Chapter 2

# FEEDING STRATEGIES FOR MAXIMISING GROSS MARGIN IN PIG PRODUCTION

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**Abstract** Nonlinear optimisation and a pig growth model are combined with the traditional use of linear programming to maximise gross margin per pig place per year for the pig producer. Emphasis in this paper is on description of the problem and analysis of the objective function.

**Keywords:** genetic algorithm, growth model, linear programming, Monte Carlo, Nelder-Mead algorithm, simulated annealing

## 1. Introduction

Production of pig meat worldwide exceeds that of any other meat. Pigs are generally fed in a controlled environment, with least cost diets determined using linear programming. Of greater importance to the pig producer than least cost diets, however, is the maximisation of gross margin (per pig place or per pig place per year). Two relatively recent developments, pig growth models and efficient methods for optimisation of nonlinear functions of high dimension, in conjunction with the traditional use of linear programming, now make gross margin maximisation possible. The method of solution involves a synthesis of optimisation techniques, linear programming being employed as a sub-routine within the framework of a wider search algorithm. The purpose of this paper is to describe this problem, together with the solution methodology and associated practical outcomes.

The format of the paper is as follows. In the next section the optimisation problem is formulated, the domain described and the objective function detailed. Successful solution approaches are described in Section 3. The nature of the objective function is discussed in Section 4. A summary then concludes the paper.

## 2. The problem

Pig farmers growing pigs from weaning until slaughter wish to maximise gross margin, namely

$$\text{Gross Return} - \text{Total Feed Costs} - \text{Weaner Cost},$$

per pig place per year. The main area over which the pig producer has control is the feeding of the pig. A single feeding regimen can be summarised, as described in DeLange, 1995, by three parameters, $d$, $r$ and $p$. A *feeding strategy* $F$ is then a finite sequence of $(d, r, p)$ triples, with each triple describing the diet for a fixed period, say one week. Here

$d$ = digestible energy density, in MegaJoules per kilogram

$r$ = minimum lysine to digestible energy ratio, in grams per MegaJoule

$p$ = proportion of the *ad libitum* daily digestible energy intake

The *ad libitum* digestible energy intake for the pig at any given liveweight is determined on farm using monitor pens of pigs. Feed supplied, feed wasted and pig liveweight are recorded in order to determine the voluntary daily feed intake of the pig.

For the purposes of this exposition we work with a feeding strategy of ten $(d, r, p)$ triples, each fed for one week, so the general point in the domain has the form

$$F = (d_1, r_1, p_1;\ d_2, r_2, p_2;\ \ldots;\ d_{10}, r_{10}, p_{10})$$

For the particular problem examined here, relevant to the New Zealand situation, a growth period of ten weeks is sufficient to include the most profitable solutions.

The energy content of the ingredients in the diet requires that $d$ be at least 12 MJ/kg and no more than 16 MJ/kg. A sensible range for $r$ is from 0.4 to 1.0 grams of lysine per MegaJoule; values outside this range are feasible, but this range has been found wide enough to encompass the most profitable diets and is in accordance with the opinion of pig nutritionists. Finally, $p$ is allowed to vary between 0.5 and 1.0; nutritionists agree that pigs require at least 50% of their maximum voluntary digestible energy intake. In summary, the general point in the domain lies in the hypercuboid in $\mathbf{R}^{30}$ given by $([12,16] \times [0.4,1] \times [0.5,1])^{10}$.

Thus the objective function of interest is $g(F)$, the gross margin per pig place per year associated with feeding strategy $F$. This is calculated in two steps:

1 Calculation of $g(F, x)$, the gross margin per pig place per year when feeding strategy $F$ is administered for $x \leq 70$ days, and then

2 Calculation of $g(F) = \max_x g(F, x)$

We now provide more detail on each of these steps.

In the first step, the gross margin per pig place per year is calculated for a given feeding strategy $F$ and number of days $x \leq 70$ for which it is fed, as detailed in Figure 1.1. This calculation successively relies on a linear program, the pig growth model and the price schedule.

