5

Problem formulation

This chapter formulates the problem of the design of an advisory system and outlines its conceptual solution. The interconnection of the managed system with its operator is modelled by the finite mixture model. Its optional parts are optimized so that the resulting mixture model is as close as possible to management aims expressed by a user's ideal pdf in a fully probabilistic sense, cf. 2.4.2. The resulting mixture is offered to the operator as the recommended ideal pdf to be followed. Unlike the user's ideal pdf, the state described by the recommended ideal pdf can be practically achieved.

The application of this conceptually simple design requires formulation and solution of a sequence of particular technical steps. Their description forms the content of this chapter.

Design conditions and adopted design principles are clarified in Section 5.1. Special attention is paid to relationships of data spaces accessible to the operator and to the advisory system. Also, necessary reductions and extensions of involved data spaces are proposed there. The learning conditions we assume are specified in Section 5.2. Dynamic predictions, forming a bridge between learning and advising parts, are discussed in Section 5.3. The design of advices offered to the operator is described in Section 5.4. Basic types of advisory systems differing in the extent of optional ingredients of the modified model are classified there. The design of the presentation part of the advisory system is in Section 5.5. Learning and design steps, which form the backbone of detailed solutions presented in Chapters 6 and 7, are summarized in Section 5.6.2.

5.1 Design principle and design conditions

This section introduces notions needed for formalization and solution of the addressed design problem.

5.1.1 Systems and data spaces

We start with inspection of relationships of the managed system, its operator, and the constructed advisory system. The following agreement introduces fixes notions.

Agreement 5.1 (Nomenclature of systems) *The system managed by the operator is called the* m-system*. The closed loop formed by the m-system and its operator (supervisor) is called the* o-system*. The* advisory system *transforming the data measured on the o-system into advices to the operator is called the* p-system*; see Fig. 5.1.*

If need be, the quantities related to m-, o-, and p-systems are distinguished by prefixes m-, o-, and p- or by subscripts m, o, and p.

Out of this chapter, we deal mostly with the data handled by the p-system. Then, their subscript p is dropped if there is no danger of misunderstanding.

Formulation and solution of the overall design need clarification of relationships among quantities dealt with by the o- and p-systems.

Agreement 5.2 (Data, observation, and action spaces) *Values of the quantities available to the operator form the* data space of the operator $d_o^*(\mathring{t})$*. This space is the Cartesian product of the* observation space of the operator $\Delta_o^*(\mathring{t})$ *— formed by the innovations* $\Delta_o(\mathring{t})$ *available to the operator; see Section 2.2 — and of the* action space of the operator $a_o^*(\mathring{t})$*, i.e.,* $d_o^*(\mathring{t}) \equiv (\Delta_o^*(\mathring{t}), a_o^*(\mathring{t}))$*.*

Values of the quantities available to the advisory system form the data space of the advisory system $d_p^*(\mathring{t})$*. This space is the Cartesian product of the* observation space of the advisory system $\Delta_p^*(\mathring{t})$ *and of the* action space of the advisory system $a_p^*(\mathring{t})$*, i.e.,* $d_p^*(\mathring{t}) \equiv (\Delta_p^*(\mathring{t}), a_p^*(\mathring{t}))$*.*

Values of the quantities available to the operator but unavailable to the advisory system form the surplus data space of the operator $d_{o+}^*(\mathring{t}) \equiv d_p^*(\mathring{t}) \cup d_o^*(\mathring{t}) \setminus d_p^*(\mathring{t})$*.*

Values of the quantities available to the advisory system but unavailable to the operator form the surplus data space of the advisory system $d_{p+}^*(\mathring{t}) \equiv d_p^*(\mathring{t}) \cup d_o^*(\mathring{t}) \setminus d_o^*(\mathring{t})$*; see Fig. 5.1.*

Note that the possible nonemptiness of the surplus data spaces singles out the addressed problem from the standard formulation of decision-making. Obviously, the design of the advisory system is meaningful only when the p- and o-data spaces overlap.

Requirement 5.1 (Overlap of data spaces) *The data spaces available to the operator* $d_o^*(\mathring{t})$ *and to the advisory system* $d_p^*(\mathring{t})$ *have a nonempty intersection* $d_{op}^*(\mathring{t}) \neq \emptyset$*. Thus,*

$$\begin{aligned} d_o^*(\mathring{t}) &= d_{op}^*(\mathring{t}) \cup d_{o+}^*(\mathring{t}), \quad d_{op}^*(\mathring{t}) \cap d_{o+}^*(\mathring{t}) = \emptyset, \\ d_p^*(\mathring{t}) &= d_{op}^*(\mathring{t}) \cup d_{p+}^*(\mathring{t}), \quad d_{op}^*(\mathring{t}) \cap d_{p+}^*(\mathring{t}) = \emptyset, \quad \textit{where} \end{aligned} \tag{5.1}$$

$d_{o+}^*(\mathring{t})$ *is the surplus data space of the operator and* $d_{p+}^*(\mathring{t})$ *is the surplus data space of the advisory system.*

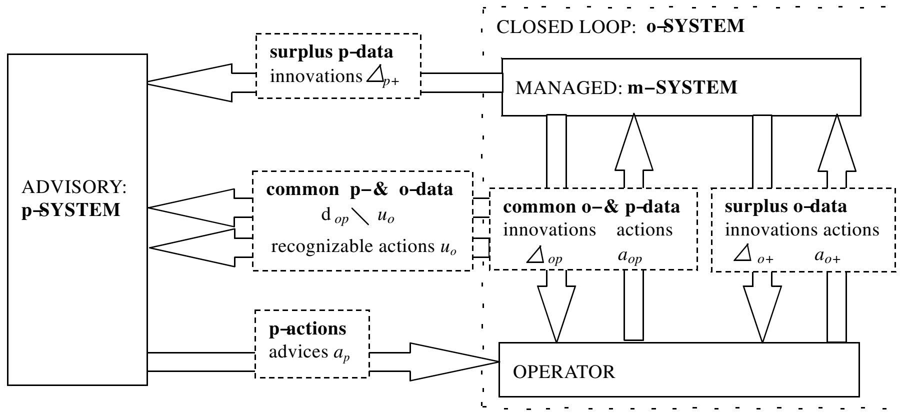

Fig. 5.1. Relationships of systems and data spaces: Agreements 5.1, 5.2, 5.7.

5.1.2 Basic scenario and design principle

The following basic scenario is considered within the time span determined by a *horizon* $\mathring{t} \leq \infty$.

The operator handling the m-system deals with a sequence $d_o(\mathring{t})$ of the o-data. A nonempty part of the data record $d_{o;t}$ is formed by operator's actions $a_{o;t} \in a_o^*$. The rest consists of innovations $\Delta_{o;t}$, i.e.,

$$d_{o;t} = (\Delta_{o;t}, a_{o;t}) \equiv \text{(o-innovations, o-actions)}.$$

Formally, the operator implements the causal strategy

$$\left\{d_o^*(t-1) \to a_{o;t}^*\right\}_{t \in t^*}.$$

The strategy of the operator is to be judged according to the expected value $\mathcal{E}$ of a loss function

$$\mathcal{Z} : \, d_o^*(\mathring{t}) \to [0, \infty] \tag{5.2}$$

that reflects the management aims. The real operator's strategy is usually chosen informally, but the joint pdf $f(d_o(\mathring{t}))$ would be the adequate description of the o-system needed for the formal evaluation of the operator's strategy; see Chapter 2.

The p-system works on a sequence of the p-data $d_p(\mathring{t})$. Each record $d_{p;t}$ contains data items $d_{pi;t}, \; i \in i^* \equiv \{1, \ldots, \mathring{d}_p\}$ with either real or discrete values. The record $d_{p;t} \equiv (\Delta_{p;t}, a_{p;t})$ includes p-innovations $\Delta_{p;t}$, observed by the p-system on the o-system and actions, of the p-system $a_{p;t}$, called *advices*.

The p-system implements its causal strategy $\left\{d_p^*(0), d_p^*(t-1) \to a_{p;t}^*\right\}_{t \in t^*}$, where $d_p(0)$ denotes experience collected before using the p-system.

The p-system strategy is designed so that the o-system accepting advices achieves the smallest expected loss (5.2).

Agreement 5.3 (Guided and unguided o-system) *The interconnection of the o-system with the p-system creates a new* guided o-system*. We shall use also the mirror term* unguided o-system *for the o-system working without a p-system.*

The adjectives guided and unguided *are also used in connection with the corresponding models, behaviors, situations, etc. For instance,* unguided model *and* guided model *mean the model of the unguided and guided o-system, respectively.*

The complete outer description of the guided o-system is given by the joint pdf $f(d_p(\mathring{t}), d_{o+}(\mathring{t}))$ of all involved data $(d_p(\mathring{t}), d_{o+}(\mathring{t}))$, cf. (5.1). The structure of the discussed interconnection is predominantly determined by the communication ways of the managed system, the operator and the advisory system. The m-system

- provides its o-innovations to the operator as responses to the o-actions,
- offers some data to the p-system that generally differ from $d_{o;t}$,
- is indirectly influenced by the p-system through the o-actions that are stimulated by advices, i.e., by actions of the p-system.

We would like to design the p-system that guides the operator to make the expectation of the loss function (5.2) as small as possible. The advisory system cannot force the operator to follow its advices. The p-system can only present a “target” to be reached. The advices will be useful if the operator is able to respect them, i.e., if the advised target is reachable by the cooperating operator. Both the design of the p-system and presentation of advices can be done in a systematic way if we adopt the fully probabilistic design; see Section 2.4.2. It determines the way we intend to stimulate the operator.

The *ideal pdf* in the sense of the fully probabilistic design, Agreement 2.7, is adopted as the optimized target offered to the operator.

The corresponding conceptual algorithm implementing this *design principle* looks as follows.

