/cn_0 \mp \sqrt{D}}{n_0^2\left(1-\mathrm{v}_{0x}/cn_0\right) \mp \sqrt{D}}, \qquad (2.7)$$

where

$$D \equiv \left(1-n_0\mathrm{v}_{0x}/c\right)^2 - \left(\left(n_0^2-1\right)/\mathcal{E}_0^2\right)\left(e^2\mathbf{A}^2\left(\tau\right)-2ec\mathbf{p}_0\mathbf{A}\left(\tau\right)\right). \qquad (2.8)$$

Further, for the consideration of radiation processes we will need the formulas for transversal velocities of the particle, which can be defined from Eqs. (2.4) and (2.6):

$$\mathrm{v}_{y,z} = \frac{c}{\mathcal{E}_0}\frac{\left(n_0^2-1\right)\left(cp_{0y,z}-eA_{y,z}\left(\tau\right)\right)}{n_0^2\left(1-\mathrm{v}_{0x}/cn_0\right)\mp\sqrt{D}}. \qquad (2.9)$$

As is seen from Eqs. (2.6)–(2.9) the expressions determining the particle energy or velocity in the wave field are, first, not single-valued and, second, may become imaginary depending on particle and wave parameters. The peculiarity arising in the induced Cherenkov process because of particle–strong wave nonlinear interaction is connected with this fact. Hence, treatment of the particle dynamics in this process should start by clarification of these questions.

2.2 Nonlinear Cherenkov Resonance and Critical Field. Threshold Phenomenon of Particle "Reflection"

To consider the behavior of a particle upon nonlinear interaction with a strong wave in a medium on the basis of Eq. (2.6) we will analyze the case where the initial velocity of the particle is directed along the wave propagation direction for which the picture of the particle nonlinear dynamics is physically more evident. In this case Eq. (2.6) becomes

$$\mathcal{E} = \frac{\mathcal{E}_0}{n_0^2 - 1}\left[n_0^2\left(1 - \frac{\mathrm{v}_0}{cn_0}\right)\right.$$

$$\left.\mp\sqrt{\left(1 - n_0\frac{\mathrm{v}_0}{c}\right)^2 - (n_0^2 - 1)\left(\frac{mc^2}{\mathcal{E}_0}\right)^2 \xi^2(\tau)}\right], \tag{2.10}$$

where $\xi^2(\tau)$ is the relativistic invariant parameter of a plane EM wave intensity, determined by Eq. (1.19).

As is seen, Eq. (2.10) is twovalence and, at first, we shall provide the unique definition of the particle energy in accordance with the initial condition. In the case of plasma ($n_0 < 1$) or vacuum ($n_0 = 1$) the term under the root is always positive, hence, in these cases one has to take before the root only the upper sign $(-)$ to satisfy the initial condition $\mathcal{E}(\tau) = \mathcal{E}_0$ when $\xi(\tau) = 0$. In the case of a vacuum, Eq. (2.10) yields results obtained in Chapter 1 (see Eq. (1.13) or Eqs. (1.24) and (1.36) for the circular and linear polarizations of the wave).

Further investigation is devoted to the case of a medium with refractive index $n_0 > 1$. In this case the nature of the particle motion essentially depends on the initial conditions and the value of the parameter $\xi(\tau)$ as far as the expression under the root in Eq. (2.10) may become negative, while the energy of the particle should be a real quantity and uniquely defined as well. To solve this problem one needs to pass the complex plane, according to which we represent Eq. (2.10) in the form of known inverse Jukowski function (to determine also the sign before the root corresponding to initial condition $\mathcal{E}(\tau)|_{\tau=-\infty} = \mathcal{E}_0$ since at $n_0 > 1$ the quantity $1 - n_0\mathrm{v}_0/c$ under the root may be negative as well):

$$\mathcal{E} = \frac{\mathcal{E}_0}{n_0^2 - 1}\left[n_0^2\left(1 - \frac{\mathrm{v}_0}{cn_0}\right) \mp \left(1 - n_0\frac{\mathrm{v}_0}{c}\right)\sqrt{1 - \frac{\xi^2(\tau)}{\xi_{cr}^2}}\right], \tag{2.11}$$

where

$$\xi_{cr} \equiv \frac{\mathcal{E}_0}{mc^2}\frac{|1 - n_0\frac{\mathrm{v}_0}{c}|}{\sqrt{n_0^2 - 1}}. \tag{2.12}$$

If $\xi_{max} < \xi_{cr}$ (ξ_{max} is the maximum value of the parameter $\xi(\tau)$) the expression under the root in Eq. (2.11) is always positive and in front of the root one has to take the upper sign $(-)$ according to the initial condition.

Then $\mathcal{E} = \mathcal{E}_0$ after the interaction ($\xi(\tau) \to 0$) and the particle energy remains unchanged.

If $\xi_{max} > \xi_{cr}$ the particle is unable to penetrate into the wave, i.e., into the region $\xi > \xi_{cr}$ since at $\xi > \xi_{cr}$ the root in Eq. (2.11) becomes a complex one. This complexity now is bypassed via continuously passing from one Riemann sheet to another, which corresponds to changing the inverse Jukowski function from "$-$" to "$+$" before the root. Hence, the upper sign ($-$) in this case stands up to the value of the wave intensity $\xi(\tau) < \xi_{cr}$, then at $\xi(\tau) = \xi_{cr}$ the root changes its sign from "$-$" to "$+$", providing continuous value for the particle energy in the field. The intensity value $\xi(\tau) = \xi_{cr}$ of the wave is a turn point for the particle motion, so that we call it the critical value.

Thus, when the maximum value of the wave intensity exceeds the critical value a transverse plane EM wave in the medium becomes a potential barrier and the "reflection" of the particle from the wave envelope ($\xi(\tau)$) takes place. If now $\xi(\tau) \to 0$, we obtain after the "reflection" for the particle energy

$$\mathcal{E} = \mathcal{E}_0 \left[1 + 2\frac{1 - n_0 \frac{\mathrm{v}_0}{c}}{n_0^2 - 1} \right]. \tag{2.13}$$

If the initial conditions are such that the wave pulse overtakes the particle ($\mathrm{v}_0 < c/n_0$), then after the "reflection" $\mathcal{E} > \mathcal{E}_0$ and the particle is accelerated. But if the particle overtakes the wave ($\mathrm{v}_0 > c/n_0$), then $\mathcal{E} < \mathcal{E}_0$ and particle deceleration takes place.

This nonlinear threshold phenomenon is bounded on the stimulated Cherenkov process. The coherent nature of the Cherenkov process is related to the existence of the critical intensity of the wave ξ_{cr}. Indeed, from Eq. (2.7) it follows that when $\xi = \xi_{cr}$ the longitudinal velocity of the particle in the field becomes equal to the phase velocity of the wave: $\mathrm{v}_x(\xi) \mid_{\xi=\xi_{cr}} = c/n_0$ irrespective of its initial velocity v_0. The latter is the Cherenkov condition of coherency in a dielectric medium. Fulfillment of the Cherenkov condition in the strong wave field leads to the nonlinear Cherenkov resonance, at which the induced absorption or emission of Cherenkov photons becomes essentially multiphoton. As a result, the particle velocity becomes greater or smaller (depending on initial velocity v_0) than the wave phase velocity and it leaves the wave, i.e., the "reflection" from the wave front occurs. In addition, the energy lost by the particle at the deceleration ($\mathrm{v}_0 > c/n_0$) is coherently transferred to the wave via induced Cherenkov radiation. As is seen from Eq. (2.13), for the initial "Cherenkov velocity" $\mathrm{v}_0 = c/n_0$ the energy of the particle after the "reflection" does not change: $\mathcal{E} = \mathcal{E}_0$, which is in congruence with the critical value of the field: $\xi_{cr} = 0$ at the initial Cherenkov velocity of the particle (see Eq. (2.7)). The latter confirms the nonlinear character of Cherenkov resonance in the strong wave field. In this case the induced Cherenkov effect will occur at $\mathrm{v}_x = \mathrm{v}_0 = \mathrm{const}$, i.e., the wave field should not change the particle initial velocity, which can take place approximately, only in the weak fields — induced Cherenkov effect in the linear theory (in accordance with the initial condition $\xi(\tau) \mid_{\tau=-\infty}= 0$ — the wave is turned on adiabatically — it is evident that in this case the linear induced Cherenkov effect is absent as well).

This threshold phenomenon of the particle "reflection" can be more clearly presented in the frame of reference connected with the wave. In this frame the electric field of the wave vanishes ($\mathbf{E}' \equiv 0$) and there is only the static magnetic field ($|\mathbf{H}'| = |\mathbf{H}| \sqrt{n_0^2 - 1}/n_0$). For not very large particle velocities in this frame the magnetic field will turn the particle back — elastic reflection from the standing wave barrier. In the opposite case the particle slips through the magnetic field. Such behavior of the particle in the intrinsic frame of the wave corresponds to the cases $\xi > \xi_{cr}$ (large velocities close to the Cherenkov one at which ξ_{cr} is small and the condition $\xi > \xi_{cr}$ is achievable) and $\xi < \xi_{cr}$ in the laboratory frame of reference, respectively (see Eq. (2.7)). Note that because of the particle reflection from the standing barrier in the frame of reference of the slowed wave we term the revealed nonlinear phenomenon a "reflection" one.

Hence, the threshold-coherent nature of spontaneous Cherenkov effect over the particle velocity ($\mathrm{v}_{th} = c/n_0$) causes the threshold for the external wave intensity ($\xi_{th} \equiv \xi_{cr}$), which in turn causes the phenomenon of particle "reflection" from the plane EM wave. It is worth emphasizing that the latter may be very small ($\xi_{cr} \to 0$) if the particle initial velocity is close to the wave phase velocity ($\mathrm{v}_0 \to c/n_0$), which means that in this case the linear theory is not applicable even for very weak wave fields ($\xi \to 0$), since the nonlinear phenomenon of particle "reflection" will take place ($\xi > \xi_{cr} \to 0$). Also, it is important that due to this phenomenon the induced process at $\xi > \xi_{cr}$ proceeds strictly in a certain direction — either radiation or absorption (inverse induced process), which has a principal meaning for induced free–free transitions related especially to problems of laser acceleration and free electron lasers.