A sample linear program is illustrated in Table 1.1. Using this we minimise the diet cost per kilogram, subject to the $d$ and $r$ values in the $(d, r, p)$ triple. Parameter $d$ is incorporated in an equality constraint for the digestible energy total. Together with parameter $r$ it also dictates the minimum lysine level in the diet. The other amino acids are constrained to be present in at least certain proportions to lysine, to ensure ideal protein balance. Parameter $p$ and the *ad libitum* digestible energy intake curve then determine the total feed intake and so the minimum daily feed cost.

The pig growth model used, from DeLange, 1995, requires three parameters particular to the pig genotype:

$$\begin{aligned} P_0 &= \text{initial mass of protein in the weaner pig} \\ Pd_{\max} &= \text{maximum daily protein deposition} \\ \min LP &= \text{minimum allowable lipid to protein ratio} \end{aligned}$$

Given a feeding regimen and initial chemical body composition the pig growth model is then capable of "growing" a pig for $x$ days, outputting backfat thickness and carcass weight.

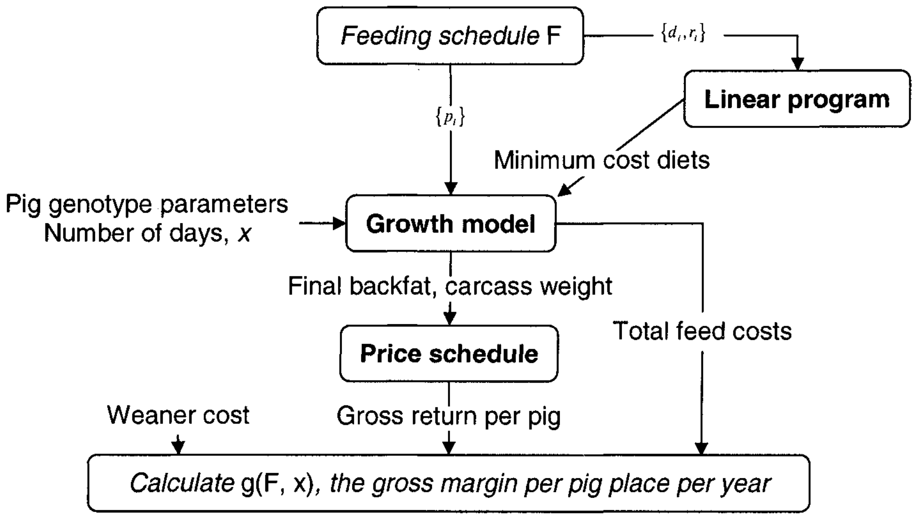


*Figure 1.1.* The gross margin per pig place per year when feeding strategy $F$ is used for $x$ days is calculated as shown in the flow chart. A linear program finds the minimum cost diets for the given feeding strategy. The pig growth model and price schedule are then used to find the market return per pig, from which the gross margin per pig place per year $g(F, x)$ may be calculated.

Price schedules for pigs at slaughter typically depend on backfat thickness and carcass weight. A 1998 New Zealand schedule is reproduced in Table 1.2. For pigs in the best categories (backfat thickness from 6mm to 9mm and carcass weight below 55kg) the producer receives NZ$3.10/kg.

In the second step, $g(F)$ is easily found as the maximum in the array

$$(g(F, 1),\ g(F, 2),\ \ldots,\ g(F, 70))$$

Figure 1.2 summarises the problem: a linear program and a pig growth model allow us to evaluate the objective function, which is then maximised over a high dimensional hypercuboid using a nonlinear program.