Algorithm 5.1 (Design principle of the advisory system)

1. *Express managing aims as the* user’s ideal pdf $^{\lfloor U}f\left(d_p(\mathring{t}), d_{o+}(\mathring{t})\right)$.
2. *Estimate an outer* multimodal *model* $f(d_p(\mathring{t}), d_{o+}(\mathring{t}))$ *describing relationships among the considered data* $(d_p(\mathring{t}), d_{o+}(\mathring{t}))$.
3. *Create the ideal pdf* $^{\lfloor I}f\left(d_p(\mathring{t}), d_{o+}(\mathring{t})\right)$ *that*
   - *is as close as possible to the user’s ideal pdf* $^{\lfloor U}f\left(d_p(\mathring{t}), d_{o+}(\mathring{t})\right)$*; the KL divergence is used as the proximity measure,*
   - *inherits those constituents of the pdf* $f\left(d_p(\mathring{t}), d_{o+}(\mathring{t})\right)$ *describing an unguided o-system that cannot be changed even by the* fully cooperating operator *that implements the* randomized *strategy recommended to him.*

The left superscript ^{I} marks the resulting ideal *pdf and its optimized constituents.*

4. *Present low-dimensional projections of the ideal pdf* ${}^{\lfloor I}f\left(d_p(\mathring{t}), d_{o+}(\mathring{t})\right)$ *as the "target" to be followed by the operator.*

Remark(s) 5.1

1. *The applicability of Algorithm 5.1 depends on the possibility of creating the involved elements practically. This nontrivial task is solved gradually in subsequent sections.*
2. *The focus on a fully probabilistic design allows us to deal with a uniform probabilistic description of learning, design, and advising.*
3. *The optimization with properly preserved elements of the m-system guarantees that a good past practice can be followed. The ideal pdf, constructed in the outlined way,*
   - *is reachable,*
   - *relates all observed consequences to their observed causes unlike human being can.*

5.1.3 Reduction of surplus data of the operator

Algorithm 5.1 deals formally with all data occurring in the guided system, i.e., with the union of p- and o-data. This subsection shows that, in the design of the p-system, we need not model the surplus o-data $d_{o+}(\mathring{t})$.

The p-system has no information on $d^*_{o+}(\mathring{t})$. Thus, it has to leave this part of the o-behavior to its fate. In other words, the strategy of the p-system, designed without considering the surplus o-data, implies that their distribution has to be accepted as the ideal one. Formally, it restricts the constructed ideal pdf ${}^{\lfloor I}f(\cdot)$ by the requirement

$$ {}^{\lfloor I}f\left(d_{o+}(\mathring{t})|d_p(\mathring{t})\right) \equiv f\left(d_{o+}(\mathring{t})|d_p(\mathring{t})\right). \tag{5.3} $$

The objective pdf $f(d_{o+}(\mathring{t})|d_p(\mathring{t}))$ (see Chapter 2) describes the data $d_{o+}(\mathring{t})$ unavailable to the p-system.

According to the concept of the fully probabilistic design, the distance of the pdf describing the inspected behavior to its ideal pdf is measured through the KL divergence (2.25). In this context, the requirement (5.3) implies a simple but important consequence.

Proposition 5.1 (Ideal pdf offered by the advisory system) *Let the ideal pdf* ${}^{\lfloor I}f(d_p(\mathring{t}), d_{o+}(\mathring{t}))$ *offered by the advisory system meet the requirement (5.3). Then,*

$$ \mathcal{D}\left(f \middle|\middle| {}^{\lfloor I}f\right) \equiv \int f(d_p(\mathring{t}), d_{o+}(\mathring{t})) \ln\left(\frac{f(d_p(\mathring{t}), d_{o+}(\mathring{t}))}{{}^{\lfloor I}f(d_p(\mathring{t}), d_{o+}(\mathring{t}))}\right) d(d_p(\mathring{t}), d_{o+}(\mathring{t})) $$

$$= \int f(d_p(\mathring{t})) \ln\left(\frac{f(d_p(\mathring{t}))}{{}^{\lfloor I}f(d_p(\mathring{t}))}\right) dd_p(\mathring{t}) \quad \textit{and} \tag{5.4}$$

$$\mathcal{D}\left({}^{\lfloor I}f \middle|\middle| f\right) \equiv \int {}^{\lfloor I}f(d_p(\mathring{t})) \ln\left(\frac{{}^{\lfloor I}f(d_0p(\mathring{t}))}{f(d_p(\mathring{t}))}\right) dd_p(\mathring{t}).$$

Thus, for our design purposes, the outer description of the o-system $f(\cdot)$ and the optimized ideal pdf ${}^{\lfloor I}f(\cdot)$ have to be specified on $d_p^*(\mathring{t})$ only.

Proof. Using the basic rules for pdfs — the chain rule and normalization; see Proposition 2.4 — we can directly verify the following identities

$$\mathcal{D}\left(f \middle|\middle| {}^{\lfloor I}f\right) \equiv \int f(d_p(\mathring{t}), d_{o+}(\mathring{t})) \ln\left(\frac{f(d_p(\mathring{t}), d_{o+}(\mathring{t}))}{{}^{\lfloor I}f(d_p(\mathring{t}), d_{o+}(\mathring{t}))}\right) d(d_p(\mathring{t}), d_{o+}(\mathring{t}))$$

$$\underbrace{=}_{\text{chain rule}} \int f(d_{o+}(\mathring{t})|d_p(\mathring{t}))f(d_p(\mathring{t}))$$

$$\times \quad \ln\left(\frac{f(d_{o+}(\mathring{t})|d_p(\mathring{t}))f(d_p(\mathring{t}))}{{}^{\lfloor I}f(d_{o+}(\mathring{t})|d_p(\mathring{t}))\ {}^{\lfloor I}f(d_p(\mathring{t}))}\right) d(d_p(\mathring{t}), d_{o+}(\mathring{t}))$$

$$\underbrace{=}_{(5.3)} \int f(d_{o+}(\mathring{t})|d_p(\mathring{t}))f(d_p(\mathring{t})) \ln\left(\frac{f(d_p(\mathring{t}))}{{}^{\lfloor I}f(d_p(\mathring{t}))}\right) d(d_p(\mathring{t}), d_{o+}(\mathring{t}))$$

$$\underbrace{=}_{\text{normalization, Proposition 2.4}} \int f(d_p(\mathring{t})) \ln\left(\frac{f(d_p(\mathring{t}))}{{}^{\lfloor I}f(d_p(\mathring{t}))}\right) dd_p(\mathring{t}).$$

The equality for $\mathcal{D}\left({}^{\lfloor I}f||f\right)$ can be proved in the same way. □

Consequently, the accepted condition (5.3) allows us to leave the surplus data space of the operator $d_{o+}^*(\mathring{t})$ completely out of our consideration. Thus, under (5.3), the design results obtained for empty and nonempty $d_{o+}^*(\mathring{t})$ are the same. Notation is, however, simpler if $d_{o+}^*(\mathring{t}) = \emptyset$. In this case, the definition (5.1) implies

$$d_o^*(\mathring{t}) = d_{op}^*(\mathring{t}) \ \Rightarrow d_o^*(\mathring{t}) \subset d_p^*(\mathring{t}). \tag{5.5}$$

From now on, <u>we adopt this formal simplification that the surplus data space of the operator $d_{o+}^*(\mathring{t})$ is empty</u> and thus the data space of the operator $d_o^*(\mathring{t})$ is a subset of the data space of the advisory system $d_p^*(\mathring{t})$.

5.1.4 Construction of a true user's ideal pdf

The considered design of an optimal advisory system assumes that the aims of management and advising can be expressed by the user's ideal pdf describing the desired behavior of the <u>p-data</u> $d_p(\mathring{t})$.

The user expresses its aims in terms of quantitative indicators, called *quality markers*, that can be evaluated using the <u>o-data</u> only. The quality markers $m(\mathring{t}) \in m^*(\mathring{t})$ qualify behavior of the o-system through a known function

$$d_o^*(\mathring{t}) \to m^*(\mathring{t}) \equiv \text{(partially) ordered space.} \tag{5.6}$$

Desired properties of quality markers, expressing management aims, are assumed to be "translated" into the *true user's ideal pdf* ${}^{\lfloor U}f(d_o(\mathring{t}))$ that characterizes desired distribution of o-data available to the operator. The following "translation" methods are at our disposal.

- A simple, typically normal, pdf is chosen. Set point for $d_{o;t}$ is defined as its mean. Covariance matrix is chosen so that the significant probability is allocated to the multivariate interval covering desirable ranges of d_o-entries.
- A simple, typically normal, model is estimated on historical records $d_o(\mathring{t}_o)$ and its mean is replaced by the desired set-point. The covariance is shrunk so that a desirable improvement is enforced.

Remark(s) 5.2

1. *A Monte Carlo-based translation, as elaborated in connection with DESIGNER project [126], can be also used.*
2. *Within this book, the true user's ideal pdf is taken as unimodal pdf. Recently, it was found that the adopted fully probabilistic design can be practically applied even when the user's ideal pdf is a finite mixture [129, 130]. This extends applicability of the fully probabilistic design on decision-making problems with multiple criteria!*

Problem 5.1 (Advising on internal quantities) *Generally, the user cares about inner, directly unobservable states, too. The operator has to guess them using observed data and also optimize behavior of these estimates. Finally, the observed-data dependent loss function is optimized. For simplicity, the advising is formulated directly as the data-driven design, cf. Agreement 2.8. The relevant theory should, however, be extended explicitly to the case when the operator deals with internal, considered but directly unobservable states, too.*

5.1.5 Extension of a true user's ideal pdf to the surplus p-data

The adopted condition (5.5) implies that an implementation of conceptual Algorithm 5.1 requires specification of a user's ideal pdf ${}^{\lfloor U}f(\cdot)$ on the data space of the advisory system $d_p^*(\mathring{t})$. The true user's ideal pdf is, however, defined "naturally" on the data space of the operator $d_o^*(\mathring{t}) \equiv d_{op}^*(\mathring{t})$; see (5.5) and Section 5.1.4. At the same time, the fact that the advisory system deals with a wider data set than the operator should be exploited for reaching a better quality of advices. Thus, it is necessary to extend the user's ideal pdf from $d_o^*(\mathring{t})$ to $d_p^*(\mathring{t})$.