Let us estimate the particle energy change due to "reflection". Note, at first, that the latter does not depend on interaction length or magnitude of the field (it is necessary only that $\xi > \xi_{cr}$). It is a nonlinear acceleration/deceleration of the shock character, which proceeds in short enough time — smaller than the wave pulse duration. As is seen from Eq. (2.13), for a certain value of the refractive index of the medium the stronger the initial velocity of the particle differs from the Cherenkov one and the closer to 1 ($n_0 - 1 << 1$), the larger is the energy change. As follows from Eq. (2.12) in these cases the strong wave fields are necessary. However, as the medium is to be dielectriclike ($n_0 > 1$) the wave intensity is confined to the threshold ionization of the medium. As is known in nonionized media a wave of intensity $\xi^2 < I/mc^2$, where I is the first ionization energy of the medium atoms (for dielectrics, the width of the forbidden zone), can propagate. In the opposite case a tunnel ionization of the atoms can take place. Consequently, the region of intensities where the "reflection" phenomenon in dielectriclike media can be applied is $\xi^2 < \xi_{max}^2 < I/mc^2$. For typical values $I \sim 10$ eV we have $\xi_{max} \sim 5 \times 10^{-3}$. To such values of the wave critical intensity correspond particle velocities near the Cherenkov one, which is possible in the case of relativistic particles in the gases ($n_0 - 1 << 1$), whereas for nonrelativistic ones, in solids ($n_0 - 1 \sim 1$). However, in the last case the negative effects of multiple scattering and ionization loss of the particle in solids also can influence. Thus, this phenomenon can be realized in the gases for relatively low densities. The optimal values of the refractive index of the gaseous media for this phenomenon are $n_0 - 1 \sim 10^{-3} \div 10^{-5}$ (e.g., for CO_2 and He at standard pressure and temperature $n_0 - 1 \sim 4.48 \times 10^{-4}$ and $\sim 3.47 \times 10^{-5}$, respectively).

As the application of large intensities is restricted with ionization threshold of the medium, we express the particle energy change due to "reflection" through the wave critical intensity. If $n_0 - 1 \equiv \mu_1 << 1$ and $1 - \mathrm{v}_0/c \equiv \mu_2 << 1$ from Eqs. (2.12) and (2.13) we have

$$\xi_{cr} \simeq \frac{|\mu_1 - \mu_2|}{2\sqrt{\mu_1 \mu_2}} ; \qquad |\Delta\mathcal{E}| \simeq \xi_{cr} mc^2 \sqrt{\frac{2}{\mu_1}} . \tag{2.14}$$

Estimations show that an electron with initial energy $\mathcal{E}_0 \sim 10$ MeV after the "reflection" from a laser pulse with $\xi \sim 5 \times 10^{-4}$ (which corresponds to the neodymium laser radiation strength $E \sim 10^7$ V/cm) in a medium with $n_0 - 1 \sim 10^{-3}$ acquires ($\mathrm{v}_0 < c/n_0$) or loses ($\mathrm{v}_0 > c/n_0$) energy $|\Delta\mathcal{E}| \sim 10$ keV. As the particle deceleration occurs because of stimulated Cherenkov radiation in this case the wave amplification takes place. Hence, as a result of the "reflection" of a beam with electron total number $\sim 5 \times 10^{14}$ an energy of ~ 1 J coherently will be radiated into the wave.

The phenomenon of charged particle "reflection" from a plane EM wave may also be used for the monochromatization of particle beams. The fact that above the critical intensity value the induced Cherenkov process occurs in only one direction — either emission or absorption — and for the initial Cherenkov velocity $\mathrm{v}_{0x} = c/n_0$ the energy of the particle after the "reflection" does not change, in principle enables conversion of the energetic or angular spreads of charged particle beams due to "reflection." The latter requires considering the general case of interaction at the arbitrary direction of particle initial motion with respect to wave propagation. So, without repeating the analysis, which has been made in the case of particle–wave parallel propagation we will present the ultimate results of the "reflection" phenomenon in the general case.

Thus, when the particle initial velocity is directed at an angle (ϑ) to the wave propagation direction the energy of the particle is given by Eq. (2.6), which at the linear polarization of the wave reads

$$\mathcal{E}(\tau) = \frac{\mathcal{E}_0}{n_0^2 - 1} \Bigg\{ n_0^2 \left(1 - \frac{\mathrm{v}_0}{cn_0}\cos\vartheta\right) \mp \Bigg[\left(1 - n_0 \frac{\mathrm{v}_0}{c}\cos\vartheta\right)^2 - \left(n_0^2 - 1\right)$$

$$\times \left(\frac{mc^2}{\mathcal{E}_0}\right)^2 \left[\xi^2(\tau)\cos^2\omega_0\tau - 2\frac{p_0 \sin\vartheta}{mc}\xi(\tau)\cos\omega_0\tau\right]\Bigg]^{1/2}\Bigg\} \tag{2.15}$$

(the wave is linearly polarized along the axis OY with vector potential $A_y = A(\tau)\cos\omega_0\tau$ and one can assume $\mathbf{p}_0 = \{p_0\cos\vartheta; p_0\sin\vartheta; 0\}$, as far as the coordinate z is free). As is seen from Eq. (2.15), in this case the "reflection" occurs from certain planes of equal phases but from the front of the wave intensity envelope as in the case $\vartheta = 0$. At the actual values of the parameters for induced Cherenkov process (ultrarelativistic particles in gaseous media with refractive index $n_0 - 1 << 1$ and not very small angles ϑ, as well as the wave intensity being confined to ionization threshold of the medium) the second term under the root is much smaller than the third one, that is, $2p_0|\sin\vartheta|/mc >> \xi_{max}$ and for the critical field in this case we have

$$\xi_{cr}(\vartheta) = \frac{c}{2\mathrm{v}_0}\frac{\mathcal{E}_0}{mc^2}\frac{\left(1 - n_0\frac{\mathrm{v}_0}{c}\cos\vartheta\right)^2}{(n_0^2-1)\,|\cos\vartheta|}\ ; \qquad \vartheta \neq 0 \tag{2.16}$$

(in the case $\vartheta = 0$, ξ_{cr} is determined by Eq. (2.12)).

If the maximal value of the wave intensity $\xi_{max} > \xi_{cr}(\vartheta)$, then the particle energy after the "reflection" is

$$\mathcal{E}(\vartheta) = \mathcal{E}_0\left[1 + \frac{2\left(1 - n_0\frac{\mathrm{v}_0}{c}\cos\vartheta\right)}{n_0^2 - 1}\right]. \tag{2.17}$$

Let the charged particle beam with an initial energetic (Δ_0) and angular (δ_0) spread interact with a plane transverse EM wave of intensity $\xi_{max} > \xi_{cr}(\vartheta)$ in a gaseous medium. To keep the mean energy $\overline{\mathcal{E}_0}$ of the beam unchanged after the interaction (at the adiabatic turning on and turning off of the wave) the axis of the beam with mean velocity $\overline{\mathrm{v}_0}$ must be pointed at the Cherenkov angle (ϑ_0) to the laser beam, i.e., $n_0(\overline{\mathrm{v}_0}/c)\cos\vartheta_0 = 1$. Under this condition the particles with velocities $\mathrm{v}_0\cos\vartheta < c/n_0$ will acquire an energy and the other particles for which the longitudinal velocities exceed the phase velocity of the wave ($\mathrm{v}_0\cos\vartheta > c/n_0$) will loss an energy according to Eq. (2.17). As a result the energies of the particles $\mathcal{E}(\vartheta)$ will approach close to the mean energy $\overline{\mathcal{E}_0}$ of the beam ($\mathcal{E}(\vartheta) \to \overline{\mathcal{E}_0}$) and the final energetic width of the beam will become less than the initial one. As there is one free parameter (for a specified velocity $\overline{\mathrm{v}_0}$ the parameters ϑ_0 and n_0 are related by Cherenkov condition) it is possible to use it to control the exchange in the energy of the particles after the "reflection" (2.17) and to reach the minimal final energy spread of the beam $\Delta << \Delta_0$ — monochromatization. Depending on the relation between the initial energetic and angular spreads and mean energy of the beam, the opposite process may occur, namely angular narrowing of the beam. Physically it is clear that with the monochromatization the angular divergence of the beam will increase and the opposite — the angular narrowing of the beam — leads to demonochromatization (in accordance with Liouville's theorem). More detailed consideration of this effect with the quantitative results can be found in the bibliography of this chapter.

To illustrate the typical picture of nonlinear interaction of a charged particle with a strong EM wave in a medium we present the graphics of numerical solutions of the Eqs. (2.2) and (2.3) for the laser pulse of finite duration, showing the behavior of particle dynamics below and above critical intensity, with the effect of acceleration. At first we will not take into account the dependence of the slowly varying intensity envelope of a laser beam from the transversal coordinates. Thus, a laser beam may be modeled as

$$E_x = 0, \quad E_z = 0, \quad E_y = \frac{E_0}{\cosh\left(\frac{\tau}{\delta\tau}\right)} \cos\omega_0\tau, \tag{2.18}$$

where $\delta\tau$ characterizes the pulse duration. The particle initial energy is taken to be $\mathcal{E}_0 = 40$ MeV and the initial velocity is directed at the angle $\vartheta = 9\times10^{-3}$ rad to the wave propagation direction ($p_{0z} = 0$). The refractive index of the gaseous medium for this calculation has been chosen to be $n_0 - 1 = 10^{-4}$. Figure 2.1 illustrates the evolution of the particle energy: the energy versus the position x is plotted for a neodymium laser ($\hbar\omega_0 \simeq 1.17$ eV) with electric field strength $E_0 = 3 \times 10^8$ V/cm and $\delta\tau = 4T$ (T is the wave period). For these parameter values the wave intensity is above the critical point and, as we see from this figure, the particle energy is abruptly changed

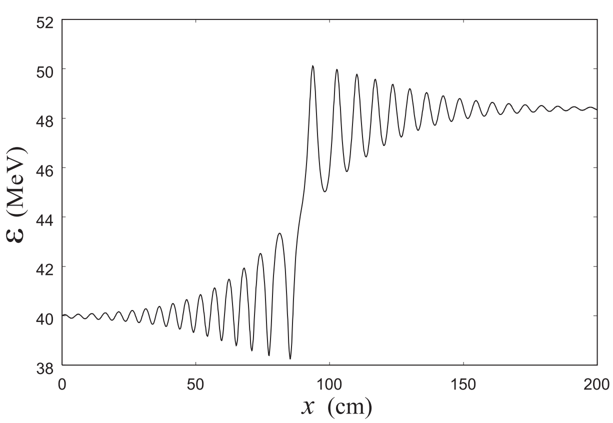

Fig. 2.1. "Reflection" of the particle. The energy versus the position x is plotted when the wave intensity is above the critical point.

corresponding to the "reflection" phenomenon. Figure 2.2a illustrates the evolution of the energies of particles with different initial interaction angles. The initial energies for all particles are $\mathcal{E}_0 \simeq 40$ MeV. Figure 2.2b illustrates the role of initial conditions: the final energy versus the interaction angle is plotted. As follows from Eq. (2.16) the critical intensity, as well as the final energy (2.17), depend on the initial interaction angle and as a consequence we have this picture. Note that the acceleration rate neither depends on the field magnitude (only should be above threshold field) nor on the interaction length.

