2

Imperfect Maintenance and Dependence

Imperfect maintenance and dependence are major concerns of this book, as stated in Chapter 1. This chapter will present a detailed introduction to imperfect maintenance and dependence, and survey typical modeling methods, their characteristics and uses. Imperfect maintenance section of this chapter updates Pham and Wang (1996). Some modeling methods on imperfect maintenance and dependence will be used in the subsequent chapters in this book.

2.1 Imperfect Maintenance

Perfect maintenance assumes that the system is “as good as new” following maintenance. However, this assumption may not be true in practice. A more realistic assumption is that, upon maintenance, the system lies in a state somewhere between ‘as good as new’ and its pre-maintenance condition, *i.e.*, maintenance is imperfect, as mentioned in Chapter 1. Kay (1976), Ingle and Siewiorek (1977), Chaudhuri and Sahu (1977), and Chan and Downs (1978) are pioneers in imperfect maintenance study. Kay (1976) and Chan and Downs (1978) study the worst PM. Ingle and Siewiorek (1977) investigate imperfect maintenance. Chaudhuri and Sahu (1977) mention the concept of imperfect PM. An early work on imperfect repair can also be found in NAPS document No. 03476-A. In fact, NAPS document No. 03476-A plays a significant role in later imperfect maintenance research. Based on this work, other researchers have proposed some imperfect maintenance models.

In the existing imperfect maintenance literature, various methods for modeling imperfect maintenance have been used and most of them are on single-unit systems. It is necessary to summarize and compare these modeling methods because it will be helpful for later study in this area, especially for a multicomponent system which is the main concern in this book. It should be pointed out that although the existing literature is mainly on a single-unit system and the modeling methods will be summarized from it, they will also be useful for modeling a multi-component system. This is because individual subsystems can be regarded as single-unit systems, and thus the methods of treating imperfect maintenance for single-unit systems may also be effective for modeling imperfect maintenance of individual subsystems, based on which imperfect maintenance of a system will be investigated, possibly together with dependence (Pham and Wang 1996).

Methods for modeling imperfect, worse and worst maintenance can be classified into seven categories. Pham and Wang (1996) summarize these methods from related papers and technical reports throughout the literature, and these seven methods and some important results for them are presented next.

2.1.1 Modeling Methods for Imperfect Maintenance

2.1.1.1 Modeling Method 1 - (p,q) Rule

Nakagawa (1979) models imperfect PM in this way: after PM a unit is returned to the 'as good as new' state (perfect PM) with probability p and returned to the 'as bad as old' state (minimal PM) with probability $q = 1 - p$. Clearly, if $p = 1$ the PM coincides with perfect one and if $p = 0$ it corresponds to minimal PM. So in this sense, minimal and perfect maintenances are special cases of imperfect maintenance and imperfect maintenance is a general maintenance. Using such a study method for imperfect maintenance and assuming that PM is imperfect, Nakagawa (1979, 1980) succeeds in obtaining optimum PM policies minimizing the *s*-expected maintenance cost rate for one-unit system under age-dependent and periodic PM policies, respectively.

Similar to Nakagawa (1979a,b), Helvic (1980) states that, while the fault-tolerant system is usually renewed after PM with probability θ_2, its operating condition sometimes remains unchanged (as bad as old) with probability θ_1 where $\theta_1 + \theta_2 = 1$.

Brown and Proschan (1983) study the following model of the imperfect repair process. A unit is repaired each time it fails. The executed repair is either a perfect one with probability p or a minimal one with probability $1 - p$. Assuming that all repair actions take negligible time, they establish ageing preservation properties of this imperfect repair model and monotonicity of various parameters and random variables associated with the failure process. They obtain an important, useful result: if the life distribution of a unit is F and its failure rate is r, then the distribution function of the time between successive perfect repairs is $F_p = 1 - (1 - F)^p$ and the corresponding failure rate $r_p = pr$. Using this result, Fontenot and Proschan (1984), and Wang and Pham (1996b) obtain optimal imperfect maintenance policies for one-component system.