By definition, the operator is not aware and consequently interested in the surplus data of the advisory system $d_{p+}(\mathring{t})$. Thus, the operator has to leave this surplus data to the fate determined by the managed system and the influence of the advisory system. It leads us to the choice

$$\begin{aligned}
{}^{\lfloor U}f(d_p(\mathring{t})) &\underbrace{=}_{\text{chain rule}} {}^{\lfloor U}f(d_{p;\mathring{t}}|d_p(\mathring{t}-1))\, {}^{\lfloor U}f(d_p(\mathring{t}-1)) \underbrace{=}_{(5.1),(5.5)} \\
&= {}^{\lfloor U}f(d_{o;\mathring{t}}|d_{p+;\mathring{t}}, d_p(\mathring{t}-1))\, {}^{\lfloor U}f(d_{p+;\mathring{t}}|d_p(\mathring{t}-1))\, {}^{\lfloor U}f(d_p(\mathring{t}-1)) \\
&\equiv {}^{\lfloor U}f(d_{o;\mathring{t}}|d_{p+;\mathring{t}}, d_p(\mathring{t}-1))\, {}^{\lfloor I}f(d_{p+;\mathring{t}}|d_p(\mathring{t}-1))\, {}^{\lfloor U}f(d_p(\mathring{t}-1)) \\
&\equiv {}^{\lfloor U}f(d_{o;\mathring{t}}|d_o(\mathring{t}-1))\, {}^{\lfloor I}f(d_{p+;\mathring{t}}|d_p(\mathring{t}-1))\, {}^{\lfloor U}f(d_p(\mathring{t}-1)) \\
&= \prod_{t \in t^*} {}^{\lfloor U}f(d_{o;t}|d_o(t-1))\, {}^{\lfloor I}f(d_{p+;t}|d_p(t-1)). \qquad (5.7)
\end{aligned}$$

The first equivalence in the third row means that $d^*_{p+;\mathring{t}}$ is left to its fate: the operator leaves the p-system to identify this part of the user's ideal pdf with the option made by the p-system. The second equivalence in the third row says that the user's ideal pdf on $d^*_{o;\mathring{t}}$ is constructed irrespective of the inaccessible values in $d^*_{p+}(\mathring{t})$. The final identity in (5.7) is adopted as a basic assumption.

Remark(s) 5.3

1. *This subsection completes construction of the user's ideal pdf on the union of data spaces needed in the first step of Algorithm 5.1. Step 2 is discussed in Section 5.2 and elaborated in Chapters 6, 8, and 10. Steps 3 and 4 are covered by Sections 5.4 and 5.5 and elaborated in Chapters 7, 9, and 11.*
2. *Operator may be aware of but uninterested in some entries of $d_{o;t}$. Then, they can be formally thought as entries in the surplus data space of an operator.*
3. *The designer may use the entries of p-data as tuning knobs in design. This option is indeed exploited when we try to create a user-friendly advisory system that controls the burden on the operator. For instance, the frequency of changes in advising is kept low.*

5.2 Learning conditions

The design of the advisory system relies on a good mixture model of the o-system. This sections discusses necessary modelling and learning conditions we suppose to be met.

Before progressing in a formal way, it is worth stressing the following facts.

The need for any advisory system arises when there are good and bad modes in managing the considered m-system.

The chance to design a good advisory system exists if both good and bad modes are reflected in its data space d^*_p. It implies that the pdf $f(d_p(\mathring{t}))$ describing $d^*_p(\mathring{t})$ completely is expected to have <u>multiple modes</u>.

The possibility to design an efficient advisory system arises if the available design experience contains well-pronounced information about all significant operation modes that may occur while the operator handles the managed system. In other words, the data providing this experience have to be rich and informative.

The top position of the operator in the control hierarchy implies (see Chapter 1) that the experience of the advisory system is to be predominantly data-based.

The rate of the operator's actions is often (much) slower than the sampling rate of the sensors and controllers within the m-system. Consequently, all considered data can be and should be grouped or reduced to a sequence generated with the rate of the operator's actions. We suppose it done.

The experience of the p-system also includes surplus data of the advisory system $d^*_{p+}(\mathring{t}) \equiv d^*_p(\mathring{t}) \setminus d^*_o(\mathring{t})$. They are directly at the disposal of the advisory system but not to the operator. These data may be invaluable in discovering various operating modes. They have to be considered in the design of any efficient p-system. Consequently, we have to deal with all data $d_p(\mathring{t})$ and thus we can mostly drop the subscript p further on and adopt the identity

$$d(\mathring{t}) \equiv d_p(\mathring{t}). \tag{5.8}$$

The general theory (see Chapter 2) and the above discussion imply that a *multiple-mode pdf* $f(d(\mathring{t})|d(0))$ is the description required for a design of the considered advisory system. The condition $d(0) \equiv d_p(0)$ stands for the experience available to the p-system before its use. Further on, $d(0)$ is fixed and formally included into $d(\mathring{t})$, Then, the notation can be simplified by "hiding" $d(0)$.

The pdf $f(d(\mathring{t}))$ can be factorized according to the chain rule $f(d(\mathring{t})) = \prod_{t \in t^*} f(d_t|d(t-1))$. The assumed low action rate of the operator implies that the faster dynamics of the system is diminished in the grouped or reduced data. Thus, slow transitions among quasi-steady-state working points are modelled. This can be well described by a *Markov model of a finite order*. Thus, we can assume that the unguided o-system is described by the joint pdf

$$f(d(\mathring{t})) = \prod_{t \in t^*} f(d_t|\phi_{t-1}), \text{where}$$

$\phi_t \in \phi^*,\ t \in t^*$, are known finite-dimensional vectors called *observable states*.

The intended *online use* of the advisory system implies that we have to deal with models containing ϕ_t that can be evaluated in a recursive manner, i.e., $\phi_t = \Phi(\phi_{t-1}, d_t)$ with a known function Φ.

The advisory system can be successful only if the relationships learned from experience are valid almost permanently. Thus, we have to assume that the outer model $\{f(d_t|\phi_{t-1})\}_{t \in t^*}$, Agreement 2.7, of the unguided o-system built on the p-data $d_t \equiv d_{p;t}$ is time invariant.

It is known that the needed multiple-mode probabilistic models can (almost) always be approximated by a *finite mixture* of unimodal models [49], called *components*. We estimate this model using — at least approximately — the standard Bayesian methodology.

For reference purposes, we summarize learning conditions adopted in the construction of the advisory system.

Requirement 5.2 (Learning conditions for the design)

1. *The data space* $d^*(\mathring{t})$ *of the advisory system (the p-system) has a nonempty intersection with the data space* $d_o^*(\mathring{t})$ *of the o-system.*
2. *The sampling rate is harmonized with the operating rate. The number* $\mathring{d}$ *of data items sent to the p-system at each time moment is fixed.*
3. *No attempt is made to influence quantities lying in the data space of the o-system and being out of the data space of the p-system.*
4. *The pdf on records* $d(\mathring{t})$*, available to the p-system, is modelled by the pdf* $f(d(\mathring{t})|\Theta) \equiv \prod_{t\in t^*} f(d_t|\phi_{t-1},\Theta)$*, where the pdf* $f(d_t|\phi_{t-1},\Theta)$*, determined by the measurable state* ϕ_{t-1} *and by the parameter* Θ*, is a time invariant finite mixture; see (5.9) below.*
5. *The approximate Bayesian learning described in Section 6.5 provides the pdf* $f(d(\mathring{t})) \equiv f(d(\mathring{t})|d(0)) = \prod_{t\in t^*} f(d_t|d(t-1))$ *needed for the design of the p-system.*

The finite mixture approximating distribution of data $d(\mathring{t})$ with multiple modes is assumed to have the form

$$f(d(\mathring{t})|\Theta) \equiv \prod_{t\in t^*} f(d_t|\phi_{t-1},\Theta) \text{ with } \textit{finite mixture}\text{s as parameterized models}$$

$$f(d_t|\phi_{t-1},\Theta) \equiv \sum_{c\in c^*} \alpha_c f(d_t|\phi_{c;t-1},\Theta_c,c),\ c^* = \{1,\ldots,\mathring{c}\},\ \mathring{c} < \infty, \tag{5.9}$$

$f(d_t|\phi_{c;t-1},\Theta_c,c)$is called *component* given by parameters Θ_c and the state

$\phi_{c;t} = \Phi_c(\phi_{c;t-1},d_t)$, i.e., the state $\phi_{c;t-1}$ can be recursively updated data d_t,

$\alpha_c \equiv$ the probabilistic *component weight*

$\Theta \equiv$ mixture parameter formed by component parameters and weights in

$$\Theta^* \equiv \left\{ \{\Theta_c \in \Theta_c^*\}_{c\in c^*},\ \alpha \equiv [\alpha_1,\ldots,\alpha_{\mathring{c}}] \in \alpha^* \equiv \left\{ \alpha_c \geq 0,\ \sum_{c\in c^*} \alpha_c = 1 \right\} \right\}.$$

The mixture parameter $\Theta = [\alpha_c,\Theta_c]_{c\in c^*}$ may also include the number of components $\mathring{c}$.

The entries of d_t can be permuted in each component and some permutations may lead to parameterizations with fewer parameters. It makes us include into the model description the permutations

$$d \to d_c \ \text{ with } \ d_{ic} = d_{j_{ic}}, \tag{5.10}$$

where j_{ic} is ith entry of the permuted indexes $[1,\ldots,\mathring{d}]$.

The assignment (5.10) is applied component-wise and together with the chain rule, Proposition 2.4, give

$$f(d_t|\phi_{c;t-1},\Theta_c,c) = \prod_{i\in i^*} f(d_{ic;t}|d_{(i+1)\cdots \mathring{d}c;t},\phi_{c;t-1},\Theta_{ic},c)$$
$$\equiv \prod_{i\in i^*} f(d_{ic;t}|\psi_{ic;t},\Theta_{ic},c). \tag{5.11}$$

The additional subscript i of the parameter Θ_{ic} indicates that only some entries of Θ_c may occur in the ith pdf predicting the ith scalar entry of $d_{ic;t}$. Similarly, the introduced *regression vector* $\psi_{ic;t}$ is generally a subvector of the vector

$$[d_{(i+1)c;t},\ldots,d_{\mathring{d}c;t},\phi'_{c;t-1}]'. \tag{5.12}$$

Now we can fix nomenclature related to the mixture.