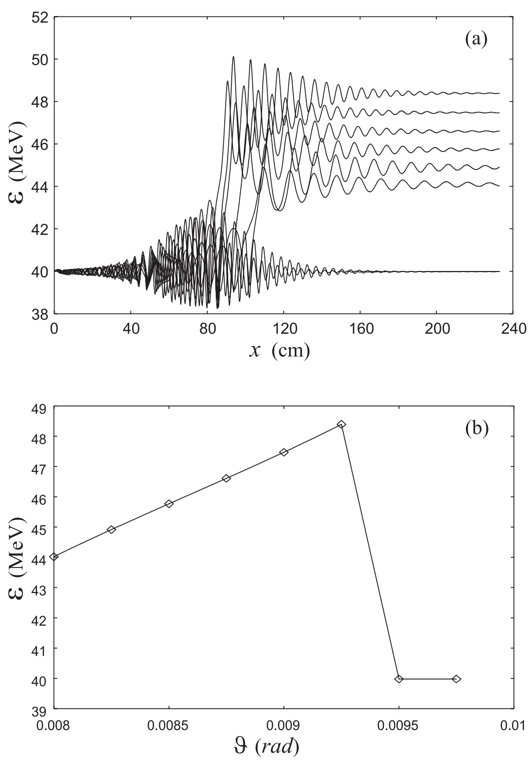

Fig. 2.2. "Reflection" of the particles with different initial interaction angles. Panel (a) displays the evolution of the energies of particles. In (b) the final energy versus the interaction angle is plotted.

To demonstrate the dependence of the considered process on transversal profile of the laser intensity for actual beams in Fig. 2.3 the evolution of the energies of particles with various initial phases (with initial energies $\mathcal{E}_0 \simeq 40$ MeV) is illustrated. The laser beam transversal profile is modeled by the Gaussian function

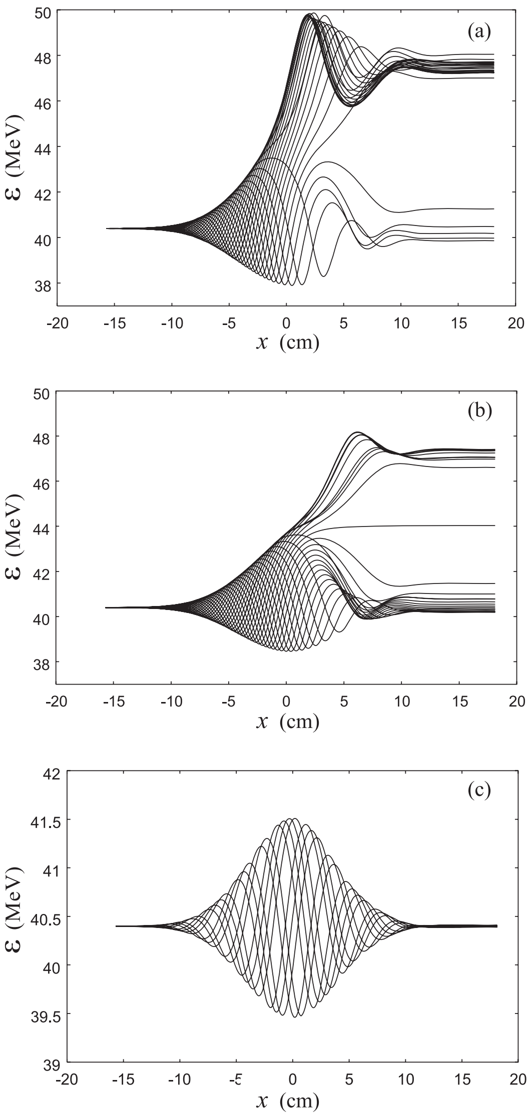

Fig. 2.3. The evolution of the energies of particles with various initial phases are shown for the laser beam with transversal intensity profile for the various entrance cordinates: (a) $z = 0$, (b) $z = d/4$, and (c) $z = d/2$.

$$E_y = E_0 \exp\left(-\frac{4}{d^2}\left(y^2 + z^2\right)\right) \frac{\cos\omega_0\tau}{\cosh\left(\frac{\tau}{\delta\tau}\right)} \tag{2.19}$$

with $d = 10^3\lambda$, $\delta\tau = 50T$. As we see from this figure the acceleration picture is essentially changed depending on the entrance coordinates of the particles. This is the manifestation of the threshold nature of the "reflection" phenomenon.

2.3 Particle Capture by a Plane Electromagnetic Wave in a Medium

If for the intensity exceeding the critical value a plane EM wave becomes a potential barrier for the external particle (with respect to the wave), then for the particle initially situated in the wave it may become a potential well and particle capture by the wave will take place. As the particle state in the wave depends on wave phases we will assume in this case a certain polarization of a monochromatic wave. Let it be linearly polarized with electric field strength along the axis OY :

$$E_y = E_0 \cos\phi; \quad \phi = \omega_0 \left(n_0 \frac{x}{c} - t \right) . \tag{2.20}$$

The solution of equations of motion (2.2) and (2.3) in the field (2.20) may be presented in the form

$$p_x(\phi) = \frac{n_0}{n_0^2 - 1} \frac{\mathcal{E}_0}{c} \left\{ \left(1 - \frac{\mathrm{v}_{0x}}{cn_0} \right) \mp \left[\left(1 - n_0 \frac{\mathrm{v}_{0x}}{c} \right)^2 - \left(n_0^2 - 1 \right) \left(\frac{mc^2}{\mathcal{E}_0} \right)^2 \right.\right.$$

$$\left.\left. \times \xi_0^2 \left(\sin\phi - \sin\phi_0 \right) \left(\sin\phi - \sin\phi_0 - 2 \frac{p_{0y}}{mc\xi_0} \right) \right]^{1/2} \right\}, \tag{2.21}$$

$$p_y(\phi) = p_{0y} - mc\xi_0 (\sin\phi - \sin\phi_0) \ ,$$

$$\mathcal{E}(\phi) = \frac{c}{n_0} p_x(\phi) + \mathcal{E}_0 \left(1 - \frac{\mathrm{v}_{0x}}{cn_0} \right) , \tag{2.22}$$

where $\xi_0 = eE_0/mc\omega_0$ is the intensity parameter of the monochromatic wave (see Eq. (1.25)), $\phi_0 = \omega(n_0 x_0/c - t_0)$ is the initial phase of the particle in the wave. Here without loss of generality it is assumed that the z component of the particle initial momentum $p_{0z} = 0$ as far as the coordinate z is free.

It is seen from Eq. (2.21) that the particle can be in the field region where

$$W(\phi) \equiv (\sin\phi - \sin\phi_0) \left(\sin\phi - \sin\phi_0 - 2 \frac{p_{0y}}{mc\xi_0} \right)$$

$$\leq \left(\frac{\mathcal{E}_0}{mc^2}\right)^2 \frac{\left(1 - n_0 \mathrm{v}_{0x}/c\right)^2}{\left(n_0^2 - 1\right) \xi_0^2} \ . \tag{2.23}$$

If the maximum value of the function $W(\phi)$

$$W_{\max}(\phi) > \left(\frac{\mathcal{E}_0}{mc^2}\right)^2 \frac{\left(1 - n_0 \mathrm{v}_{0x}/c\right)^2}{\left(n_0^2 - 1\right) \xi_0^2} \ , \tag{2.24}$$

then the region (2.23) will be a potential well for the particle and the capture of the latter by the transverse EM wave will take place. The equilibrium phases of the wave (ϕ_s) correspond to the extrema of the function $W(\phi)$:

$$\sin\phi_s = \sin\phi_0 + \frac{p_{0y}}{mc\xi_0} \ ; \qquad \cos\phi_s \neq 0 \ , \tag{2.25}$$

$$\cos\phi_s = 0 \ ; \qquad \sin\phi_s \neq \sin\phi_0 + \frac{p_{0y}}{mc\xi_0} \ . \tag{2.26}$$

The particle moves with a Cherenkov velocity $\mathrm{v}_{xs} = c/n_0$ when it is in the equilibrium phases ϕ_s. Equation (2.22) together with Eqs. (2.25) and (2.26) determine the equilibrated values of the particle transverse momentum p_{ys}. In particular, $p_{ys} = 0$ corresponds to the case (2.25). The motion of the particle in these phases will be stable when

$$|\sin\phi_0 + \frac{p_{0y}}{mc\xi_0}| < 1. \tag{2.27}$$

If the initial velocity of the particle is equal to the Cherenkov one ($\mathrm{v}_{0x} = c/n_0 = \mathrm{v}_{xs}$), then from Eq. (2.24) we have the following condition for the particle capture by the wave:

$$\frac{p_{0y}}{mc\xi_0} < 1 + |\sin\phi_0 + \frac{p_{0y}}{mc\xi_0}|. \tag{2.28}$$

At the fulfillment of Eq. (2.27) the condition of particle capture (2.28) always holds, and therefore the condition of stable motion (2.27) thus determines the capture of the particle in the considered regime. In particular, as is seen from Eqs. (2.25) and (2.27), when $p_{y0} = 0$, then $\phi_s = \phi_0$ and any phase is equilibrated. In this case the phase $\cos\phi_0 = 0$ ($Ey = 0$) is unstable. This is physically clear in the wave frame where the magnetic field of the wave corresponding to this phase is zero: $\mathbf{H}' = 0$, while the stability in the capture regime is due to particle rotation around the vector of the magnetic field (when $p_{ys} = 0$). If the particle initial velocity differs from the Cherenkov value $\mathrm{v}_{0x} = \mathrm{v}_0 = c/n_0 + \Delta\mathrm{v}$, then in the capture regime the particle will undergo stable oscillations close to the equilibrated Cherenkov value. From

Eq. (2.24) one can obtain the following condition for the capture of such particle:

$$|\Delta \mathrm{v}| < \frac{c}{n_0}\frac{mc^2}{\mathcal{E}_0}\xi_0\sqrt{(n^2-1)}\,(1+|\sin\phi_0|)\ . \tag{2.29}$$

The spread tolerances of the unequilibrated particle's initial phase and velocity can be defined from the condition (2.29) ($\Delta\mathrm{v} = (c/n_0\omega_0)|d\phi/dt|$).

Note that the needed value of the field for the particle capture by the wave defined from Eq. (2.29) is the critical value of the field (2.12) for the "reflection" of the external particle ($\phi_0 = 0$).