## 3. Solution algorithms

For this problem the objective function is of the "black box" variety; it is not known analytically. Many objective function evaluations are thus required by the solution algorithms, involving substantial computation. The domain is also of moderately high dimension. Simulated annealing ( Metropolis et al., 1953) and genetic algorithms (Holland, 1975) are both successful stochastic approaches to the problem, while a satisfactory de-

*Table 1.1.* The linear program providing the least cost diet.

| | *Barley* | | *Soybean* | | $\cdots$ | | *Meat and Bone* | | |
|---|---|---|---|---|---|---|---|---|---|
| minimise: | $0.25B$ | $+$ | $0.715S$ | $+$ | $\cdots$ | $+$ | $0.5M$ | | |
| Digestible energy | $13.2B$ | $+$ | $15.86S$ | $+$ | $\cdots$ | $+$ | $12.5M$ | $=$ | $d$ |
| Lysine lower bound | $3.19B$ | $+$ | $27.33S$ | $+$ | $\cdots$ | $+$ | $16.19M$ | $\geq$ | $dr$ |
| Balanced | $2.85B$ | $+$ | $10.74S$ | $+$ | $\cdots$ | $+$ | $20.3M$ | $\geq$ | $0.62dr$ |
| amino | $1.2B$ | $+$ | $4.68S$ | $+$ | $\cdots$ | $+$ | $1.09M$ | $\geq$ | $0.19dr$ |
| acid | $2.42B$ | $+$ | $11.48S$ | $+$ | $\cdots$ | $+$ | $9.95M$ | $\geq$ | $0.32dr$ |
| lower | $8.34B$ | $+$ | $36.75S$ | $+$ | $\cdots$ | $+$ | $18.91M$ | $\geq$ | $0.95dr$ |
| bounds | $3B$ | $+$ | $16.22S$ | $+$ | $\cdots$ | $+$ | $9.54M$ | $\geq$ | $0.67dr$ |
| | $6.85B$ | $+$ | $33.25S$ | $+$ | $\cdots$ | $+$ | $21.5M$ | $\geq$ | $dr$ |
| | | | | | $\vdots$ | | | | |
| Mineral | $0.5B$ | $+$ | $3S$ | $+$ | $\cdots$ | $+$ | $105M$ | $\geq$ | 8 |
| bounds | $0.5B$ | $+$ | $3S$ | $+$ | $\cdots$ | $+$ | $105M$ | $\leq$ | 13 |
| | $3.1B$ | $+$ | $2.6S$ | $+$ | $\cdots$ | $+$ | $52M$ | $\geq$ | 7 |
| | $3.1B$ | $+$ | $2.6S$ | $+$ | $\cdots$ | $+$ | $52M$ | $\leq$ | 11 |
| | $0.2B$ | $+$ | $0.1S$ | $+$ | $\cdots$ | $+$ | $5.5M$ | $\geq$ | 10 |
| | $0.2B$ | $+$ | $0.1S$ | $+$ | $\cdots$ | $+$ | $5.5M$ | $\leq$ | 20 |
| | | | | | $\vdots$ | | | | |
| Ingredient | $-0.4B$ | $+$ | $0.6S$ | $-$ | $\cdots$ | $-$ | $0.4M$ | $\leq$ | 0 |
| upper | $-0.1B$ | $-$ | $0.1S$ | $-$ | $\cdots$ | $-$ | $0.1M$ | $\leq$ | 0 |
| bounds | $-0.3B$ | $-$ | $0.3S$ | $-$ | $\cdots$ | $-$ | $0.3M$ | $\leq$ | 0 |
| | $-0.4B$ | $-$ | $0.4S$ | $-$ | $\cdots$ | $-$ | $0.4M$ | $\leq$ | 0 |
| | $-0.4B$ | $-$ | $0.4S$ | $-$ | $\cdots$ | $-$ | $0.4M$ | $\leq$ | 0 |
| | | | | | $\vdots$ | | | | |
| Diet mass | $B$ | $+$ | $S$ | $+$ | $\cdots$ | $+$ | $M$ | $=$ | 1 |

*Table 1.2.* A New Zealand price schedule giving prices in cents per kg for pigs at slaughter, as at 24 August 1998. A levy of \$9.40 per carcass is deducted.

