Preface

Since the last century, the postulational method and an abstract point of view have played a vital role in the development of modern mathematics. The experience gained from the earlier concrete studies of analysis point to the importance of passage to the limit. The basis of this operation is the notion of distance between any two points of the line or the complex plane. The algebraic properties of underlying sets often play no role in the development of analysis; this situation naturally leads to the study of metric spaces. The abstraction not only simplifies and elucidates mathematical ideas that recur in different guises, but also helps economize the intellectual effort involved in learning them. However, such an abstract approach is likely to overlook the special features of particular mathematical developments, especially those not taken into account while forming the larger picture. Hence, the study of particular mathematical developments is hard to overemphasize.

The language in which a large body of ideas and results of functional analysis are expressed is that of *metric spaces*. The books on functional analysis seem to go over the preliminaries of this topic far too quickly. The present authors attempt to provide a leisurely approach to the theory of metric spaces. In order to ensure that the ideas take root gradually but firmly, a large number of examples and counterexamples follow each definition. Also included are several worked examples and exercises. Applications of the theory are spread out over the entire book.

The book treats material concerning metric spaces that is crucial for any advanced level course in analysis. Chapter 0 is devoted to a review and systematisation of properties which we shall generalize or use later in the book. It includes the Cantor construction of real numbers. In Chapter 1, we introduce the basic ideas of metric spaces and Cauchy sequences and discuss the completion of a metric space. The topology of metric spaces, Baire's category theorem and its applications, including the existence of a continuous, nowhere differentiable function and an explicit example of such a function, are discussed in Chapter 2. Continuous mappings, uniform convergence of sequences and series of functions, the contraction mapping principle and applications are discussed in Chapter 3. The concepts of connected, locally connected and arcwise connected spaces are explained in Chapter 4. The characterizations of connected subsets of the reals and arcwise connected subsets of the plane are also in Chapter 4. The notion of compactness, together with its equivalent characterisations, is included in Chapter 5. Also contained in this chapter are characterisations of compact subsets of special metric spaces. In Chapter 6, we discuss product metric spaces and provide a proof of Tychonoff's theorem.

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