Consider now the particle capture in equilibrium phases (2.26). With the help of Eqs. (2.22) and (2.23) one can show that the particle motion at the phases $\cos\phi_0 = 0$ will be stable when

$$p_{ys}\sin\phi_s > 0\ ;\quad \phi_s = (2k+1)\pi/2\ ;\quad k = 0; \pm 1; \pm 2; \ldots . \tag{2.30}$$

For the capture of initial Cherenkov particle ($\mathrm{v}_{0x} = c/n_0$) at the phases $\phi_s = (2k+1)\pi/2$ from Eq. (2.24) one can obtain the following condition:

$$W_{\max}(\phi) = 4|\sin\phi_0 + \frac{p_{0y}}{mc\xi_0}| > 0,$$

which always holds. Therefore, the particle capture in this case is determined by condition (2.30). If $p_{y0}\sin\phi_0 > 0$, the phase ϕ_0 is an equilibrated one for any value of the particle transverse momentum ($p_{0y} = p_{ys}$). But if $\mathrm{v}_{0x} = c/n_0 + \Delta\mathrm{v}_x$ the condition for capture is

$$|\Delta \mathrm{v}_x| < \frac{2c}{n_0}\sqrt{n^2-1}\frac{mc^2}{\mathcal{E}_0}\xi_0|\sin\phi_0 + \frac{p_{0y}}{mc\xi_0}|^{1/2}. \tag{2.31}$$

From Eq. (2.31) the critical value of the field can be defined for unequilibrated particle "capture" at the wave phases $\phi_0 = (2k+1)\pi/2$.

If $\cos\phi_0 \neq 0$ from Eq. (2.24) one can obtain that when $p_{0y}/mc\xi_0 > 2$ the Cherenkov particle capture is defined again by condition (2.30).

2.4 Laser Acceleration in Gaseous Media. Cherenkov Accelerator

The phenomenon of charged particle "reflection" and capture by a transverse EM wave can be used for particle acceleration in laser fields. As the application of large intensities in this process is restricted because of the medium ionization the acceleration owing to "reflection" in the medium with refractive index $n_0 = \text{const}$ — single “reflection” — is relatively small. However, if the refractive index decreases along the wave propagation direction in such a way that the condition of particle synchronous motion with the wave $\mathrm{v}_x(x) = c/n_0(x)$ takes place continuously, the phase velocity of the wave will increase all the time and the particle being in front of the wave barrier (at $\xi > \xi_{cr}$) will continuously be “reflected”, i.e., continuously accelerated. The law $n_0 = n_0(x)$ must have an adiabatic character not to allow the particle to leave the wave after the single “reflection”. Such variation law of the refractive index can be realized in a gaseous medium adiabatically decreasing the pressure.

For particle acceleration one can also use the capture regime. In this case in the medium with $n_0 = \text{const}$ the particle energy does not change on average (particle makes stable oscillations around the equilibrium phases in the wave moving with average velocity $< \mathrm{v}_x >= c/n_0$). However, if one decreases the refractive index along the propagation direction of the wave, so that the particle does not leave the equilibrium phases, then the wave will continuously accelerate the particle. Then, to realize the capture regime (2.25) one needs $p_{0y}/mc\xi_0 < 2$. For not very strong fields this is sufficiently strict confinement on the transverse momentum of the particle. On the other hand, to accelerate the particle significantly large transverse momenta are needed. Therefore, this regime can be used to pass the particles through the matter and, also, to separate the particles by velocities (parameter ξ defines the region of particle velocities captured by the wave (see Eq. (2.29)).

For particle acceleration by laser fields one can use the capture regime (2.26) corresponding to large transverse momenta of the particle $p_{0y}/mc\xi_0 > 2$. So, we will consider the general case of particle capture with arbitrary initial momentum $\mathbf{p}_0$ and laser acceleration in gaseous medium with varying refractive index $n_0(x)$.

We will use the particle equations of motion (2.2) and (2.3) in the field (2.20) where the refractive index $n_0 \to n_0(x)$ and consequently the wave phase is determined as follows:

$$\phi(x,t) = \frac{\omega_0}{c} \int n_0(x) dx - \omega_0 t. \tag{2.32}$$

Then from the equations

$$\frac{d\phi_s}{dt} = 0 \ , \qquad \frac{d^2\phi_s}{dt^2} = 0 \tag{2.33}$$

defining wave equilibrium phases we obtain the variation laws for equilibrium velocity of the particle and refractive index of the medium, respectively:

$$\mathrm{v}_{xs}(x) = \frac{c}{n_0(x)}, \tag{2.34}$$

$$\frac{dn_0(x)}{dx} = -\frac{n_0^3(x)}{c^2}\left(\frac{d\mathrm{v}_x}{dt}\right)_s . \tag{2.35}$$

From Eq. (2.2) and the equation for the particle energy variation

$$\frac{d\mathcal{E}}{dt} = e\mathrm{v}_y E_0 \cos\phi(x,t) \tag{2.36}$$

one can obtain the acceleration of the particle in the longitudinal direction

$$\frac{d\mathrm{v}_x}{dt} = \frac{ecn_0(x)}{\mathcal{E}}\left[1 - \frac{\mathrm{v}_x}{cn_0(x)}\right]\mathrm{v}_y E_0 \cos\phi(x,t). \tag{2.37}$$

The equation of motion (2.3) determines in general for an arbitrary $n_0(x)$ the integral of motion (2.5), from which for the equilibrium transverse momentum of the particle we have (again without loss of generality it is assumed that the z component of the particle initial momentum $p_{0z} = 0$ since the coordinate z is free)

$$p_{ys} = p_{0y} - mc\xi_0 \left(\sin\phi_s - \sin\phi_0\right). \tag{2.38}$$

Defining within Eq. (2.38) the equilibrium transverse velocity of the particle $\mathrm{v}_{ys}(x) = c^2 p_{ys}/\mathcal{E}_s(x)$ and substituting together with Eq. (2.34) into Eq. (2.37) for the equilibrium value of the particle longitudinal acceleration we obtain

$$\left(\frac{d\mathrm{v}_x}{dt}\right)_s = c\omega_0\xi_0 \frac{p_{ys}}{mc}\cos\phi_s \left(\frac{mc^2}{\mathcal{E}_s(x)}\right)^2 \frac{n_0^2(x)-1}{n_0(x)}. \tag{2.39}$$

Substituting Eq. (2.39) into Eq. (2.35) we will have the equation which determines the variation law of the medium refractive index:

$$\frac{dn_0(x)}{dx} = -\frac{\omega_0}{c}\xi_0 \frac{p_{ys}}{mc}\cos\phi_s \left(\frac{mc^2}{\mathcal{E}_s(x)}\right)^2 n_0^2(x)\left[n_0^2(x)-1\right]. \tag{2.40}$$

It is seen from this equation that for the particle acceleration in the capture regime via decreasing refractive index of the medium ($dn_0(x)/dx < 0$) one needs $p_{ys}\cos\phi_s > 0$ (equilibrium transverse momentum of the particle must be directed along the vector of the wave electric field). In the opposite case the continuous deceleration of the particle will take place accompanied by induced Cherenkov radiation (regime of continuous amplification of the wave by the particle beam at $dn_0(x)/dx > 0$).

The energy of equilibrium particle acquired on the distance x is defined by

$$\mathcal{E}_s^2(x) = \frac{n_0^2(x)}{n_0^2(x) - 1} \left(m^2 c^4 + c^2 p_{ys}^2 \right). \tag{2.41}$$

Integrating Eq. (2.40) within Eq. (2.41) the ultimate formula for the variation law of the medium refractive index becomes

$$\frac{1}{2} \left[\frac{n_0(0)}{n_0^2(0) - 1} - \frac{n_0(x)}{n_0^2(x) - 1} \right] + \frac{1}{4} \ln \left[\frac{n_0(x) + 1}{n_0(x) - 1} \cdot \frac{n_0(0) - 1}{n_0(0) + 1} \right]$$

$$= -\frac{mc^2 \xi_0 \omega_0 p_{ys} \cos \phi_s}{m^2 c^4 + c^2 p_{ys}^2} x \,. \tag{2.42}$$

Equation (2.41) in the general case defines the particle acceleration in the capture regime when the medium refractive index falls along the wave propagation according to law (2.42). It defines the longitudinal dimension of such "Cherenkov accelerator" as well. The transverse dimension of the latter is defined by

$$\mathcal{E}_s(y) = \mathcal{E}_s(0) + mc\omega_0 \xi_0 (y - y_0) \cos \phi_s. \tag{2.43}$$

Here $\mathcal{E}_s(0)$ and y_0 are the initial equilibrium values of the energy and transverse coordinate of the particle ($y - y_0$ is the transverse dimension of "Cherenkov accelerator"). As is seen from Eq. (2.43) the particle acceleration takes place if $(y - y_0) \cos \phi_s > 0$, and in the opposite case deceleration occurs ($\mathcal{E}_s(y) < \mathcal{E}_s(0)$) in accordance with what was mentioned above. For relativistic particles, when $n_0(x) \sim 1$ and $n_0(x) - 1 << n_0(0) - 1$, from Eq. (2.42) we have

$$n_0(x) - 1 \simeq \frac{m^2 c^4 + c^2 p_{ys}^2}{4mc^2 \xi_0 \omega_0 p_{ys} \cos \phi_s} \frac{1}{x}. \tag{2.44}$$

As as this formula is valid at the large variation of the medium refractive index $n_0(x) - 1$, then according to Eq. (2.41) it corresponds to large acceleration of the particle: $\mathcal{E}_s(x) >> \mathcal{E}_s(0)$. In particular, Eq. (2.41) determines the initial value of the refractive index $n_0(0)$ as a function of the initial value of the equilibrium energy of the particle $\mathcal{E}_s(0)$:

$$n_0(0) - 1 = \frac{\mathcal{E}_s(0) - \sqrt{\mathcal{E}_s^2(0) - c^2 p_{ys}^2 - m^2 c^4}}{\sqrt{\mathcal{E}_s^2(0) - c^2 p_{ys}^2 - m^2 c^4}} \tag{2.45}$$

(since $\phi_s = \text{const}$, then $p_{ys} = \text{const}$ according to Eq. (2.38)). From the comparison of Eqs. (2.44) and (2.45) ($n_0(x) - 1 << n_0(0) - 1$; $n_0(0) \sim 1$) one can find the longitudinal dimension of acceleration on which the decreasing law of refractive index (2.44) is valid:

$$x >> \frac{\mathcal{E}_s^2(0) - c^2 p_{ys}^2 - m^2 c^4}{2mc^2 \xi_0 \omega_0 p_{ys} \cos\phi_s}. \tag{2.46}$$

The energy of the equilibrium particle acquired on such distances is

$$\mathcal{E}_s(x) \simeq \sqrt{2mc^2 \xi_0 \omega_0 |p_{ys} \cos\phi_s| x}\ ; \qquad \mathcal{E}_s(x) >> \mathcal{E}_s(0). \tag{2.47}$$

The estimations show that, for example, at electric field strengths of laser radiation $E \sim 10^8$ V/cm an electron with initial energy $\mathcal{E}_s(0) \sim 5$ MeV acquires energy $\mathcal{E}_s(x) \sim 50$ MeV already at the distance $x \sim 1$ cm. The transverse dimension of acceleration $y - y_0$ is of the order of a few millimeters and the longitudinal dimension of the system is of the order of the transverse one (a few times larger). At the distance $x \sim 1$ m the particle energy gain is of the order of 1 GeV . Note that because of multiple scattering on the atoms of the medium the particles can leave the regime of stable motion as a result of change of p_{ys}. The analysis shows that the multiple scattering essentially falls in the above-mentioned gaseous media (see Section 2.2) for laser field strengths $E > 10^7$ V/cm.

