Preface

When constructing a mathematical model for a problem in natural science, one often needs to combine methods from different fields of mathematics. Stochastic differential equations, for instance, are used to model the effect of noise on a system evolving in time. Being at the same time differential equations and random, their description naturally involves methods from the theory of dynamical systems, and from stochastic analysis.

Much progress has been made in recent years in the quantitative description of the solutions to stochastic differential equations. Still, it seems to us that communication between the various communities interested in these models (mathematicians in probability theory and analysis, physicists, biologists, climatologists, and so on) is not always optimal. This relates to the fact that researchers in each field have developed their own approaches, and their own jargon. A climatologist, trying to use a book on stochastic analysis, in the hope of understanding the possible effects of noise on the box model she is studying, often experiences similar difficulties as a probabilist, trying to find out what his latest theorem on martingales might imply for action-potential generation in neurons.

The purpose of this book is twofold. On the one hand, it presents a particular approach to the study of stochastic differential equations with two timescales, based on a characterisation of typical sample paths. This approach, which combines ideas from singular perturbation theory with probabilistic methods, is developed in Chapters 3 and 5 in a mathematically rigorous way. On the other hand, Chapters 4 and 6 attempt to bridge the gap between abstract mathematics and concrete applications, by illustrating how the method works, in a few specific examples taken from physics, biology, and climatology. The choice of applications is somewhat arbitrary, and was mainly influenced by the field of speciality of the modellers we happened to meet in the course of the last years.

The present book grew out of a series of articles on the effect of noise on dynamical bifurcations. In the process of writing, however, we realised that many results could be improved, generalised, and presented in a more concise way. Also, the bibliographical research for the chapters on applications revealed many new interesting problems, that we were tempted to discuss as well. Anyone who has written a book knows that there is no upper bound on the amount of time and work one can put into the writing. For the sake of timely publishing, however, one has to stop somewhere, even though the feeling remains to leave something unfinished. The reader will thus find many open problems, and our hope is that they will stimulate future research.

We dedicate this book to Erwin Bolthausen and Hervé Kunz on the occasion of their sixtieth birthdays. As our thesis advisors in Zürich and Lausanne, respectively, they left us with a lasting impression of their way to use mathematics to solve problems from physics in an elegant way. Our special thanks go to Anton Bovier, who supported and encouraged our collaboration from the very beginning. Without his sharing our enthusiasm and his insightful remarks, our research would not have led us that far.

Many more people have provided help by giving advice, by clarifying subtleties of models used in applications, by serving on our habilitation committees, or simply by showing their interest in our work. We are grateful to Ludwig Arnold, Gérard Ben Arous, Jean-Marie Barbaroux, Dirk Blömker, Fritz Colonius, Predrag Cvitanović, Jean-Dominique Deuschel, Werner Ebeling, Bastien Fernandez, Jean-François Le Gall, Jean-Michel Ghez, Maria Teresa Giraudo, Peter Hänggi, Peter Imkeller, Alain Joye, Till Kuhlbrodt, Robert Maier, Adam Monahan, Khashayar Pakdaman, Etienne Pardoux, Cecile Penland, Pierre Picco, Arkady Pikovsky, Claude-Alain Pillet, Michael Rosenblum, Laura Sacerdote, Michael Scheutzow, Lutz Schimansky-Geier, Igor Sokolov, Dan Stein, Alain-Sol Sznitman, Peter Talkner, Larry Thomas, and Michael Zaks.

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