Agreement 5.4 (Nomenclature related to mixtures)

Pdfs: The pdf $f(d_t|\phi_{c;t-1},\Theta_c,c)$ in (5.9) is called the parameterized component *of a mixture and α_c is the* weight of the cth parameterized component.

The pdf $f(d_{ic;t}|\psi_{ic;t},\Theta_{ic},c)$ in (5.11) is called the parameterized factor.

A parameterized factor occurring in several components is the common parameterized factor.

Factors are called the adjacent factors *if $d_{(i+1)c;t}$ is in the regression vector of the factor predicting $d_{ic;t}$ or vice versa.*

The predictive pdf $f(d_t|d(t-1),c)$ is called the component.

The predictive pdf $f(d_{ic;t}|d_{(i+1)\cdots\mathring{d}c;t},d(t-1),c)$ is called the factor.

The pdf $f(\Theta_{ic}|d(t))$ is called the factor estimate.

The pdf $f(\Theta_c|d(t))$ is called the component estimate.

The pdf $f(\alpha|d(t))$ is called the component-weight estimate.

The pdf $f(\Theta|d(t))$ is called the mixture estimate.

The estimate is called the prior estimate *if $t=0$.*

The estimates are called posterior estimates *if $t\in t^*$. Often, the posterior estimate is called the estimate only.*

Data: The vector d_t containing data measured at time t is called the data record.

The predicted scalar quantity $d_{ic;t}$ is called the output of the factor.

The entry $d_{i;t}$, $i=1,\ldots,\mathring{d}$, of the data record d_t is also called the ith data channel *or simply the ith* channel. *These terms are used in the implementation context.*

Data entries that never play the role of the output of a factor are called nonmodelled data.

The vector $\phi_{c;t-1}$, that can be updated recursively using the newest data d_t, is the observable state of the parameterized component.

The parameterized factor is determined by the regression vector $\psi_{ic;t}$ *formed by a subselection of entries from the vector $[d_{(i+1)\cdots\mathring{d}c;t},\phi'_{c;t-1}]'$ (5.12).*

The coupling $\Psi_{ic;t}\equiv[d_{ic;t},\psi'_{ic;t}]'$ is called the data vector *of the factor.*

The data vector Ψ *is in the* phase form *if it consists of a selection of entries from the data record* d_t *and its several delayed values* $d_t, \ldots, d_{t-\partial}$, $\partial \in \partial^* \equiv \{0, 1, \ldots, \mathring{\partial}\}$, $\mathring{\partial} < \infty$. *The corresponding state vector* ϕ_{t-1} *is also said to be in the phase form.*

Structures: Model structure is defined in a hierarchical way starting from the simplest elements, i.e., factors.

The list of regressors, *meaning the entries of the regression vector of a parameterized factor, is called the* structure of the parameterized factor.

The structure of the parameterized factor in the phase form *is the list of pairs* (j, ∂_j) *stating that* $d_{j;t-\partial_j}$, $\partial_j \in \partial^*$, *belongs to the data vector* Ψ_t.

The structure of the parameterized component *is an ordered list of factors creating it. The order characterizes the chosen permutation (5.10).*

Structure of the parameterized mixture *is the list of parameterized components creating it.*

Remark(s) 5.4

1. *The introduced factors, predicting individual entries of the modelled data,*
   - *provide flexibility of the parametric description,*
   - *allow us to jointly describe continuous and discrete valued quantities,*
   - *permit us to respect dependencies reflected in several components,*
   - *open a way for use of different models for different entries of* d_t.
2. *The adopted dynamic mixture model is not sufficiently general. The component weights should also depend on the state vector. The choice is driven by our inability to estimate this "natural" and more realistic model. This important aspect is discussed more deeply in Section 5.3. The restrictive assumption is partially relaxed in Chapter 13.*
3. *The modelled p-data may contain a part that is not in the data space of the o-system. Some of them might just bring complementary information and their evolution need not be modelled. They are used only as entries in the state vectors* ϕ_t. *In this way, the introduced nonmodelled data may arise. It is worth stressing that their use is limited more or less to one-step-ahead predictions. Otherwise, such quantities have to be modelled as their predictions are needed.*
4. *Redundancy and contradictions in specification of structures have to be checked when being defined.*
5. *Let us assume that*

$$d_t \sim \mathcal{N}_{d_t}\left(\begin{bmatrix} \theta \\ 0 \end{bmatrix}, \begin{bmatrix} r_1 & r_{12} \\ r_{12} & r_2 \end{bmatrix} \right),$$

where the symbol $\sim$ *expresses that the two-dimensional vector* d_t *is distributed according to the normal pdf* $\mathcal{N}_d(\mu, R)$ *with the expectation* μ *and covariance matrix* R. *The scalar parameter* $\theta \neq 0$ *determines* μ.

It is straightforward to show that both possible factorized parameterizations differ in the number of parameters whenever $r_{12} \neq 0$. *This provides* *the counterexample to the conjecture that the number of parameters to be estimated does not depend on the order of factors. Thus, the notation of the component structure as an* ordered list of factors *is meaningful.*

6. *Formally, the measured data, the mixture model and a suitably specified prior pdf allow us to get the needed model of the unguided o-system $f(d(\mathring{t}))$ through the Bayesian estimation and prediction. Practically, the estimation of mixture models on tens of thousands of records d_t with tens of entries is computationally very intensive. Thus, we are forced to use an approximations developed in detail in Section 6.5.*

5.3 Mixtures as approximate models and predictors

The considered mixture has dynamic components but constant weights. They express that the cth component of the o-system is active for $100 \times \alpha_c\%$ of the operating time. The adopted model (5.9) allows mutually independent as well as past-state-independent changes of active components at any time moment. For dynamic mixtures, this is an "unnatural" choice. The restriction to constant component weights has a pragmatic motivation. The assumption of constant α and a proposal as to how to weaken it are discussed here. A more systematic attempt to avoid this restriction is given in Section 13.2.

Let us consider that the parameterized model is a projection of a more complex model labelled by Θ and by an additional parameter $M \in M^* \equiv \cup_{c \in c^*} M_c^*$ where the finite collection of sets $\{M_c^*\}_{c \in c^*}$ covers M^*, i.e. $M_c^* \cap M_{\tilde{c}}^* = \emptyset$ for $c \neq \tilde{c}$. Then,

$$\begin{aligned} f(d_t|d(t-1), \Theta) &= \int f(d_t, M|d(t-1), \Theta)\, dM \\ &= \int f(d_t|d(t-1), \Theta, M) f(M|d(t-1), \Theta)\, dM \\ &= \sum_{c \in c^*} \int_{M_c^*} f(d_t|d(t-1), \Theta, M) f(M|d(t-1), \Theta)\, dM. \end{aligned} \quad (5.13)$$

Let us assume that for each $c \in c^*$ there is an $M_c \in M_c^*$ for which the following approximation

$$f(d_t|d(t-1), \Theta, M) \approx f(d_t|d(t-1), \Theta, M_c) \equiv f(d_t|d(t-1), \Theta_c, c)$$

is good for all $M \in M_c^*$ and for all possible data sequences $d(t),\ t \in t^*$. Using this assumption, we arrive at the finite mixture with data-dependent weights

$$f(d_t|d(t-1), \Theta) \approx \sum_{c \in c^*} f(d_t|d(t-1), \Theta_c, c) \tilde{\alpha}_c(d(t-1), \Theta), \quad (5.14)$$

where

$$\tilde{\alpha}_c(d(t-1),\Theta) = \int_{M_c^*} f(M|d(t-1),\Theta)\,dM$$
$$= \text{Probability}(M \in M_c^*|d(t-1),\Theta) \equiv f(M_c^*|d(t-1),\Theta).$$

The approximating mixture on the right-hand side of (5.14) has to be pdf. The individual components are pdfs; thus the right-hand side of (5.14) becomes pdf iff we assume $\sum_{c\in c^*}\tilde{\alpha}_c(d(t-1),\Theta) = 1$ for all $d(t-1)$. In order to make the consequences of this condition explicit, we write $\tilde{\alpha}_c(d(t-1),\Theta)$ as a normalized product of constant probabilistic weights α_c and of a nonnegative parameterized functions of data $\beta_c(d(t-1),\Theta)$

$$\tilde{\alpha}_c(d(t-1),\Theta) = \frac{\alpha_c\beta_c(d(t-1),\Theta)}{\sum_{\tilde{c}\in c^*}\alpha_{\tilde{c}}\beta_{\tilde{c}}(d(t-1),\Theta)}.$$

The mixtures with constant component weights are obtained if the functions $\{\beta_c(\cdot)\}_{c\in c^*}$ are constant. If they really depend on data, we get the mixture

$$f(d_t|d(t-1),\Theta) = \sum_{c\in c^*}\alpha_c\frac{f(d_t|d(t-1),\Theta_c,c)\beta_c(d(t-1),\Theta)}{\sum_{\tilde{c}\in c^*}\alpha_{\tilde{c}}\beta_{\tilde{c}}(d(t-1),\Theta)}. \tag{5.15}$$

For nontrivial $\beta_c(\cdot)$, the components of the mixture (5.15) do not belong to the exponential family so that their efficient estimation on large data sets is extremely difficult. Consequently, no efficient algorithm for estimation of the overall mixture is known except a novel attempt in Section 13.2.

We show, however, that our restricted model (5.9) with constant weights can be interpreted as a limit of the model (5.14).

Proposition 5.2 (Constant weights approximation (5.14))
Weights of the approximating pdf (5.14) converge almost surely to constant values. Thus, the adopted model with constant weights can be viewed as an asymptotic version of the "correct" approximating pdf (5.14).