To illustrate the particle acceleration in the capture regime we will represent the results of numerical solution of Eqs. (2.2) and (2.3) in the field of an actual laser beam with the electric field strength

$$E_y = E_0 \exp\left(-\frac{4}{d^2}\left(y^2 + z^2\right)\right) \frac{\cos\left(\frac{\omega_0}{c}\int n_0(x)dx - \omega_0 t + \varphi_0\right)}{\cosh\left(\frac{\frac{1}{c}\int n_0(x)dx - t + \varphi_0/\omega_0}{\delta\tau}\right)}, \tag{2.48}$$

$$E_x = 0, \quad E_z = 0,$$

where $\delta\tau$ characterizes the pulse duration and φ_0 is the initial phase. Simulations have been made for neodymium laser ($\hbar\omega_0 \simeq 1.17$ eV) with electric field strength $E_0 = 3 \times 10^8$ V/cm and $\delta\tau = 1000T$, $d = 5 \times 10^3 \lambda$. The variation law for the refractive index of the medium is defined in self-consistent manner (see Eqs. (2.35) and (2.37)), that may be approximated by the function

$$n(x) = \frac{n_0 + n_f}{2} + \frac{(n_f - n_0)}{2} \tanh(\kappa x), \tag{2.49}$$

where n_0, n_f are the initial and final values of the refractive index and κ characterizes the decreasing rate.

Figure 2.4 illustrates the evolution of the particle energy in the capture regime. The particle initial energy is taken to be $\mathcal{E}_0 = 50.5$ MeV and the initial velocity is directed at the angle $\vartheta = 9 \times 10^{-3}$ rad to the wave propagation direction ($p_{0z} = 0$). The initial value of the refractive index has been chosen

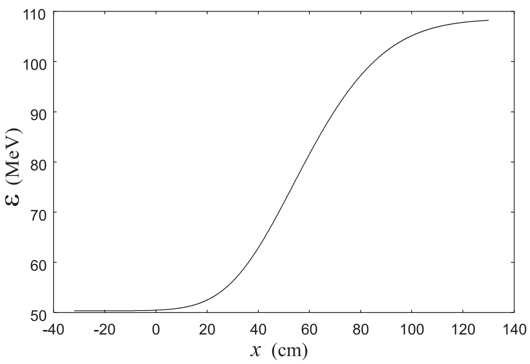

Fig. 2.4. The evolution of the particle energy in the capture regime with variable refractive index.

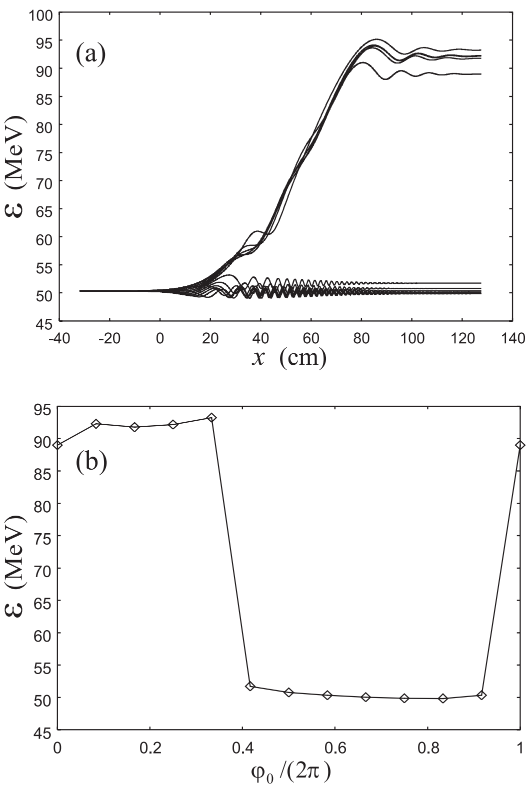

Fig. 2.5. Acceleration of the particles in the capture regime. Panel (a) displays the evolution of the energies of particles with various initial phases. The initial entrance coordinate is $z = 0$. In (b) the final energy versus the initial phase is plotted.

to be $n_0 - 1 \simeq 10^{-4}$. As we see in the capture regime with variable refractive index, one can achieve considerable acceleration.

To show the role of initial conditions in Fig. 2.5a the evolution of the energies of particles with the same initial energies $\mathcal{E}_0 = 50.5$ MeV ($\vartheta = 9 \times 10^{-3}$ rad) and various initial phases is illustrated. The initial entrance coordinate is $z = 0$. Figure 2.5b displays the role of initial conditions: the final energy versus the initial phase is plotted. In Fig. 2.6 the parameters are the same as in Fig. 2.5a except the initial entrance coordinate, which is taken to be $z = 0.25$ mm. As we see from these figures the captured particles are accelerated, while the particles situated in the unstable phases (or if the conditions for capture are not fulfilled) after the interaction remain with the initial energy.

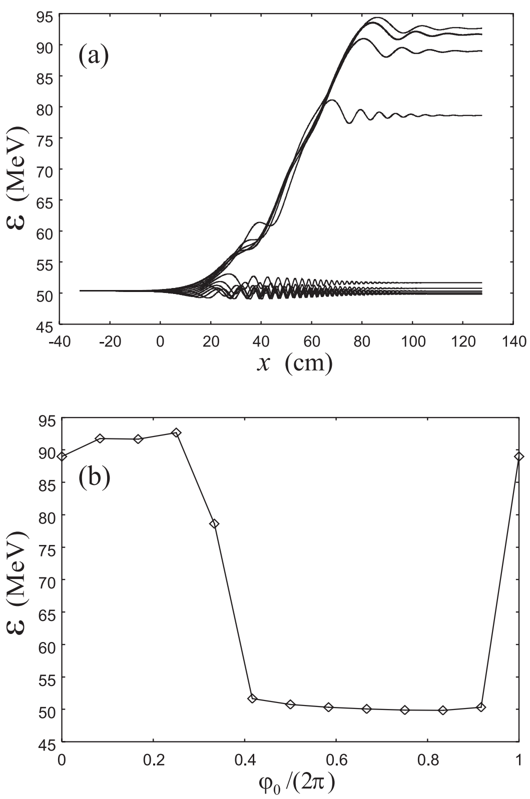

Fig. 2.6. Acceleration of the particles in the capture regime. Panel (a) displays the evolution of the energies of particles with various initial phases. The initial entrance coordinate is $z = 0.25$ mm. In (b) the final energy versus the initial phase is plotted.

2.5 Nonlinear Compton Scattering in a Medium

"Reflection" and capture phenomena are essentially changing the picture of Compton scattering in a medium. The existence of the critical field in a medium with refractive index $n(\omega) > 1$ confines the intensity of external wave on which Compton scattering of a charged particle proceeds. Therefore, one can consider the Compton effect in dielectriclike media only if the wave intensity does not exceed the critical value. On the other hand, as was mentioned above the multiphoton absorption and radiation due to the nonlinear Cherenkov resonance in the field just occurs at wave intensities close to the critical one. Hence, it is important to consider the nonlinear Compton effect in a gaseous medium where the induced Cherenkov radiation will accompany and interfere with the Compton radiation at external wave intensities close to the critical value. At the latter the nonlinear Compton effect (high harmonic radiation) will take place even in very weak wave fields ($\xi \lesssim \xi_{cr} << 1$) in contrast to nonlinear Compton effect in vacuum where for the radiation already of the second harmonic with considerable intensity, superstrong fields ($\xi > 1$) are required, as has been shown in Chapter 1.

The energy radiated by a charged particle in a medium at a frequency ω in the domain $d\omega$ and solid angle dO is given by

$$d\varepsilon_{\mathbf{k}} = \frac{e^2 n(\omega)}{4\pi^2 c^3}\omega^2 d\omega dO \left| \int_{-\infty}^{+\infty} [\nu \mathbf{v}] \exp\left[i\mathbf{k}\mathbf{r}(t) - i\omega t\right] dt \right|^2, \tag{2.50}$$

where $\mathbf{k} = \nu n(\omega)\,\omega/c$ is the radiation wave vector in the medium (ν is a unit vector along the radiation direction) and $n(\omega)$ is the refractive index of the medium at frequency ω.

The particle law of motion $\mathbf{r}(t)$ in the plane monochromatic EM wave of circular polarization is determined by analogy with Eqs. (1.27)–(1.29) and is written as

$$x(t) = \mathrm{v}_x t,$$

$$y(t) = -\xi \frac{c}{\omega_0} \frac{mc^2}{\mathcal{E}\left(1 - n_0 \frac{\mathrm{v}_x}{c}\right)} \cos\omega_0 \left(1 - n_0 \frac{\mathrm{v}_x}{c}\right) t, \tag{2.51}$$

$$z(t) = \xi \frac{c}{\omega_0} \frac{mc^2}{\mathcal{E}\left(1 - n_0 \frac{\mathrm{v}_x}{c}\right)} \sin\omega_0 \left(1 - n_0 \frac{\mathrm{v}_x}{c}\right) t.$$

Here it is assumed that the initial velocity of the particle is directed along the wave propagation ($\mathrm{v}_0 = \mathrm{v}_{0x}$) at which the particle longitudinal velocity v_x and energy $\mathcal{E}$ do not vary in time since it depends only on the wave intensity ξ^2 (see Eqs. (2.7) and (2.10)) and for the circular polarization of the wave $\xi^2 = \mathrm{const}$ (the strong wave intensity effect is responsible for permanent renormalization of these quantities in the field). Then, in the equations for particle energy and velocity (2.7)–(2.10) one should take only the sign minus before the root in accordance with the above discussion.