| | *Carcass Weight (kg)* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Fat (mm)* | 35.0 and under | 35.1 to 40.0 | 40.1 to 45.0 | 45.1 to 50.0 | 50.1 to 55.0 | 55.1 to 60.0 | 60.1 to 65.0 | 65.1 to 70.0 | 70.1 to 75.0 | 75.1 to 80.0 | 80.1 and over |
| $< 6$ | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 |
| 6–9 | 310 | 310 | 310 | 310 | 310 | 295 | 295 | 280 | 280 | 280 | 275 |
| 10–12 | 285 | 285 | 285 | 285 | 285 | 280 | 280 | 280 | 280 | 280 | 275 |
| 13–15 | 190 | 190 | 190 | 215 | 215 | 245 | 245 | 245 | 245 | 245 | 245 |
| 16–18 | 150 | 150 | 150 | 150 | 150 | 165 | 165 | 165 | 165 | 165 | 165 |
| $> 18$ | 120 | 120 | 120 | 120 | 120 | 135 | 135 | 135 | 135 | 135 | 135 |

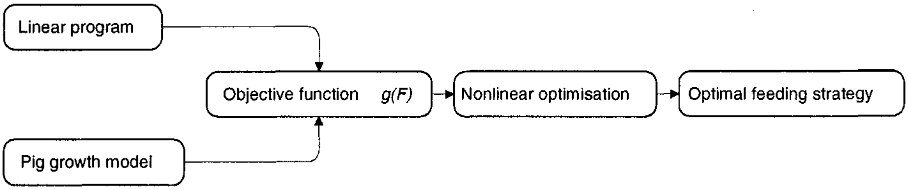


*Figure 1.2.* The components involved in maximisation of gross margin per pig place per year.

terministic approach is provided by the Nelder-Mead algorithm (Nelder and Mead, 1965). They typically yield, for the particular problem used, a gross margin of around NZ\$280 per pig place per year. Pure random search, however, is not successful. Figure 1.3 shows the progress of a genetic algorithm on the problem, compared with that of Pure Random Search. In 30 seconds a genetic algorithm can find a feeding strategy equal to that reached by pure random search in over 100 hours! Pure random search, in that time, reaches a solution of approximately NZ\$250 per pig place per year.

Table 1.3 displays the final results of a successful run. It lists the values of $d$, $r$ and $p$ in the best solution; the cost in cents per kilogram of each diet is also given. Slaughter date $x$ was selected as 63 days, so the parameters for the tenth week were not used and are not shown. The linear program then provides the ingredients for each diet in the feeding strategy, together with the optimal proportion of each ingredient by weight. For example, a kilogram of the first diet in Table 1.3 contains 510g barley, 280g soybean and 100g wheat by-product, with the remainder made up of meat and bone meal, soya oil, salt and synthetic amino acids. (The unusual diet administered in week 7 allows the pig to maintain near maximum protein deposition with virtually zero lipid deposition, permitting continued weight gain but restricting fat content in the pig. Not all solutions found by the optimisation techniques contain this kind of aberration.)

## 4. The nature of the objective function

A question which always vexes the global optimiser is whether the optimum has been reached. In order to investigate this question in this context, further information about the nature of the objective function is obtained in two ways:

1 By examining the value of the objective function along randomly taken domain cross-sections through the best known solution. This provides evidence that the optimum has been reached and suggests