Proof. For a fixed c and Θ, it holds that

$$\begin{aligned}
\mathcal{E}[\tilde{\alpha}_c(d(t),\Theta)|d(t-1),\Theta] &\equiv \int f(M_c^*|d(t),\Theta)f(d_t|d(t-1),\Theta)\,dd_t \\
\underbrace{=}_{\text{chain rule}} &\int \frac{f(M_c^*,d_t|d(t-1),\Theta)}{f(d_t|d(t-1),\Theta)} f(d_t|d(t-1),\Theta)\,dd_t \\
\underbrace{=}_{\text{canceling}} &\int f(M_c^*,d_t|d(t-1),\Theta)\,dd_t \\
\underbrace{=}_{\text{marginalization}} &\quad f(M_c^*|d(t-1),\Theta) \equiv \tilde{\alpha}_c(d(t-1),\Theta).
\end{aligned}$$

Thus, $\{\tilde{\alpha}_c(d(t),\Theta), d(t)\}_{t\in t^*}$ is a martingale, which is moreover nonnegative and bounded by 1 as $\tilde{\alpha}_c(d(t),\Theta)$ is a probability. Thus, the martingale convergence theorem [81] applies and $\tilde{\alpha}_c(d(t),\Theta)$ converges almost surely to a constant probability. □

The mixture serves mainly as the predictor of the future behavior of the o-system. The assumed invariance of weights may deteriorate its quality substantially. It can be seen on a simple mixture with a well-separated pair of components of similar symmetric shapes and equal weights. For such a mixture, the expected value, which is taken as a good point estimate of the future values d_t, sits in the improbable area between them. The following proposition shows how the problem can be resolved in a generic way that fits into the considered context.

Proposition 5.3 (Mixtures on grouped data) *Let us decompose data sequence $d(\mathring{t})$ into adjacent nonoverlapping groups of a length $n > 1$. Thus, we consider the following probabilistic description of the whole sequence (with a negligible exception of initial and terminal groups of the length n)*

$$f(d(\mathring{t})|\Theta) = \prod_{\tau=1}^{\frac{\mathring{t}}{n}} f(d_{[(\tau-1)n+1]\cdots\tau n}|d((\tau-1)n),\Theta). \tag{5.16}$$

Let the individual parameterized pdfs have the mixture form with constant component weights

$$f(d_{[(\tau-1)n+1]\cdots\tau n}|d((\tau-1)n),\Theta) = \sum_{c\in c^*} \alpha_c f(d_{[(\tau-1)n+1]\cdots\tau n}|d((\tau-1)n),\Theta_c,c). \tag{5.17}$$

Then, the predictor of $d_{\tau n}$ based on $d(\tau n-1)$ (notice the braces!) has data-dependent weights. Specifically,

$$f(d_{\tau n}|d(\tau n-1),\Theta) = \sum_{c\in c^*} \tilde{\alpha}_c(d(\tau n-1),\Theta) f(d_{\tau n}|d(\tau n-1),\Theta_c,c)$$
$$\tilde{\alpha}_c(d(\tau n-1),\Theta) = \frac{\alpha_c f(d_{[(\tau-1)n+1]\cdots(\tau n-1)}|d((\tau-1)n),\Theta_c,c)}{\sum_{\tilde{c}\in c^*} \alpha_{\tilde{c}} f(d_{[(\tau-1)n+1]\cdots(\tau n-1)}|d((\tau-1)n),\Theta_{\tilde{c}},\tilde{c})}. \tag{5.18}$$

Proof. The derived formula is directly implied by the chain rule for pdfs. □

Remark(s) 5.5

Note that we predict the data at the end of the grouping interval. It is sufficient for the assumed low rate of operator interventions for which the predictions serve. The other data records in the group can be predicted similarly, but the very first data record in the group is still predicted with constant weights.

Problem 5.2 (Mixtures at factor level) *The mixture modelling of grouped data indicates that mixtures can be used at various levels of decomposition* *of the pdf $f(d(\mathring{t}))$. For instance, modelling of factors by mixtures could bring additional freedom. For instance, non-normal factors can be approximated or modes living at the factor level can be modelled.*

Problem 5.3 (Shifted and repeatedly used predictors) *Poorer predictions of "earlier" entries in the group can be suppressed by dealing with n mutually shifted predictors or by using the single one working on shifted data. Experiments indicate that these techniques are worth elaborating.*

Problem 5.4 (Rational approximations) *The discussed way of getting data-dependent weights seems to be acceptable. In spite of this, a theoretical and algorithmic solution admitting rational forms instead of finite mixtures would be highly desirable. An attempt presented in Section 13.2 is based on the technique given in [131]. Alternatively, the approximate estimation developed in Section 6.5 could be extended by applying it both to the numerator and the denominator of the "rational" approximation.*

5.4 Design of advisory systems

Here, elements related to the optimization part of the basic design Algorithm 5.1 are presented.

5.4.1 Types of advisory systems

Under the adopted notations and assumptions, we are able to specify a <u>formal</u> description of advisory systems and to distinguish their basic types.

Agreement 5.5 (Fixed and adaptive advisory systems) *The advisory system is a system that constructs the ideal pdf ${}^{\lfloor I}f(d(\mathring{t})|\mathcal{P})$ and presents its projections to the operator. The following types of advisory systems are distinguished according to the exploited experience $\mathcal{P}$.*

The fixed advisory system *constructs the ideal pdf in <u>offline mode</u> using the experience $\mathcal{P} \equiv d(0)$, cf. Chapter 2, obtained before the <u>use of the advisory</u> system in its online mode. Thus, its construction is formally described by the mapping*

$$ {}^{\lfloor U}f(d(\mathring{t})),\ f(\Theta|d(0)) \rightarrow \left\{ {}^{\lfloor I}f(d(\mathring{t})|d(0)) \right\}. $$

The pdf ${}^{\lfloor U}f(d(\mathring{t}))$ is the user's ideal pdf obtained by extending the true user's ideal pdf, Section 5.1.4, in the way described in Section 5.1.5. The involved parameter estimate $f(\Theta|d(0))$ is based on the experience collected before online usage of the advisory system. Thus, the fixed advisory system exploits the unguided model $f(d(\mathring{t})|d(0))$ that results from the estimation based on the data produced <u>without</u> the use of the p-system, cf. Agreement 5.3.

The adaptive advisory system *extends its experience during its online use. Thus, its construction is formally described by the sequence of mappings*

$$\left\{ {}^{\lfloor U}f(d_{t+1}, \cdots, d_{\mathring{t}}|d(t)),\ f(\Theta|d(t)) \to \left\{ {}^{\lfloor I}f(d_{t+1}, \cdots, d_{\mathring{t}}|d(t)) \right\} \right\}_{t \in \{0\} \cup t^*}.$$

Thus, the adaptive advisory system exploits the guided model $f(d_t, \cdots, d_{\mathring{t}}|d(t))$ that results from the estimation based also on the data produced with the use of the p-system, cf. Agreement 5.3.

The use of the advisory system in the online mode consists of a sequential presentation of the optimized ideal pdf ${}^{\lfloor I}f(\cdot)$. It is evaluated at measured and (or) contemplated arguments $d_o(t)$ as well as at the measured values $d_{p+}(t),\ t \in t^*$.

As anticipated by conceptual Algorithm 5.1, only some optional parts of the estimated model are optimized. Basic variants are discussed now.

Generally, the p-system may not be aware which quantities in $d_o^*(\mathring{t})$ are operator actions. For instance, the operator can change both pressure and temperature of a managed gas system. The advisory system measures changes of both of them but may have no information on the command button pressed by the operator. Note that such a situation is more frequent in medical or societal applications.

This incomplete knowledge is another important difference that makes the design of the advisory system a very specific decision-making problem. The following agreement singles out the case when this information lack is complete.

Agreement 5.6 (Academic advisory system) *The advisory system designed without knowing which entries of $d_{o;t}$ belong to the action space of the operator $a_o^*(\mathring{t})$ is called the* academic advisory system. *The corresponding design is called the* academic design.

Agreement 5.7 (Industrial and simultaneous advisory systems) *The advisory system designed with knowledge of a nonempty part of the action space of the operator, say $u_o^*(\mathring{t}) \subset a_o^*(\mathring{t})$, $u_o^*(\mathring{t}) \neq \emptyset$, is called the* industrial advisory system.

The o-actions $u_{o;t}$ are called the recognizable actions*; see Fig. 5.1.*

The design of recognizable actions is called industrial design. *The joint academic and industrial design is called* simultaneous design.

Remark(s) 5.6
Obviously, the industrial advisory system is to be optimized through the simultaneous design whenever possible. Sometimes, however, the weights have physical meaning and cannot be influenced by operator's strategy.

5.4.2 Advices as actions of the p-system

During development of the advisory system, basic types of its actions emerged. Here we classify them (a wider discussion follows).

Agreement 5.8 (Nomenclature of actions of the p-system) *The actions available to p-systems*

$$a_{p;t} \equiv (c_t, u_{o;t}, z_t, s_t) \quad \textit{are interpreted as follows.} \tag{5.19}$$

Recommended pointers $\{c_t\}_{t\in t^*}$ *are pointers to the components that are recommended to be kept active at respective time moments. Recommended pointers are* academic advices. *Academic advices cannot be directly communicated to the operator and have to be reflected in the ideal pdf* ${}^{\lfloor I}f(d_{o;t}|d(t-1))$ *offered to him.*

Recommended recognizable actions $\{u_{o;t}\}_{t\in t^*}$ *guide the operator in selecting recognizable actions. These advices result from the industrial or simultaneous designs. The recommended recognizable actions can be interpreted as ordinary inputs of the o-system with the operator serving as an imperfect actuator. Ideally, the recommended recognizable actions should be directly fed into the o-system. This insures that the identical notation is being used for the* recommended *recognizable actions and the recognizable actions.*

Priority actions $\{z_t\}_{t\in t^*}$ *select entries of* $\{d_t\}_{t\in t^*}$ *whose ideal behavior is shown to the operator. These advices result from optimized* assigning priorities. *The priority action* z_t *is* $\mathring{z}$ *vector* ($\mathring{z} \leq \mathring{d}_o$) *of differing indexes* $z_{i;t} \in \{1,\ldots,\mathring{d}_o\}$, $i \in \{1,\ldots,\mathring{z}\}$. *The operator gets the marginal pdfs* $\left\{{}^{\lfloor I}f(d_{z_t;t}|d(t-1)\right\}_{t\in t^*}$ *of the ideal pdf resulting from the previous design.*

Signaling actions $\{s_t \in s^* \equiv \{0,1\}\}_{t\in t^*}$ *stimulate the operator to take some measures when behavior of the o-system significantly differs from the desired one. These advices result from optimized* signaling. *The operator gets probabilistic information whether an intervention is needed or not. It is coded, for instance, as traffic lights.*

5.4.3 Unguided and guided models for respective designs

We assume that the learning part of the advisory system provides a good model $f(d_t|d(t-1))$ of the o-system uninfluenced by the p-system, i.e., the model of the unguided o-system.