Substituting Eqs. (2.7), (2.9), and (2.51) into Eq. (2.50) and integrating, the following ultimate formula for the spectral power of the Compton radiation of the s-th harmonic in a medium is obtained:

$$dP_{\mathbf{k}}^{(s)} = \frac{e^2 n(\omega)}{2\pi c} \frac{\omega^2}{\omega_0 \left(1 - n_0 \frac{\mathrm{v}_x}{c}\right)} \left\{ \left[n(\omega) \frac{\mathrm{v}_x}{c} - \cos\theta \right]^2 \frac{J_s^2(\alpha)}{n^2(\omega)\sin^2\theta} \right.$$

$$\left. + \xi^2 \left(\frac{mc^2}{\mathcal{E}}\right)^2 J_s'^2(\alpha) \right\} \delta\left[\omega \frac{1 - n(\omega)\frac{\mathrm{v}_x}{c}\cos\theta}{\omega_0\left(1 - n_0\frac{\mathrm{v}_x}{c}\right)} - s\right] d\omega dO, \tag{2.52}$$

where θ is the angle between the radiation direction and axis OX , and the argument of the Bessel function

$$\alpha = \xi \frac{mc^2}{\mathcal{E}} \frac{\omega n(\omega)\sin\theta}{\omega_0\left(1 - n_0\frac{\mathrm{v}_x}{c}\right)}. \tag{2.53}$$

The δ-function in Eq. (2.52) determines the conservation law of the Compton radiation process in a medium (radiation spectrum)

$$\omega = s\omega_0 \frac{1 - n_0\frac{\mathrm{v}_x}{c}}{1 - n(\omega)\frac{\mathrm{v}_x}{c}\cos\theta}. \tag{2.54}$$

First, let us consider the cases of limit intensities of the wave $\xi = 0$ and $\xi = \xi_{cr}$. If in Eq. (2.52) $\xi \to 0$, then the radiation power will differ from zero only for the $s = 0$ harmonic. In that case, the conservation law of Compton process (2.54) becomes the condition of Cherenkov radiation ($\mathrm{v}_x \to \mathrm{v}_{0x} = \mathrm{v}_0$) and Eq. (2.52) after the integration over θ passes to the Tamm–Frank formula

$$dP_\omega^{(0)} = \frac{e^2\mathrm{v}_0}{c^2}\left(1 - \frac{c^2}{n^2(\omega)\mathrm{v}_0^2}\right)\omega d\omega. \tag{2.55}$$

In the other limit case of $\xi = \xi_{cr}$, the longitudinal velocity of the particle $\mathrm{v}_x = c/n_0$ and Eq. (2.54) allows the nonzero frequencies of radiation either for infinitely large harmonics ($s = \infty$) or when the condition

$$1 - n(\omega)\frac{\mathrm{v}_x}{c}\cos\theta = 0 \tag{2.56}$$

is fulfilled. However, it is easy to see that at the satisfaction of condition (2.56) the radiation power becomes zero. Hence, at the value of external wave intensity $\xi = \xi_{cr}$ only the harmonics $s = \infty$ are radiated the power of which differs from zero at the value of the Bessel function argument $\alpha = s$, which gives

$$1 - \frac{\mathbf{k}\mathbf{v}_{cr}}{\omega} = 0 \; ; \qquad \mathbf{k} = \nu n(\omega)\frac{\omega}{c},$$

where

$$\mathbf{v}_{cr} = \left\{ \frac{c}{n_0}, 0, c\sqrt{n_0^2 - 1}\frac{1 - n_0\frac{\mathrm{v}_0}{c}}{n_0^2\left(1 - \frac{\mathrm{v}_0}{cn_0}\right)} \right\}.$$

In that case, Eq. (2.52) again passes to the Tamm–Frank formula (2.55) for a particle moving with the velocity $\mathrm{v}_0 = \mathrm{v}_{cr} > c/n(\omega)$. In this case the radiation of fundamental frequency ω_0 exists as well. So, only in limit cases $\xi = 0$ and $\xi = \xi_{cr}$ does Compton radiation fully turn into Cherenkov radiation and at the values of external wave intensity $0 < \xi < \xi_{cr}$ the radiation of the particle involves superposition of Compton and Cherenkov radiation.

The nonlinear scattering in laser fields of moderate intensities, that is, radiation of high harmonics at $\xi << 1$, is of great interest. In considering this process it is possible even at weak wave fields of intensities $\xi \approx \xi_{cr} << 1$ due to the Cherenkov resonance, i.e., when the radiation is close to the Cherenkov cone with the incident wave. In accordance with Eq. (2.52) significant nonlinearity in the radiation process arises when the argument of the Bessel function $\alpha \sim s$ ($s >> 1$). As is seen from Eqs. (2.53) and (2.54) such large values of α can be reached due to $\mathrm{v}_x \to c/n_0$, i.e., if the intensity of an incident wave is close to the critical value ($\xi \to \xi_{cr}$) and radiation is close to the Cherenkov cone $(1 - n(\omega)(\mathrm{v}_x/c)\cos\theta \to 0)$.

To determine the conditions and quantitative results for high harmonics ($s >> 1$) radiation, one should substitute in Eq. (2.53) the concrete expressions of the particle longitudinal velocity v_x and energy $\mathcal{E}$ in the field. From Eqs. (2.7) and (2.10) we have

$$\alpha = \frac{mc^2}{\mathcal{E}_0}\frac{n(\omega)\omega\sin\theta}{\omega_0\left(1 - n_0\frac{\mathrm{v}_0}{c}\right)\sqrt{1 - \frac{\xi^2}{\xi_{cr}^2}}}\xi. \tag{2.57}$$

In Eq. (2.57), the radiation angle ($\sin\theta$) should be defined from the condition $\theta \simeq \theta_c$, where θ_c is the Cherenkov angle. At fundamental frequency ω_0 the Cherenkov angle $\theta_c << 1$, whereas at other frequencies ω it may not be small depending on the medium dispersion and, consequently, the conditions of nonlinearity will be different. However, the number of harmonics at all frequencies is large enough. The harmonic $s = 0$ at fundamental frequency ω_0 cannot be radiated since $\mathrm{v}_x < c/n_0$. The first harmonic ($s = 1$) at frequency ω_0 is radiated at the angle $\theta = 0$. The negative harmonics ($s = -1, -2, ...$) correspond to anomalous Compton scattering in a medium with refractive index $n(\omega) > 1$. At frequencies $\omega \neq \omega_0$ the harmonic $s = 0$ corresponds to Cherenkov radiation; however, the power of the radiation differs from the Tamm–Frank formula because of the oscillatory character of the particle motion in the wave field (influence of Compton effect).

2.6 Radiation of a Particle in Capture Regime. Cherenkov Amplifier

Consider the radiation of the particle captured by a plane monochromatic wave in a gaseous medium. We will assume that the particle initial velocity is directed along the wave propagation and has a value close to the Cherenkov one:

$$\mathrm{v}_0 = \mathrm{v}_{0x} = \frac{c}{n_0}\,(1+\mu)\,; \qquad \mu << 1. \tag{2.58}$$

From the equations of motion (2.2) and (2.3) it follows that at $\mu = 0$

$$\mathrm{v}_x = \mathrm{v}_{x0} = \frac{c}{n_0}\ , \qquad \mathrm{v}_y = 0\ , \qquad x = x_0 + \frac{c}{n_0}t, \tag{2.59}$$

where x_0, $y_0 = 0$, $z_0 = 0$ are the initial coordinates of the particle at the moment $t = 0$ in the wave of linear polarization

$$E = E_y = E_0 \cos\left(\omega_0 n_0 \frac{x}{c} - \omega_0 t\right). \tag{2.60}$$

The solution of Eqs. (2.2) and (2.3) at $\mu << 1$ can be represented as

$$\mathrm{v}_x(t) = \frac{c}{n_0}\,(1+\mu u_x(t))\ , \qquad \mathrm{v}_y(t) = c\mu u_y(t) \tag{2.61}$$

and after the linearization of these equations by parameter μ we have the following set of equations for the functions $u_x(t)$ and $u_y(t)$:

$$\frac{du_x}{dt} = \frac{e\left(n_0^2-1\right)^{3/2}}{n_0^2 mc} E_0 \cos\phi_0 \cdot u_y\ ,$$

$$\frac{du_y}{dt} = -\frac{e\left(n_0^2-1\right)^{1/2}}{mc} E_0 \cos\phi_0 \cdot u_x\ . \tag{2.62}$$

Integrating this set of equations at the initial conditions $u_{x0} = 1$ and $u_{y0} = 0$ in accordance with Eq. (2.59), for the particle velocity in the capture regime we obtain

$$\mathrm{v}_x(t) = \frac{c}{n_0}\left(1 + \mu\cos\Omega_0 t\right),$$

$$\mathrm{v}_y(t) = -\frac{c}{\left(n_0^2 - 1\right)^{1/2}}\mu\sin\Omega_0 t, \tag{2.63}$$

$$\Omega_0 = \frac{e\left(n_0^2 - 1\right)E_0|\cos\phi_0|}{n_0 mc}. \tag{2.64}$$

In the derivation of Eqs. (2.63) and (2.64) the following approximation has been made (due to the small parameter μ):

$$\mu\frac{\omega_0}{\Omega_0} << 1, \tag{2.65}$$

which is violated for the wave phase $\cos\phi_0 = 0$. This is connected with the fact that the stability in the capture regime is provided by the action of magnetic field $\mathbf{H}'$ in the frame of reference connected with the wave and $\mathbf{H}' = 0$ in the phase $\cos\phi_0 = 0$, so that this phase is unstable.

As is seen from Eq. (2.63) the particle velocity in the wave oscillates with the frequency Ω_0, which depends on the initial phase ϕ_0. In the particle beam case the various particles being initially in different phases of the wave well will have diverse velocities and space bunching of the particles will occur as a result of which the current density of the beam will be modulated. Equation (2.64) shows that the modulation frequency $\Omega_0 \simeq \omega_0\left(n_0^2 - 1\right)\xi|\cos\phi_0|$ and as even for the strong laser fields $\xi << 1$ (and $n_0^2 - 1 << 1$), then $\Omega_0 << \omega_0$.