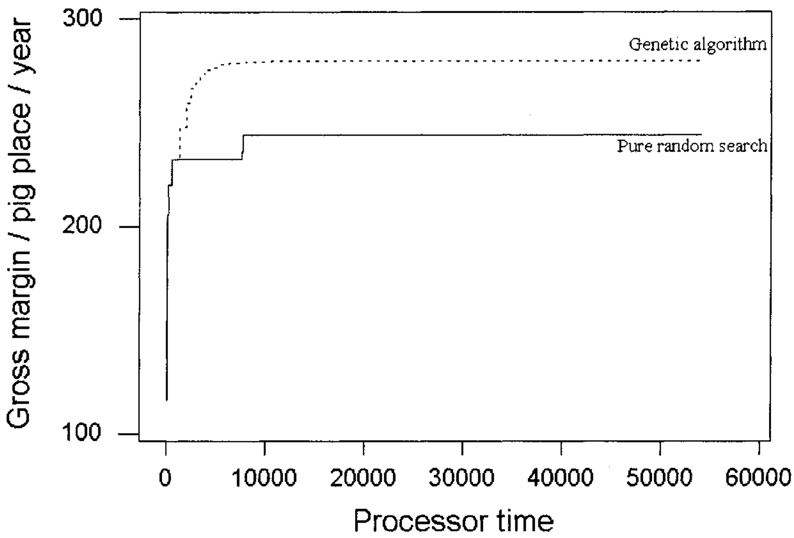


*Figure 1.3.* Comparison of the time in seconds required by a genetic algorithm and pure random search to attain certain dollar values of gross margin per pig place per year.

*Table 1.3.* Weekly values of $d$, $r$ and $p$ and cost per kilogram of diet for a solution found using a genetic algorithm.

| *Week* | $d$ | $r$ | $p$ | *Cost (c/kg)* |
|---|---|---|---|---|
| 1 | 14.51 | 0.791 | 0.961 | 41.24 |
| 2 | 14.57 | 0.627 | 1.000 | 35.93 |
| 3 | 14.40 | 0.554 | 1.000 | 33.10 |
| 4 | 13.14 | 0.568 | 0.840 | 30.61 |
| 5 | 14.34 | 0.493 | 0.873 | 31.00 |
| 6 | 14.18 | 0.478 | 0.850 | 30.20 |
| 7 | 13.58 | 0.758 | 0.510 | 37.55 |
| 8 | 13.14 | 0.550 | 0.670 | 30.07 |
| 9 | 14.14 | 0.528 | 0.680 | 31.66 |

that the objective function takes the form of a single peaked, but very craggy, high-dimensional volcano.

2 By comparing the performance of pure random search with the expected performance if the form was broadly that of this "craggy volcano".

We now report on these two investigations into the nature of the objective function.

Figure 1.4 shows the typical shape of the objective function along a random cross-section through the putative argmax. Overall, the function appears to be unimodal. Jumps in the function are caused by the step function nature of the price schedule and the fact that the pig must grow a whole number of days.

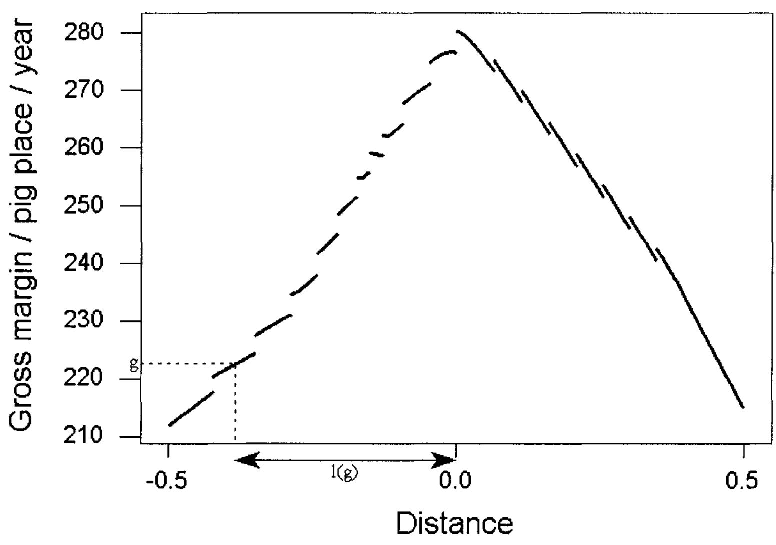


*Figure 1.4.* The approximately volcano shape of a section through the objective function at the best known solution.