A systematic design of the p-system, as described in Algorithm 5.1, requires a model relating its advices to responses of the o-system, i.e., the model of the guided o-system.

The needed but *speculative class of models* is proposed gradually in subsequent sections. The term “speculative” underlines that the model is based on hypothesis that the operator fully cooperates and is able to drive the o-system to the desired state. Possible adverse consequences of this speculation can be suppressed by using an adaptive advisory system with models having advices as an explicit part of experience. Initially, and often permanently, we have to rely on such speculative models as the basis of the systematic academic, industrial and simultaneous design, respectively. Speculative guided models are also used for assigning priorities and signalling.

The consistent transition from the unguided model to the guided one has the following common structure.

The considered data split

$$d_t \equiv d_{p;t} = (\Delta_{p;t}, a_{p;t}) \equiv (\text{p-innovations}, \text{p-actions}) \equiv (\text{p-innovations}, \text{advices}).$$

The corresponding factorization of the unguided model by the chain rule reads

$$f(d_t|d(t-1)) = f(\Delta_{p;t}|a_{p;t}, d(t-1))f(a_{p;t}|d(t-1)).$$

Assuming (speculating) that the recommended actions and actions realized by the o-system coincide, the first factor on the right-hand side describes the reaction of the o-system on advices. It is given by its physical nature and cannot be changed. The second factor describes the rule of generating $a_{p;t}$. Exactly this rule should be changed by the optimized advising. This gives the general form of the optimized guided model

$$\begin{aligned} {}^{\lfloor I}f(d_t|d(t-1)) &= f(\Delta_{p;t}|a_{p;t}, d(t-1))\, {}^{\lfloor I}f(a_{p;t}|d(t-1)) \\ &= \frac{f(d_t|d(t-1))}{\int f(d_t|d(t-1))\, d\Delta_{p;t}}\, {}^{\lfloor I}f(a_{p;t}|d(t-1)). \end{aligned} \quad (5.20)$$

As discussed in Section 5.4.1, respective designs differ in the available advices, and consequently they lead to different guided models.

Remark(s) 5.7
Note that conditioning used in (5.20) is potential source of computational difficulties. We can say beforehand that for the academic and simultaneous designs the obtained guided models are relatively simple, unlike for the industrial design and assigning priorities. The complexity of signalling related model is somewhere in between these two cases.

5.4.4 Academic design

We have at disposal the multiple-mode model of the unguided o-system $f(d(\mathring{t}))$. As discussed above; see Agreement 5.5, the advisory system maps $f(d(\mathring{t}))$ on an ideal pdf ${}^{\lfloor I}f(d(\mathring{t}))$, whose projections are presented to the operator, preferably in a graphic form.

The joint pdf $f(d(\mathring{t}))$ describes the probability distribution of achievable modes within the data space of the advisory system. Thus, the reachable ideal pdf should be created from these modes. The selection of modes leading to a higher management quality should be advised. The academic design selects the recommended mode through the recommended pointer $c_t \in c^*$ to a particular component (mode) by defining the ideal pdf

$${}^{\lfloor I}f(d(\mathring{t}), c(\mathring{t})) \equiv \prod_{t \in t^*} {}^{\lfloor I}f(d_t, c_t|d(t-1)) \equiv \prod_{t \in t^*} f(d_t|d(t-1), c_t)\, {}^{\lfloor I}f(c_t|d(t-1)).$$

The pdfs $f(d_t|d(t-1), c_t),\ c_t \in c^*$ are estimated components and the optional probabilities ${}^{\lfloor I}f(c_t|d(t-1))$ describe the randomized causal strategy

$$\{d^*(t-1) \to c_t \in c^*\}_{t\in t^*} \quad \text{to be designed.} \tag{5.21}$$

Remark(s) 5.8

1. *Usage of the notation* ${}^{\lfloor I}f(d(\mathring{t}), c(\mathring{t}))$ *is a bit inconsistent as* $c(\mathring{t})$ *is a part of* $d(\mathring{t})$*. It helps, however, to focus attention on the discussed advices. Further on, both variants are used. Context should prevent possible misunderstandings.*
2. *The optimized ideal pdf* ${}^{\lfloor I}(d_t|d(t-1))$ *is projected on low-dimensional pdfs on subsets of* $d^*_{o;t}$*. They describe entries of o-data and are also called advisory mixtures. This agreement is used for all basic designs.*

The recommended pointer $c_t \in d^*_{p+;t}$, thus, a projection on d^*_o has to be presented to the operator. The corresponding marginal predictive pdf has the form

$${}^{\lfloor I}f(d_{o;t}|d(t-1)) = \sum_{c_t\in c^*} {}^{\lfloor I}f(c_t|d(t-1)) f(d_{o;t}|d(t-1), \Theta_{c_t}, c_t). \tag{5.22}$$

The advising strategy $\left\{ {}^{\lfloor I}f(c_t|d(t-1))\right\}_{t\in t^*}$ is optimized in the sense of the fully probabilistic design; see Section 2.4.2. For that, we have to specify a user *ideal pdf for the interconnection of the o- and p-systems* ${}^{\lfloor U}f(d(\mathring{t}), c(\mathring{t})) \equiv {}^{\lfloor I}f(d_t|d(t-1))\, {}^{\lfloor U}f(c_t|d(t-1))$. The constructed ideal pdf ${}^{\lfloor I}f(d(\mathring{t}), c(\mathring{t})) = \prod_{t\in t^*} {}^{\lfloor I}f(d_t|d(t-1))\, {}^{\lfloor U}f(c_t|d(t-1))$ should be as close as possible to the user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}), c(\mathring{t}))$.

The recommended pointers c_t belong to $d^*_{p+;t}$ so that the general extension of the user's ideal pdf could be used; see Section 5.1.5. Practical reasons make us to take this part of the user's ideal pdf as tuning knob of the design. For instance, it is reasonable to inhibit fast changes of values of $c(\mathring{t})$ in order to get relatively stable advices given to the operator. This can be achieved by selecting such a user's ideal pdf $\prod_{t\in t^*} {}^{\lfloor U}f(c_t|d(t-1))$ that assigns high probabilities to sequences $c(\mathring{t})$ with small differences $c_t - c_{t-1}$. Also, an offline analysis may discourage operating at some dangerous modes of $f(d(\mathring{t}))$ completely. For that, it is sufficient to restrict support of ${}^{\lfloor U}f(c_t|d(t-1))$ on pointers to the nondangerous components. Such considerations determine the discussed part of the user's ideal pdf

$$\left\{ {}^{\lfloor U}f(c_t|d(t-1))\right\}_{t\in t^*}.$$

With it, the complete user's ideal pdf gets the form

$${}^{\lfloor U}f(d(\mathring{t})) = \prod_{t\in t^*} {}^{\lfloor I}f(d_{p+;t}|d(t-1))\, {}^{\lfloor U}f(d_{o;t}|d_o(t-1))\, {}^{\lfloor U}f(c_t|d(t-1)). \tag{5.23}$$

It follows from the discussed extension of the true user's ideal pdf ${}^{\lfloor U}f(d_o(\mathring{t}))$ to ${}^{\lfloor U}f(d(\mathring{t}))$ (see Section 5.1.5) from the chain rule and the fact, that the user can deal with quantities which he is aware of.

This selection of the extended user's ideal pdf completes the formulation of the academic design that makes the general fully probabilistic design formally applicable; see Proposition 2.11. The practical evaluation requires approximations that are discussed in Chapter 7.

As mentioned above, some components might be handled as "dangerous". We take a component as dangerous if a permanent operation on it leads to an unacceptable behavior of the o-system.

Agreement 5.9 (Dangerous component) *Let* $f(\Psi|c)$ *be the* steady-state pdf *of the data vector* Ψ *assigned to the permanent activity of the cth component of the mixture*

$$f(\Psi|c) = \int f(\Psi|\tilde{\Psi}, c) f(\tilde{\Psi}|c)\, d\tilde{\Psi}. \tag{5.24}$$

Here, $f(\Psi|\tilde{\Psi}, c) \equiv f(\Psi_t = \Psi|\Psi_{t-1} = \tilde{\Psi}, c)$ *is a formal, state-space version of the cth component that describes the evolution of the data vector* Ψ_t*.*

Let us consider average marker *of the form*

$$\frac{1}{\mathring{t}} \sum_{t \in t^*} m(\Psi_t) \tag{5.25}$$

given by a partial quality marker $m(\Psi_t)$*. Then, the component c is called dangerous if the probability of a given set* $\bar{m}^*$ *of non-acceptable values of* $m(\Psi)$

$$\int \chi_{\bar{m}^*}(m(\Psi)) f(\Psi|c)\, d\Psi \tag{5.26}$$

is too high. The symbol $\chi_{x^*}(\cdot)$ *means indicator of the set* x^**.*

5.4.5 Industrial design

The industrial design optimizes recommended recognizable actions $u_{o;t}$. Ideally, these actions are directly fed into the o-system and their consequences are predicted by the model of the unguided o-system. They are similar to ordinary inputs of the o-system with the operator serving as an imperfect actuator.

The constructed randomized strategy is described by the pdfs $\left\{ {}^{\lfloor I}f(u_{o;t}|d(t-1)) \right\}_{t \in t^*}$. These pdfs replace $\left\{ f(u_{o;t}|d(t-1)) \right\}_{t \in t^*}$ forming a part of the estimated unguided model $f(d_t|d(t-1))$. Thus, the ideal pdf generated by this design has the form

$$\begin{aligned} {}^{\lfloor I}f(d_t|d(t-1)) &= f(\Delta_t|u_{o;t}, d(t-1))\, {}^{\lfloor I}f(u_{o;t}|d(t-1)) = \\ &= {}^{\lfloor I}f(u_{o;t}|d(t-1)) \frac{\sum_{c_t \in c^*} \alpha_{c_t} f(\Delta_t|u_{o;t}, d(t-1), c_t) f(u_{o;t}|d(t-1), c_t)}{\sum_{c_t \in c^*} \alpha_{c_t} f(u_{o;t}|d(t-1), c_t)}. \end{aligned} \tag{5.27}$$

The strategy determining the optimal ${}^{\lfloor I}f(u_{o;t}|d(t-1))$ is obtained through the fully probabilistic design, Proposition 2.11. The needed user's ideal pdf

$$^{\lfloor U}f(d(\mathring{t})) = \prod_{t\in t^*} {}^{\lfloor U}f(\Delta_t|u_{o;t}, d(t-1))\, {}^{\lfloor U}f(u_{o;t}|d(t-1)),$$

that includes the target for the recognizable actions, is constructed exactly as described in Section 5.1.5.