To calculate the power of noncoherent radiation by Eq. (2.50) one needs the particle law of motion $\mathbf{r}(t)$ in the capture regime. Defining the latter by integration of Eq. (2.63) with the initial conditions $x(t)\mid_{t=0} = x_0$, $y(t)\mid_{t=0} = 0$

$$x(t) = x_0 + \frac{c}{n_0}t + \mu\frac{c}{n_0\Omega_0}\sin\Omega_0 t\ ,$$

$$y(t) = -\mu\frac{c}{\left(n_0^2 - 1\right)^{1/2}\Omega_0}\left(1 - \cos\Omega_0 t\right) \tag{2.66}$$

and expanding the exponent of Eq. (2.50) into the series over the small parameter μ (taking into account as well that $\mu\omega/\Omega_0 << 1$), after the calculations we will have the following formula for differential power of noncoherent radiation in the capture regime:

$$dP_{\mathbf{k}} = dP_{\mathbf{k}}^{(0)} + dP_{\mathbf{k}}^{(+)} + dP_{\mathbf{k}}^{(-)}\ , \tag{2.67}$$

$$dP_{\mathbf{k}}^{(0)} = \frac{e^2 n(\omega)}{2\pi c n_0^2}\omega^2 \sin^2\theta \cdot \delta\left[\omega \frac{n(\omega)}{n_0}\cos\theta - \omega\right] d\omega dO, \tag{2.68}$$

$$dP_{\mathbf{k}}^{(\pm)} = \mu^2 \frac{e^2 n(\omega)}{8\pi c}\frac{\omega^2}{n_0\left(n_0^2-1\right)}\delta\left[\omega\frac{n(\omega)}{n_0}\cos\theta - \omega \pm \Omega_0\right]$$

$$\times\left\{\left[n_0^2 + \left(\frac{n_0^2}{2}-1\right)\sin^2\theta\right] \pm 2\frac{n(\omega)}{n_0}\left(\frac{n_0^2}{2}-1\right)\frac{\omega}{\Omega_0}\cos\theta\sin^2\theta\right.$$

$$\left. + \frac{n^2(\omega)}{n_0^2}\frac{\omega^2}{\Omega_0^2}\sin^2\theta\left[\frac{n_0^2}{2} + \left(\frac{n_0^2}{2}-1\right)\cos^2\theta\right]\right\} d\omega dO, \tag{2.69}$$

where θ is the angle between the radiation direction and axis OX. The term $dP_{\mathbf{k}}^{(0)}$ corresponds to Cherenkov radiation by the particle moving with the velocity $\mathrm{v} = c/n_0$ in the wave and the terms $dP_{\mathbf{k}}^{(\pm)}$ determine the radiation due to oscillatory motion of the particle. According to the δ-functions in Eqs. (2.68) and (2.69) for the radiation angles we have

$$\cos\theta_0 = \frac{n_0}{n(\omega)}, \qquad \cos\theta_\pm = \frac{n_0}{n(\omega)}\left(1 \mp \frac{\Omega_0}{\omega}\right). \tag{2.70}$$

Note that the approximation $\mu\omega/\Omega_0 << 1$ applied in the calculations is necessary only to obtain ultimate analytical formulas (in the general case the particle velocity is expressed by elliptic functions and analytical solution of the problem is complicated).

Integrating Eqs. (2.68) and (2.69) over the solid angle for the spectral distribution of the radiation we obtain

$$dP_\omega^{(0)} = \frac{e^2}{c n_0}\left[1 - \frac{n_0^2}{n^2(\omega)}\right]\omega d\omega\ , \tag{2.71}$$

$$dP_\omega^{(\pm)} = \mu^2 \frac{e^2}{4c}\frac{1}{n_0\left(n_0^2-1\right)}\left\{n_0^2 + \frac{n_0^2 + n^2(\omega) - 2}{2}\right.$$

$$\left.\times\left[\frac{\omega^2}{\Omega_0^2} - \frac{n_0^2}{n^2(\omega)}\left(1 \mp \frac{\Omega_0}{\omega}\right)^2\right]\right\}\omega d\omega. \tag{2.72}$$

In Eq. (2.72)

$$\omega = \pm\frac{\Omega_0}{1 - \frac{n(\omega)}{n_0}\cos\theta}. \tag{2.73}$$

As Ω_0 depends on initial phase ϕ_0 (see Eq. (2.64)), in the case of a particle beam captured by a wave of linear polarization at a certain angle θ a whole spectrum of frequencies will be radiated, in contrast to common Cherenkov radiation at which only a definite frequency is radiated at that certain angle.

Let us compare the radiation at the fundamental frequency ω_0 with the common Cherenkov radiation at the same frequency (in the absence of the external wave). In this case $dP_{\omega_0}^{(0)} = 0$ and for $dP_{\omega}^{(-)}$ the conservation law for the radiation of frequency ω_0 is violated (see the second expression in Eq. (2.70)). From Eq. (2.72) at $\omega = \omega_0$ we have

$$dP_{\omega_0}^{(+)} = \frac{e^2}{2cn_0}\mu^2\frac{\omega_0}{\Omega_0}\omega_0 d\omega. \tag{2.74}$$

If one substitutes $\mathrm{v} = c(1+\mu)/n_0$ in the Tamm–Frank formula (2.55), then with the linear approximation by parameter μ we will have

$$dP_{\omega_0} = \frac{2e^2}{cn_0}\mu\omega_0 d\omega. \tag{2.75}$$

A comparison of Eqs. (2.74) and (2.75) shows that the radiation of the particle at the fundamental frequency ω_0 in the capture regime is much smaller than the spontaneous Cherenkov radiation (because of condition (2.65)). Such a decrease of radiation is connected with the violation of coherency due to oscillation of particle velocity in the wave field.

The fundamental frequency ω_0 in the capture regime is radiated at the angle $\theta \simeq \sqrt{2\Omega_0/\omega_0}$ (see Eq. (2.73)). The common Cherenkov angle is $\theta_c \simeq \sqrt{\mu/2}$ and as far as $\mu << \Omega_0/\omega_0$ then $\theta >> \theta_c$, i.e., the radiation angle at the frequency of stimulating wave in the capture regime is much larger than the spontaneous Cherenkov angle in the absence of the external wave.

At the other frequencies $\omega \neq \omega_0$ the radiation is mainly determined by $dP_{\omega}^{(0)}$, which practically coincides with the Tamm–Frank formula.

Consider now the case of circular polarization of the incident wave

$$E_y = E_0 \cos\left(\frac{\omega_0 n_0}{c}x - \omega_0 t\right), \qquad E_z = E_0 \sin\left(\frac{\omega_0 n_0}{c}x - \omega_0 t\right). \tag{2.76}$$

Linearizing the equations of motion (2.2) and (2.3) in the field (2.76) under the condition (2.58) for the particle velocity in the capture regime we obtain

$$\mathrm{v}_x = \frac{c}{n_0}\left(1 + \mu\cos\Omega_0' t\right),$$

$$\mathrm{v}_y = -\mu\frac{c}{\left(n_0^2-1\right)^{1/2}}\cos\phi_0\cdot\sin\Omega_0' t\ , \tag{2.77}$$

$$\mathrm{v}_z = -\mu \frac{c}{\left(n_0^2 - 1\right)^{1/2}} \sin\phi_0 \cdot \sin \Omega_0' t \ ,$$

where the oscillation frequency in the wave well Ω_0' does not depend on the initial phase ϕ_0 in contrast to the case of the linearly polarized wave. If we calculate the radiation power by Eqs. (2.77), then the same formulas (2.67)–(2.73) for the case of wave linear polarization will be obtained. The only difference is that Ω_0' is constant for all particles situated at the difference phases in the wave well, and at the certain angle only one frequency will be radiated in this case.

Equations (2.63) and (2.77) show that the energy of the particle in the field

$$\mathcal{E} = \mathcal{E}_0 + \mu \frac{\mathcal{E}_0}{n_0^2 - 1} \cos \Omega_0 t \ ; \qquad \mathcal{E}_0 = \frac{mc^2 n_0}{\left(n_0^2 - 1\right)^{1/2}} \tag{2.78}$$

oscillates between the values

$$\mathcal{E}_{\min} = \mathcal{E}_0 \left(1 - \frac{\mu}{n_0^2 - 1}\right) ; \qquad \mathcal{E}_{\max} = \mathcal{E}_0 \left(1 + \frac{\mu}{n_0^2 - 1}\right) ,$$

consequently the exchange of the energy is

$$\Delta\mathcal{E} = 2\mu \frac{mc^2 n_0}{\left(n_0^2 - 1\right)^{3/2}} . \tag{2.79}$$

According to Eqs. (2.78) the particle captured by the wave periodically acquires and loses such energy $\Delta\mathcal{E}$. Due to the induced Cherenkov effect the energy lost by the particle is coherently radiated into the wave (particularly for this reason the above-considered noncoherent radiation at the frequency of stimulating wave ω_0 is sufficiently suppressed) and the amplification of the initial wave will take place. Hence, the particle capture phenomenon may in principle serve as a FEL mechanism (Cherenkov amplifier). For the latter one needs to solve the self-consistent problem on the basis of the set of Maxwell–Vlasov equations.

Let us now consider the amplitude of the wave field to be a slowly varying function of the space-time coordinates (x, t) with respect to the phase. The problem will be investigated first for the circular polarization of the wave

$$E_y(x,t) = E(x,t) \cos\left(\frac{\omega_0 n_0 x}{c} - \omega_0 t\right) ,$$

$$E_z(x,t) = E(x,t) \sin\left(\frac{\omega_0 n_0 x}{c} - \omega_0 t\right) \tag{2.80}$$

with the boundary conditions

$$E_y(0,t) = E_0 \cos\omega_0 t \ , \qquad E_z(0,t) = -E_0 \sin\omega_0 t. \tag{2.81}$$

Related to particles we will assume that it crosses the boundary of the medium $x = 0$ at the moment $t = t_0$ with the initial velocity (2.58). Linearizing the equations of motion (2.2) and (2.3) in the field (2.80) for a single particle velocity in the field we obtain

$$\mathrm{v}_y = -\frac{c}{\left(n_0^2-1\right)^{1/2}}\mu\cos(\omega_0 t_0)\sin\left[\frac{e\left(n_0^2-1\right)}{mcn_0}\int_{t_0}^{t}E(t',x)dt'\right],$$

$$\mathrm{v}_z = \frac{c}{\left(n_0^2-1\right)^{1/2}}\mu\sin(\omega_0 t_0)\sin\left[\frac{e\left(n_0^2-1\right)}{mcn_0}\int_{t_0}^{t}E(t',x)dt'\right]. \tag{2.82}$$

To define the electric current of the particle stream we assume that the space is continuously filled with the charged particles. Then at the moment t_0 in the point x will be situated only the particles for which $t_0 = t-\ n_0 x/c$ (with accuracy $\mu\omega_0/\Omega_0 << 1$). Hence, for the electric current of the particle stream we will have

$$j_y(x,t) = -\mu\frac{ec\rho_0}{\left(n_0^2-1\right)^{1/2}}\cos\left(\frac{\omega_0 n_0 x}{c}-\omega_0 t\right)$$

$$\times\sin\left[\frac{e\left(n_0^2-1\right)}{mcn_0}\int_{t-n_0x/c}^{t}E(t',\frac{c}{n_0}(t'-t)+x)dt'\right], \tag{2.83}$$

$$j_z(x,t) = -\mu\frac{ec\rho_0}{\left(n_0^2-1\right)^{1/2}}\sin\left(\frac{\omega_0 n_0 x}{c}-\omega_0 t\right)$$

$$\times\sin\left[\frac{e\left(n_0^2-1\right)}{mcn_0}\int_{t-n_0x/c}^{t}E(t',\frac{c}{n_0}(t'-t)+x)dt'\right],$$

where ρ_0 is the mean density of the particles in the initial stream, which will be assumed constant (since $\mu << 1$ the variation ρ_0 is small and can be neglected).

