

Prologue

You may think of the dialogue you are about to read, as I often did while writing it, as being between a “master” and a “pupil”—the master in his middle years, well-versed in mathematics and as devoted and passionate about his craft as any artist is about his art; the pupil on the threshold of adulthood, articulate in speech, adventuresome of mind, and enthusiastically receptive to any knowledge the more learned teacher may care to impart.

Their conversation—the exact circumstances of which are never described—is initiated by the master, one of whose tasks is to persuade his disciple that the concept of number is more subtle than might first be imagined. Their mathematical journey starts with the teacher guiding the student, by way of questions and answers, through a beautifully simple geometrical demonstration (believed to have originated in ancient India), which establishes the existence of a certain number, the understanding of whose nature is destined to form a major part of the subsequent discussion between the enquiring duo.

Strong as the master’s motivation is to have the younger person glimpse a little of the wonder of mathematics, stronger still is his desire to see that his protégé gradually becomes more and more adept at mathematical reasoning so that he may experience the pure pleasure to be had from simply “finding things out” for himself. This joy of discovery is soon felt by the young learner, who having embarked upon an exploration, is richly rewarded when, after some effort, he chances upon a sequence of numbers that he surmises is inextricably linked to the mysterious number lately revealed by the master. Enthralled by this fortunate occurrence, he immediately finds himself in the grip of a burning curiosity to know more about this number and its connection with the sequence that has already captivated him. Thus begins this tale told over five chapters.

I have made every effort to have the first four chapters as self-contained as possible. The use of mathematical notation is avoided

whenever words can achieve the same purpose, albeit in a more lengthy manner. When mathematical notation is used, nothing beyond high school algebra of the simplest kind is called on, but in ways that show clearly the need for this branch of mathematics. While the algebra used is simple, it is often clever, revealing that a few tools handled with skill can achieve a great deal. If readers were to appreciate nothing more than this aspect of algebra—its power to prove things in general—then this work will not have been in vain.

Unfortunately, to have the fifth chapter completely self-contained would have meant sacrificing exciting material, something I didn't wish to do, preferring to reward the reader for the effort taken to reach this point, when it is hoped he will understand enough to appreciate the substance of what is being related.

Throughout the dialogue, so as to distinguish between the two speakers, the following typographical conventions are used:

The Master's Voice—assured, but gently persuasive—is set in this mildly bold typeface, and is firmly fixed at the left edge of the column.

The Pupil's Voice—deferential, but eager and inquiring—is set in this lighter font, and is moved slightly inward from the margin.

The best conversations between teachers and students are both serious and playful, and my hope is that the readers of this book will sense that something of that spirit, of real learning coupled with real pleasure, coexist in this dialogue.

David Flannery
September, 2005

The Square Root of 2

A Dialogue Concerning a Number and a Sequence

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2006, XII, 260 p. 31 illus., Hardcover

ISBN: 978-0-387-20220-4