Remark(s) 5.9

1. *Note that $u_{o;t}$ belongs to $d^*_{o;t}$, thus it is the only advice that can be* <u>*directly*</u> *presented to the operator.*
2. *The estimated strategy, generating the recognizable actions of the unguided o-system and described by the marginal pdfs $f(u_{o;t}|d(t-1), c_t)$ of individual components, influences the resulting ideal pdf; see (5.27). This effect is specific for nontrivial mixtures, in which the strategies used at various components do not cancel in the inspected conditional pdf (5.27).*
3. *Properties of the components creating the ideal pdf are influenced by the advising strategy $\{ {}^{\lfloor I}f(u_{o;t}|d(t-1))\}_{t\in t^*}$ influencing the recognizable actions $u_o(\mathring{t})$ applied. It may, for instance, convert the dangerous components in nondangerous ones and vice versa.*

5.4.6 Simultaneous academic and industrial design

The simultaneous design optimizes both recommended pointers to components and recommended recognizable actions. It should lead to a better advising strategy than a sequential use of academic and industrial designs. The simultaneous design is surprisingly simpler than the industrial one; see Chapter 7.

The ideal pdf ${}^{\lfloor I}f(d_t|d(t-1))$, generated by the fully probabilistic design, Proposition 2.11, minimizes the KL divergence to the user's ideal pdf

$$\begin{aligned} &^{\lfloor U}f(d(\mathring{t})) = \prod_{t\in t^*} {}^{\lfloor U}f(\Delta_{o;t}|u_{o;t}, d_o(t-1))\, {}^{\lfloor I}f(\Delta_{p+;t}|u_{o;t}, d(t-1)) \\ &\times {}^{\lfloor U}f(u_{o;t}|d(t-1))\, {}^{\lfloor U}f(c_t|u_{o;t}, d(t-1)), \end{aligned}$$

which includes the user's ideal pdf ${}^{\lfloor U}f(u_{o;t}|d_o(t-1))$ for the recognizable actions $u_{o;t}$ as well as the target probability ${}^{\lfloor U}f(c_t|u_{o;t}, d(t-1))$ for the recommended pointers. The latter element is discussed in connection with the academic design, Section 5.4.4. In the simultaneous design, it may depend on $u_{o;t}$, too.

The result is similar to (5.27) with the component weights replaced by the designed probabilities ${}^{\lfloor I}f(c_t|u_{o;t}, d(t-1))$ of recommended actions. Specifically, it holds that

$$
{}^{\lfloor I}f(d_t|d(t-1)) = {}^{\lfloor I}f(\varDelta_t|u_{o;t}, d(t-1))\, {}^{\lfloor I}f(u_{o;t}|d(t-1)) \equiv {}^{\lfloor I}f(u_{o;t}|d(t-1))
$$
$$
\times \frac{\sum_{c_t \in c^*} {}^{\lfloor I}f(c_t|u_{o;t}, d(t-1)) f(\varDelta_t|u_{o;t}, d(t-1), c_t) f(u_{o;t}|d(t-1), c_t)}{\sum_{c_t \in c^*} {}^{\lfloor I}f(c_t|u_{o;t}, d(t-1)) f(u_{o;t}|d(t-1), c_t)}. \quad (5.28)
$$

Remark(s) 5.10

It is worth repeating that the elements in the formula (5.28) with the superscript ${}^{\lfloor I}$ *are optimized. The elements without the superscript* ${}^{\lfloor I}$ *are those obtained through estimation of the unguided o-system. They reflect those parts of operating practice that are not expected to be changed by the advising.*

5.5 Interactions with the operator

The discussed advisory systems can be seen as specific versions of a high-level control system. The ideal pdf ${}^{\lfloor I}f(d(\mathring{t}))$ resulting from the designs discussed above has to be, however, perceived by a human being. This implies a non-standard task, namely, the optimization of the *presentation of advices*. Essentially, low-dimensional projections of the high-dimensional ideal pdf have to be shown to the operator. This calls for generating additional actions of the p-system caring about interaction of the p-system with the operator. Specifically, it has to be taken into account that

- A few selected quantities can only be shown to the operator including those which should be changed the most urgently in order to minimize risk of malfunctioning or maximize benefit. In other words, presentation priorities have to be dynamically assigned to the o-data.
- The information load on the operator has to be controlled by demanding changes of the o-actions only when needed.
  In other words, signaling that controls dynamically operator's attention has to be designed.

These problems are addressed in Sections 5.5.1 and 5.5.2 under typical advising scenarios.

5.5.1 Assigning priorities

Presentation of quantities worth the operator interest is simple when priorities of critical quantities to be shown are fixed by technological prescriptions.

The situation is also simple if a full question and answer mode of the dialog is adopted. In this case, the operator can ask the p-system:

> What happens to a quantity $d_{i;t}$ if I assign a value $\bar{d}_{j;t}$ to the quantity $d_{j;t}$?

In this case, the optimized (guided) predictive pdf ${}^{\lfloor I}f(d_{i;t}|\bar{d}_{j;t}, d(t-1))$ is simply shown. This pdf obtained via marginalization and conditioning from the optimized guided pdf ${}^{\lfloor I}f(d_t|d(t-1))$.

A similar simple case arises when a few recognizable actions $u_{o;t}$ are to be recommended only. Then, the marginal pdfs ${}^{\lfloor I}f(u_{io;t}|d(t-1))$, $i = 1, \ldots, \mathring{u}_o$, of the optimized pdf ${}^{\lfloor I}f(u_{o;t}|d(t-1))$ are evaluated and shown to the operator.

The situation becomes more complex when there is a need to show only marginal pdfs of a few critical quantities contained in the extensive full data record d_t. It leads to the following specific decision problem.

The p-system is given task to generate a $\mathring{z}$-vector of priority actions z_t with entries $z_{i;t} \in \{1, \ldots, \mathring{d}_o\}$, $i \in i^* \equiv \{1, \ldots, \mathring{z}\}$, $\mathring{z} < \mathring{d}_o$. The value of $z_{i;t} = j$ means that recommendations on jth entry of $d_{o;t}$ should be shown to the operator.

Note that the number of shown quantities $\mathring{z}$ has to be small, say 5, in order to respect limited perceiving abilities of human beings.

The ideal pdf ${}^{\lfloor I}f(d(\mathring{t}))$ resulting from academic, industrial or simultaneous designs as well as the user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}))$ are assumed to be fixed and available when designing the presentation strategy $\left\{ {}^{\lfloor I}f(z_t|d(t-1)) \right\}_{t \in t^*}$.

Similarly as in the academic design, the target pdf for the adopted fully probabilistic design ${}^{\lfloor U}f(d(\mathring{t}))$ is extended by the factor $\prod_{t \in t^*} {}^{\lfloor U}f(z_t|d(t-1))$. This tuning knob allows us to respect technological preferences and (or) to restrict rate of changes of the quantities selected for the presentation to the operator.

Using the redundant notation $d(\mathring{t}), z(\mathring{t})$ instead of $d(\mathring{t})$ (cf. Remark 5.8) the optimized model, describing the influence of presentation actions, gets the form

$$
{}^{\lfloor I}f(d(\mathring{t}), z(\mathring{t})) = \prod_{t \in t^*} {}^{\lfloor I}f(d_t|z_t, d(t-1))\, {}^{\lfloor I}f(z_t|d(t-1)), \tag{5.29}
$$

where the pfs $\left\{ {}^{\lfloor I}f(z_t|d(t-1)) \right\}_{t \in t^*}$ describe the randomized *presentation strategy* to be designed.

Let us assume again that the operator cooperates fully when given the ideal pdf for entries of d_t with indexes z_t. Then, the operator is expected to act so that the behavior of the o-system has the distribution with the marginal pdf ${}^{\lfloor I}f(d_{z_t;t}|d(t-1))$, while entries not shown follow the model of the unguided o-system. Here, the pdf ${}^{\lfloor I}f(d_t|d(t-1))$ is the ideal pdf offered by the advisory system that results from the previous optimization.

Thus, the compromise ${}^{\lfloor I}f(d_t|z_t, d(t-1))$ between

- the unguided model of the o-system $f(d_t|d(t-1))$, acting without the p-system, and
- the optimized guided model ${}^{\lfloor I}f(d_t|d(t-1))$, acting fully according to the p-system that presents $d_{z_t;t}$

looks as follows (5.20):

$$
{}^{\lfloor I}f(d_t|z_t, d(t-1)) = f(d_t|d(t-1)) \frac{{}^{\lfloor I}f(d_{z_t;t}|d(t-1))}{f(d_{z_t;t}|d(t-1))}. \tag{5.30}
$$

In this model, the marginal pdf $f(d_{z_t;t}|d(t-1))$ of the presented quantities $d_{z_t;t}$ — computed from the estimated mixture $f(d_t|d(t-1))$ — is replaced by the corresponding marginal pdf ${}^{\lfloor I}f(d_{z_t;t}|d(t-1))$ gained from the previously designed ideal pdf ${}^{\lfloor I}f(d_t|d(t-1))$, which is generally also mixture. Thus, the model (5.30) is rather complex ratios of mixtures. Consequently, the fully probabilistic design must be approximated in order to get a feasible presentation strategy; cf. Chapter 7.

Moreover, the number of variants to be compared $\begin{pmatrix} \mathring{d}_o \\ \mathring{z} \end{pmatrix}$ during optimization is mostly very large. It motivates us to select $\mathring{z} = 1$ and to use ${}^{\lfloor I}f(z_t = i|d(t-1)),\ i \in \{1, \ldots, \mathring{d}_o\}$ as a degree of the presentation priority assigned to the ith entry of d_t.