Through examination of many such cross-sections an average relationship is set up between $g$, the gross margin per pig place per year, and $l(g)$, the distance between the corresponding place in the domain and the best known solution (as illustrated in Figure 1.4). The expected number of iterations until convergence to a particular level for pure random search will be the reciprocal of the probability that a given iteration falls in the associated level set. This probability is the relative volume of the level set. Assuming roughly hyper-spherical level sets for the objective function, the probability is

$$\frac{\pi^{15} l(g)^{30}}{15! V}$$

where $V$ is the volume of the hypercuboid domain.

Figure 1.5 compares this theoretical expected number of pure random search iterations to reach a given gross margin per pig place per year with a step function recording progress of a particular run. Evidently

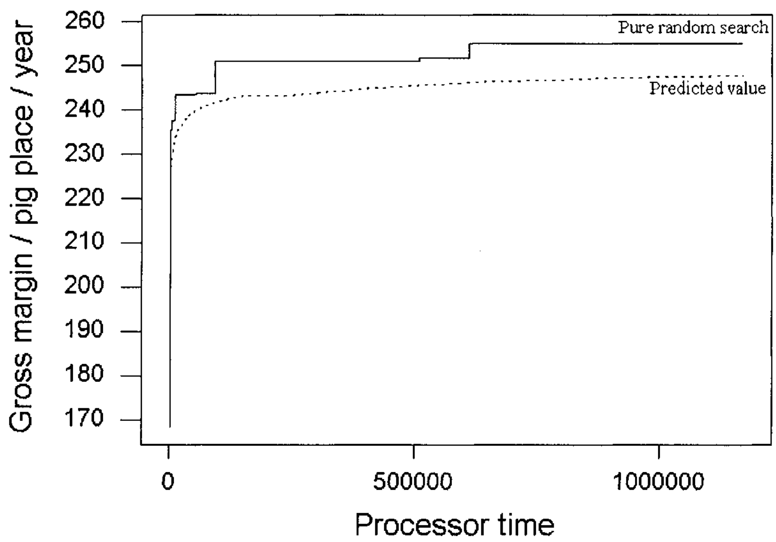


*Figure 1.5.* A comparison of the expected number of iterations to reach a value of the objective function (dotted line), under the assumption of hyperspherical level sets, with the number required in a particular run of pure random search (solid line).

the forms of the two curves are similar. The actual run is higher than the curve based on the estimated relationship between $g$ and $l(g)$ with the assumption of hyperspherical level sets. This may well indicate that smaller peaks exist. There are also some cross-sections where the objective function does not decrease very rapidly. Overall there is evidence that the level sets have slightly greater relative volume than predicted.

It's time to ask an important question: Is this a theoretical solution with no practical value? In practice, feeding regimens cannot be administered precisely. Allowing for an achievable tolerance in feeding of 0.05 in each parameter of each diet, however, it is evident that the optimal solution is still only one part of the space in $10^{40}$. An intelligent search is needed in order to find solutions approaching this optimum, as the comparison with pure random search has shown. Even with only three distinct diets (as is common in practice today), the optimal solution is one part in $10^{12}$. A useful, practical solution region remains a needle in a high dimensional haystack. As the world moves to continuous feeding, the dimension of the problem will increase and the need for intelligent optimisation methods will become even more pressing.

## 5. Summary

The development of pig growth models, together with the availability of nonlinear optimisation tools on high speed computers, has made it possible to extend the traditional use of optimisation in pig nutrition. Linear programming to determine least cost diets is now a component of a larger optimisation routine, where, for example, the challenge of diet formulation to maximise gross return per pig place per year can be answered. Optimisation methods show considerable promise for increasing the efficiency of the worldwide industry of pig feeding.

## Acknowledgments

Professor Paul Moughan is thanked for his encouragement and pertinent questions during the development of this material at Massey University. The contributions of Lindsay Alexander in providing programming advice and of Christopher Clark in developing relevant software are also much appreciated.

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http://www.springer.com/978-0-387-30408-3

Global Optimization
Scientific and Engineering Case Studies
Pintér, J.D. (Ed.)
2006, XXIII, 546 p., Hardcover
ISBN: 978-0-387-30408-3