Because we are investigating the induced radiation, the field of the scalar potential and longitudinal radiation field along the axis OX will not be considered here. Substituting Eqs. (2.83) into the Maxwell equation and taking into account the slow variation of the radiation field amplitude:

$$\left|\frac{\partial E}{\partial t}\right| << \omega_0|E| \ , \qquad \left|\frac{\partial E}{\partial x}\right| << \frac{\omega_0 n_0}{c}|E| \ ,$$

we obtain the equation of the self-consistent field:

$$\frac{\partial E}{\partial x} + \frac{n_0}{c}\frac{\partial E}{\partial t} = \frac{2\pi e\rho_0}{n_0\left(n_0^2-1\right)^{1/2}}\mu$$

$$\times \sin\left[\frac{e\left(n_0^2-1\right)}{mcn_0}\int_{t-n_0x/c}^{t} E(t', \frac{c}{n_0}(t'-t)+x)dt'\right]. \tag{2.84}$$

Equation (2.84) has a simpler form over wave coordinates $\tau = t - n_0x/c$, $\eta = x$. Then, for the field amplitude $E(t,x) = f(\tau,\eta)$ we have

$$\frac{\partial}{\partial \eta}f(\tau,\eta) = \frac{2\pi e\rho_0}{n_0\left(n_0^2-1\right)^{1/2}}\mu \sin\left[\frac{e\left(n_0^2-1\right)}{mc^2}\int_0^{\eta} f(\tau,\eta')d\eta'\right]. \tag{2.85}$$

The simple analytic solution can be received at the incident monochromatic wave: $f(\tau,0) = E_0$. In this case, it follows from Eq. (2.84) that $f(\tau,\eta)$ does not depend on τ, i.e., $f(\tau,\eta) = f(\eta)$, and for the quantity

$$\varphi = \frac{e\left(n_0^2-1\right)}{mc^2}\int_0^{\eta} f(\eta')d\eta' \tag{2.86}$$

we have the nonlinear equation of anharmonic oscillator

$$\varphi'' = \frac{2\pi e^2\rho_0\left(n_0^2-1\right)^{1/2}}{mc^2n_0}\mu\sin\varphi, \tag{2.87}$$

the general solution of which is the incomplete elliptic integral of the first kind

$$\frac{1}{2}\left(n_0^2-1\right)\frac{eE_0x}{mc^2} = \int_0^{\varphi/2}\frac{dz}{\sqrt{1+\zeta^2\sin^2 z}},$$

$$\zeta^2 = \frac{8\pi\mu}{n_0\left(n_0^2-1\right)^{3/2}}\frac{mc^2\rho_0}{E_0^2}. \tag{2.88}$$

In the linear case when $\varphi << 1$ from Eq. (2.88) we have

$$E(x) = E_0\left[\begin{array}{ll}\cosh\left(\frac{x}{l_c}\right), & \mu > 0,\\ \cos\left(\frac{x}{l_c}\right), & \mu < 0.\end{array}\right. \tag{2.89}$$

Hence, for $\mu > 0$, which corresponds to particles' initial velocity $\mathrm{v}_0 > c/n_0$, exponential amplification of the incident wave occurs. For $\mu < 0$, that is, $\mathrm{v}_0 < c/n_0$, the amplification vanishes on average. The quantity in Eq. (2.89)

$$l_c = \left(\frac{mc^2 n_0}{2\pi e^2 \mu \rho_0 \left(n_0^2 - 1\right)^{1/2}} \right)^{1/2} \tag{2.90}$$

is the coherent length of amplification. Equation (2.85) is an analogue of the equation of the quantum amplifier. The role of inverse population in atomic systems here performs detuning of the Cherenkov resonance $\mathrm{v}_0 - c/n_0$ (parameter μ).

Analysis of the obtained formulas shows that the linear regime takes place at the electric field strengths of amplifying radiation

$$E \lesssim e\lambda_0 \rho_0 \left(\frac{mc^2}{\mathcal{E}_0} \right)^3$$

(λ_0 is the wavelength of incident wave) and at the coherent length of amplification

$$l_c \lesssim \frac{mc^2}{e^2 \lambda_0 \rho_0} \left(\frac{\mathcal{E}_0}{mc^2} \right)^2 .$$

In the saturation regime from Eq. (2.85) we have

$$E(x) = E_0 + \mu \frac{2\pi mc^2 \rho_0}{n_0 \left(n_0^2 - 1\right)^{3/2}} \frac{1}{E_0} \left\{ 1 - \cos \left[\left(n_0^2 - 1\right) \frac{eE_0 x}{mc^2} \right] \right\} . \tag{2.91}$$

The wave energy gain found from Eq. (2.91) corresponds to the particle energy exchange in the capture regime (in a unit volume) according to Eq. (2.79):

$$\Delta W = \rho_0 \Delta \mathcal{E} = \frac{2\mu \rho_0 \mathcal{E}_0}{n_0^2 - 1} . \tag{2.92}$$

The saturation regime and Eq. (2.91) is valid when the electric field strengths of amplifying radiation

$$E \gtrsim e\lambda_0 \rho_0 \frac{\mathcal{E}_0}{mc^2} .$$

Consider now the case of linear polarization of incident wave

$$E_y = E(x,t) \cos \left(\frac{\omega_0 n_0 x}{c} - \omega_0 t \right) . \tag{2.93}$$

By analogy with the previous case for the velocity of a single particle in the field (2.93) we obtain

$$\mathrm{v}_x = \frac{c}{n_0}\left(1 + \mu \cos\left[\int_{t_0}^{t} \Omega_0(t', x)dt'\right]\right),$$

$$\mathrm{v}_y = -\frac{c}{\left(n_0^2 - 1\right)^{1/2}} \mu \sin\left[\int_{t_0}^{t} \Omega_0(t', x)dt'\right], \tag{2.94}$$

where the modulation frequency

$$\Omega_0(t, x) = \frac{e\left(n_0^2 - 1\right)}{mcn_0} E(x, t) \cos\omega_0 t_0 \tag{2.95}$$

already depends on initial phase $\phi_0 = \omega_0 t_0$. Therefore, in the particle beam case, all harmonics will be radiated in contrast to circular polarization of the wave. By calculating the electric current of the particle stream and expanding into series over Bessel functions we find that the induced radiation stipulated by the y component of the current (coherent radiation) will include only the odd harmonics and the noncoherent part of the radiation stipulated by the x component of the current (longitudinal field along the axis OX) will include only the even harmonics. As in the previous case we will consider the coherent radiation. Then, substituting y component of the current

$$j_y(x, t) = -\mu \frac{ec\rho_0}{\left(n_0^2 - 1\right)^{1/2}} \sum_{s=-\infty}^{+\infty} i^{s-1} J_s(\alpha) \exp\left[is\omega_0\left(\frac{n_0 x}{c} - t\right)\right],$$

$$s = 2k - 1\ ;\ k = 0, \pm 1, \pm 2, \ldots,$$

$$\alpha(x, t) = \frac{e\left(n_0^2 - 1\right)}{mcn_0} \int_{t - n_0 x/c}^{t} E(t', \frac{c}{n_0}(t' - t) + x)dt' \tag{2.96}$$

into the Maxwell equation for the slowly varying amplitude of the self-consistent field we will have the equation

$$2is\omega_0\left(\frac{n_0}{c}\frac{\partial E_s}{\partial x} + \frac{n_s^2}{c^2}\frac{\partial E_s}{\partial t}\right) + \frac{s^2\omega_0^2}{c^2}\left(n_s^2 - n_0^2\right) E_s$$

$$= i^s \frac{4\pi e\rho_0 s\omega_0}{c\left(n_0^2 - 1\right)^{1/2}} \mu J_s(\alpha), \tag{2.97}$$

where n_s is the medium refractive index at the s-th harmonic of the fundamental frequency ω_0 ($n_s \equiv n(s\omega_0)$).

Consider Eq. (2.97) with regard to the presence and absence of synchronism. In the last case, when $n_s \neq n_0$ taking into account the slow variation of the field amplitude from Eq. (2.97) we obtain

$$E_s = i^s \mu \frac{4\pi e c \rho_0}{\left(n_0^2 - 1\right)^{1/2}} \frac{1}{s\omega_0} \frac{1}{n_s^2 - n_0^2} J_s(\alpha). \tag{2.98}$$

As is seen from this formula in the absence of synchronism, there is a weak dependence of radiation field on harmonics' number.

In the case of synchronism ($n_s = n_0$), Eq. (2.97) becomes

$$\frac{\partial E_s}{\partial x} + \frac{n_0}{c}\frac{\partial E_s}{\partial t} = i^{s-1} \mu \frac{2\pi e \rho_0}{n_0 \left(n_0^2 - 1\right)^{1/2}} J_s(\alpha). \tag{2.99}$$

For the first harmonic (fundamental coherent radiation) the results repeat almost exactly the case of wave circular polarization (Eqs. (2.88)–(2.90)), the only difference being that the coherence length in this case is $\sqrt{2} l_c$.

To determine the radiation on the other harmonics in the case of synchronism consider the problem in the given field. Then, for large x when

$$\frac{e\left(n_0^2 - 1\right) E_0 x}{mc^2} >> 1$$

for the harmonics' amplitudes we have

$$E_s = i^{s-1} \mu \frac{2\pi m c^2 \rho_0}{n_0 \left(n_0^2 - 1\right)^{3/2}} \frac{1}{E_0}. \tag{2.100}$$

Hence, the radiation intensity on the harmonics

$$I_s = \frac{c}{8\pi} |E_s|^2 \simeq e^2 c \frac{(\lambda_0^3 \rho_0)^2}{\lambda_0^4} \left(\frac{\mathcal{E}_0}{mc^2}\right)^2. \tag{2.101}$$

Equation (2.101) as well as Eq. (2.92) and estimation formulas are obtained when $\mu \sim \xi (mc^2/\mathcal{E}_0)^2$, which is defined from the condition of particle capture. As in the linear regime the coherence length increases as energy squared, and the losses of the particles in the medium depend on energy logarithmically, then the energy increase for amplification of weak signals does not give an essential advantage. The optimal energy is $\mathcal{E}_0 \sim mc^2$. Then $l_c \sim (r_0 \lambda_0 \rho_0)^{-1}$, where $r_0 = e^2/mc^2$ is the electron classical radius. The estimations show that for the amplification of optical radiation in the capture regime with $n_0 = \text{const}$, electron beams of large densities are necessary. The situation considerably will be improved if media with varying refractive index $n_0(x)$ are used. Then along the direction of increase of $n_0(x)$ the particles will be continuously decelerated, and the wave continuously amplified (a regime inverse to the one considered in Section 2.4).

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