Problem 5.5 (Alternative design of presentation priorities) *This part of the design is, a bit surprisingly, the hardest one. It calls for an alternative formulation. For instance, it would be possible to perform design predecessors with alternative fixed choices of presented quantities and then to compare the predicted quality of the guided closed loop behavior. This formulation is worth considering.*

5.5.2 Stimulating the operator

Operator may actively call the p-system for advices. Typically, however, his attention has to be attracted when the state of the managed system requires it. The problem how to attract the operator's attention is addressed here. In modelling of signalling influence, we proceed similarly as in previous sections.

The p-system is given the task to generate actions $\{s_t\}_{t\in t^*}$, $s_t \in \{0, 1\}$. The value $s_t = 0$ means that system is in a good state and no extra operator activity is needed. The value of $s_t = 1$ urgently demands operator actions. We search for an admissible *signaling strategy* described by the causal rules

$$\{d^*(t-1) \to s_t^* \equiv \{0, 1\}\}_{t\in t^*}\,.$$

The user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}))$ as well as the ideal pdf ${}^{\lfloor I}f(d(\mathring{t}))$, resulting from academic or industrial or simultaneous design, are assumed to be fixed here. The user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}))$ is extended to signaling actions $s(\mathring{t})$ by the pf ${}^{\lfloor U}f(s_t|d(t-1))$ reflecting the desired damping of the stimulation.

Stimulation, when respected by the operator,

- results into the guided behavior of the o-system, i.e., $f(d_t|s_t = 1, d(t-1)) \equiv {}^{\lfloor I}f(d_t|d(t-1))$ if $s_t = 1$,
- leaves the o-system unguided, i.e., $f(d_t|s_t = 0, d(t-1)) \equiv f(d_t|d(t-1))$ if $s_t = 0$

Thus, the resulting model of the optimized guided o-system has the form

$${}^{\lfloor I}f(d(\mathring{t}), s(\mathring{t})) = \prod_{t\in t^*} {}^{\lfloor I}f(d_t|s_t, d(t-1))\, {}^{\lfloor I}f(s_t|d(t-1)),$$

where the probabilities $\left\{ {}^{\lfloor I}f(s_t|d(t-1)) \right\}_{t\in t^*}$ describe the constructed *signaling strategy*. The influence of the signalling action is described by the model

$$^{\lfloor I}f(d_t|s_t, d(t-1)) = \delta_{s_t,0} f(d_t|d(t-1)) + (1-\delta_{s_t,0})\, {}^{\lfloor I}f(d_t|d(t-1)).$$

As $s_t \in d^*_{p+;t}$, it cannot be directly provided to the operator. Instead,

$$\hat{s}_t = \mathcal{E}[\delta_{s_t,0}|d(t-1)] = {}^{\lfloor I}f(s_t = 0|d(t-1))$$

is shown, usually, as a colored traffic light. Here, *Kronecker symbol* is used

$$\delta_{s,\tilde{s}} = \begin{cases} 1 \text{ if } & s = \tilde{s} \\ 0 \text{ otherwise} \end{cases}. \tag{5.31}$$

5.6 Design summary

For reference purposes, a review of the proposed models, expressing the expected influence of particular advises, is given, Subsection 5.6.1. The overall design scenario in Subsection 5.6.2 anticipates the decision subtasks to be solved on the way towards the constructed advisory system.

5.6.1 Influence of advices on the o-system

During the discussion of particular designs, the following overall model of the interconnecting of the o-system and the p-system has arisen. Recall, the superscript ${}^{\lfloor I}$ indicates that the corresponding object results from the optimization of advices $a_{p;t} = (c_t, u_{o;t}, z_t, s_t)$. The overall model has the form

$f(d_{o;t}|a_{p;t}, d(t-1)) \equiv \delta_{s_t,0} f(d_{o;t}|d(t-1)) + (1-\delta_{s_t,0})\, {}^{\lfloor I}f(d_{o;t}|d(t-1))$

s_t is 0 if operator does not use recommended actions, otherwise s_t is 1.

He reacts on signaling chosen according to ${}^{\lfloor I}f(s_t|d(t-1))$.

The first predictor above is obtained in learning of the unguided system

$$f(d_{o;t}|d(t-1)) \equiv \sum_{c\in c^*} \alpha_c f(d_{o;t}|d(t-1), c) \text{ and} \tag{5.32}$$

$${}^{\lfloor I}f(d_{o;t}|d(t-1)) \equiv f(d_{o;t}|d(t-1)) \frac{{}^{\lfloor I}f(d_{z_t;t}|d(t-1))}{f(d_{z_t;t}|d(t-1))}, \text{ where}$$

z_t is the presentation action chosen according to ${}^{\lfloor I}f(z_t|d(t-1))$

$$\begin{aligned} {}^{\lfloor I}f(d_{o;t}|d(t-1)) &\equiv {}^{\lfloor I}f(u_{o;t}|d(t-1)) \times \\ &\times \frac{\sum_{c_t\in c^*} {}^{\lfloor I}f(c_t|u_{o;t}, d(t-1)) f(\mathit{\Delta}_{o;t}|u_{o;t}, d(t-1), c_t) f(u_o|d(t-1), c_t)}{\sum_{c_t\in c^*} {}^{\lfloor I}f(c_t|u_{o;t}, d(t-1)) f(u_o|d(t-1), c_t)}. \end{aligned}$$

The probabilities $\left\{ {}^{\lfloor I}f(s_t|d(t-1)) \right\}_{t\in t^*}$, $s_t \in s^* \equiv \{0,1\}$, describe signaling strategy. They result from the corresponding, fully probabilistic, signaling design. This design is the last in the sequence of respective designs.

The probabilities $\left\{ {}^{\lfloor I}f(z_t|d(t-1)) \right\}_{t\in t^*}$,

$$z_t \in z^* \equiv \left\{ [z_1, \ldots, z_{\mathring{z}}],\ z_i \in \{1, \ldots, \mathring{d}_o\} \right\}$$

describe presentation strategy. They result from the presentation design made after finishing some of the designs listed below.

The pdfs $\left\{ {}^{\lfloor I}f(u_{o;t}|d(t-1)) \right\}_{t\in t^*}$, $u_{o;t} \in u_o^*$, describe strategy generating recommended recognizable actions. They result from industrial or simultaneous design. The industrial design relies on availability of the component weights $f(c_t|u_{o;t}, d(t-1))$. They are obtained either from learning, then $f(c_t|u_{o;t}, d(t-1)) = \alpha_c$, or from the previous academic design, then $f(c_t|u_{o;t}, d(t-1)) = {}^{\lfloor I}f(c_t|u_{o;t}, d(t-1))$. The simultaneous design — when adopted — is the first in the sequence of designs.

The probabilities $\left\{ {}^{\lfloor I}f(c_t|d(t-1)) \right\}_{t\in t^*}$, $c_t \in c^* \equiv \{1, \ldots, \mathring{c}\}$, describe strategy generating academic advices. They result from the academic or simultaneous designs that start the overall sequence of designs.

Remark(s) 5.11

Recall that the adopted models relating advices to the behavior of the guided o-system are of a speculative nature. They serve as the necessary departing point, but the model should be corrected through the use of the p-system: we should estimate model $f(d_t|a_{p;t}, d(t-1), \Theta)$ describing explicitly dependence of data d_t on the adopted advices $a_{p;t}$. For it, the use of the adaptive version of the p-system becomes highly desirable.

5.6.2 Overall scenario and design subtasks

The solution of the overall design problem includes a number of particular steps anticipated in the algorithm below. The algorithm serves us as a design guide. Its steps are discussed in Chapters 6 and 7 at the general level and specialized to specific mixture elements in subsequent Chapters. For fixed advisory system, all steps, except steps 16c and 17, are made in offline mode. For adaptive advisory system, steps 7, 14, 15 have to be run in online mode, too. Note that the recursively performed estimation step 7 is computationally cheap and redesigns 14, 15 can be performed with much slower rate without a significant harm.

Algorithm 5.2 (Design of the advisory system)

1. *Collect the learning data that have to reflect all modes of operating.*
2. *Select quality markers and express their desirable values as the true user's ideal pdf ${}^{\lfloor U}f(d_o(\mathring{t}))$.*
3. *Collect prior physical information on the managed system, operating and measuring conditions.*
4. *Preprocess the data available for learning by*

   a) *grouping data records according to the operator's perceiving and acting rates,*
   b) *reducing dimensionality using expert knowledge that should exclude surely irrelevant record entries,*
   c) *removing outliers,*
   d) *replacing missing data,*
   e) *suppressing high-frequency noise.*
5. *Select the largest acceptable structure of the estimated mixture.*
6. *Select a prior pdf initializing estimation of the mixture describing the o-system.*
7. *Estimate the mixture describing the o-system; if need be, go back to step 6 or even go to step 4.*
8. *Estimate structure of the mixture in its full hierarchy from factors to the whole mixture and, if need be, go back to step 6.*
9. *Reduce dimensionality of the problem by removing data that do not influence even indirectly quantities in* $^{\lfloor U}f(d_o(\mathring{t}))$ *and, if need be, go back to Steps 4 or 6.*
10. *Use physical prior information for individual factors and, if need be, go back to Step 6.*
11. *Validate quality of the obtained model using expert opinion as well as independent testing data. And, if the result is unsatisfactory, repeat whole learning procedure, possibly from Step 3.*
12. *Analyze individual components and qualify recommendable and dangerous ones.*
13. *Select basic advising scenario (academic, industrial or simultaneous).*
14. *Design the advising strategy.*
15. *Design presentation and signaling strategies.*
16. *Validate advising strategy by*
   a) *comparing real actions of the operator with those generated by advising system (without closing the advising loop),*
   b) *judging proximity of good modes in data with the behavior stimulated by a good operator,*
   c) *using advising system at the full scale.*
17. *Use the p-system in the fixed or adaptive mode by feeding the currently measured data into the advising mixture as well as into presentation and signaling strategies. Show the low-dimensional projection of the advising mixture to the operator. The shown quantities are selected by the presentation strategy and the call for o-actions is driven by the signaling strategy.*

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