Preface

Fields are sets in which all four of the rational operations, memorably described by the mathematician Lewis Carroll as "perdition, distraction, uglification and derision", can be carried out. They are assuredly the most natural of algebraic objects, since most of mathematics takes place in one field or another, usually the rational field $\mathbb{Q}$, or the real field $\mathbb{R}$, or the complex field $\mathbb{C}$. This book sets out to exhibit the ways in which a systematic study of fields, while interesting in its own right, also throws light on several aspects of classical mathematics, notably on ancient geometrical problems such as "squaring the circle", and on the solution of polynomial equations.

The treatment is unashamedly unhistorical. When Galois and Abel demonstrated that a solution by radicals of a quintic equation is not possible, they dealt with permutations of roots. From sets of permutations closed under composition came the idea of a permutation group, and only later the idea of an abstract group. In solving a long-standing problem of classical algebra they laid the foundations of modern abstract algebra. It is surely reasonable now to suppose that anyone setting out to study Galois Theory will have a significant experience of the language and concepts of abstract algebra, and assuredly one can use this language to present the arguments more coherently and concisely than was possible for Galois (who described his own manuscript as *ce gâchis*[1]!) I hope that I have done so, but the arguments in Chapters 7 and 8 still require concentration and dertermination on the part of the reader.

Again, on this same assumption (that my readers have had some exposure to abstract algebra), I have chosen in Chapter 2 to examine the properties and interconnections of euclidean domains, principal ideal domains and unique factorisation domains in abstract terms before applying them to the crucial ring of polynomials over a field.

[1] this mess

All too often mathematics is presented in such a way as to suggest that it was engraved in pre-history on tablets of stone. The footnotes with the names and dates of the mathematicians who created this area of algebra are intended to emphasise that mathematics was and is created by real people. Foremost among the people whose work features in this book are two heroic and tragic figures. The first, a Norwegian, is Niels Henrik Abel, who died of tuberculosis at the age of 26; the other, from France, is Evariste Galois, who was killed in a duel at the age of 20. Information on all these people and their achievements can be found on the St Andrews website `www-history.mcs.st-and.ac.uk/history/`.

The book contains many worked examples, as well as more than 100 exercises, for which solutions are provided at the end of the book.

It is now several years since I retired from the University of St Andrews, and I am most grateful to the University, and especially to the School of Mathematics and Statistics, for their generosity in continuing to give me access to a desk and a computer. Special thanks are due to Peter Lindsay, whose answers to stupid questions on computer technology were unfailingly helpful and polite. I am grateful also to my colleague Sophie Huczynka and to Fiona Brunk, a final year undergraduate, for drawing attention to mistakes and imperfections in a draft version. The responsibility for any inaccuracies that remain is mine alone.

John M. Howie
University of St Andrews
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