"Though what I'm saying is perhaps not new, I have felt it quite vividly on this new occasion."

J. W. Goethe, in a letter from Naples, 17 May 1787

Foreword

The present textbook is my best effort to write a lively, problem-oriented and understandable introduction to classical modern algebra. Besides careful exposition, my goals were to lead the reader right away to interesting subject matter and to assume no more background than that provided by a first course in linear algebra.

In keeping with these goals, the exposition is by and large geared toward certain motivating problems; relevant conceptual tools are introduced gradually as needed. This way of doing things seems more likely to hold the reader's attention than a more or less systematic stringing together of theorems and proofs. The pace is more leisurely and gentle in the beginning, later faster and less cautious, so the book lends itself to self-study.

This first volume, primarily about fields and Galois theory, in order to deal with the latter introduces just the necessary amount of group theory. It also covers basic applications to number theory, ring extensions and algebraic geometry. I have found it advantageous for various reasons to bring into play early on the notion of the algebraic closure of a field. Naturally, Galois' beautiful results on solvable groups of prime degree could not be left out, nor could Dedekind's Galois-theoretical arithmetic reduction principle. Infinite Galois extensions are not neglected either. Finally, it seemed appropriate to include the fundamentals of transcendental extensions.

At the end of the volume there is a collection of exercises, interspersed with remarks that enrich the text. The problems chosen are of widely varying degrees of difficulty, but very many of them are accompanied by hints — sometimes amounting to an outline of the solution — and in any case there are no outright riddles. These exercises are of course meant to allow readers to practice their grasp of the material, but they serve another important purpose as well: precisely because the main text was kept short and to the point, without lots of side-results, the appendix will give the reader a better idea of the wealth of consequences and applications derived from the theory.

The linear algebra facts used, when not totally elementary, are accompanied by references to my *Lineare Algebra*, now published by Spektrum Akademischer Verlag and abbreviated LA I and LA II. This has not been translated, but equivalent spots in other linear algebra textbooks are not hard to find. Theorems and lesser results are numbered within each chapter in sequence, the latter being marked F1, F2, . . . — the F is inherited from the German word Feststellung. Allusions to historical matters are made only infrequently (but certainly not at random). When a theorem or other result bears the name of a mathematician, this is sometimes a matter of tradition more than of accurate historical origination.

The first German edition of this book appeared in 1987. I thank my colleagues who, already back at the writing stage, favored it with their interest and gave me encouragement — none more than the late H.-J. Nastold, with whom I had many fruitful conversations, W. Lütkebohmert, who once remarked that there was no suitable textbook for the German Algebra I course, O. Willhöft, who suggested several good problems, and H. Schulze-Relau and H. Epkenhans, whose critical perusal of large portions of the manuscript was a great help. The second (1991) and third (1995) editions benefited from the remarks of numerous readers, to whom I am likewise thankful, in particular R. Alfes, H. Coers, H. Daldrop and R. Schopohl. The response and comments on the part of students were also highly motivating. Special thanks are due to the publisher BI-Wissenschaftsverlag (later acquired by Spektrum) and its editor H. Engesser, who got me going in the first place.

The publication of this English version gives me great pleasure. I'm grateful to Springer-Verlag New York and its mathematics editor Mark Spencer, for their support and competent handling of the project. And not least for seeing to it that the translation be done by Silvio Levy: I have observed the progress of his task with increasing appreciation and have incorporated many of the changes he suggested, in a process of collaboration that led to noticeable improvements. Further perfecting is of course possible, and readers' suggestions and criticism will continue to be welcome and relevant for future reprints.

Münster, July 2005 Falko Lorenz

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Lorenz, F.
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