Later on, we will refer to this method for modeling imperfect maintenance as the (p,q) rule, that is, after maintenance (corrective or preventive) a system becomes "as good as new" with probability p and "as bad as old" with probability $1 - p$. In fact, this modeling method is getting popular: more and more imperfect maintenance models have used this rule in recent years.

Bhattacharjee (1987) obtains the same results as Brown and Proschan (1983), and some new results for Brown-Proschan model of imperfect repair via a shock model representation of the sojourn time.

Lim *et al.* (1998) extend the Brown and Proschan (1983) imperfect repair model, and propose a new Bayesian imperfect repair model where the probability of perfect repair, *P*, is considered to be a random variable. Assuming that *P* has a prior distribution II(*p*), they obtain the distribution of waiting times between two successive perfect repairs and its corresponding failure rate. Lim *et al.* (1998) discuss the posterior distribution of *P* and its estimators, and study some preservation properties for certain nonparametric classes of life distributions and the monotonicity properties for several parameters. Cha and Kim (2001) model Bayesian availability where *P* is not fixed but a random variable with a prior distribution.

Li and Shaked (2003) equip the Brown and Proschan (1983) imperfect repair model with PM, and obtain stochastic maintenance comparisons for the numbers of failures under different policies via a point-process approach. They also obtain some results involving stochastic monotonicity properties of these models with respect to the unplanned complete repair probability.

2.1.1.2 Modeling Method 2 - $(p(t), q(t))$ Rule

Block *et al.* (1985) extend the above Brown-Proschan imperfect repair model with the (p,q) rule to the age-dependent imperfect repair for one-unit system: an item is repaired at failure (corrective maintenance). With probability $p(t)$, the repair is a perfect repair; with probability $q(t) = 1 - p(t)$, the repair is a minimal one, where t is the age of the item in use (the time since the last perfect repair). Block *et al.* (1985) prove that if the item's life distribution F is a continuous function and its failure rate is r, the successive perfect repair times form a renewal process with interarrival time distribution

$$F_p = 1 - \exp\left\{\int_0^t p(x)[1 - F(x)]^{-1} F(dx)\right\}$$

and the corresponding failure rate $r_p(t) = p(t)r(t)$. In fact, similar results can be found in Beichelt and Fischer (1980), and NAPS Document No. 03476-A. Block *et al.* (1985) prove that the ageing preservation results of Brown and Proschan (1983) hold under suitable hypotheses on $p(t)$. Later on, we will call this imperfect maintenance modeling method as the $(p(t), q(t))$ rule.

Using this $(p(t), q(t))$ rule, Block *et al.* (1988) investigate a general age-dependent PM policy, where an operating unit is replaced when it reaches age T; if it fails at age $y < T$, it is either replaced by a new unit with probability $p(t)$, or it undergoes minimal repair with probability $q(t) = 1 - p(t)$. The cost of the i^{th} minimal repair is a function, $c_i(y)$, of age y and number of repairs. After a perfect maintenance, planned or unplanned (preventive), the procedure is repeated.

Both Brown and Proschan (1983) model and Block *et al.* (1985) model assume that the repair time is negligible. It is worthwhile to mention that Iyer (1992) obtains availability results for imperfect repair using the $(p(t), q(t))$ rule given that the repair time is not negligible. His realistic treatment method will be helpful for later research.

Sumita and Shanthikumar (1988) propose and study an age-dependent counting process generated from a renewal process and apply that counting process to the age-dependent imperfect repair for the one-unit system.

Whitaker and Samaniego (1989) propose an estimator for the life distribution when the above model by Block *et al.* (1985) is observed until the time of the m^{th} perfect repair. This estimator was motivated by a nonparametric maximum likelihood approach, and was shown to be a 'neighborhood MLE'. They derive large-sample results for this estimator. Hollander *et al.* (1992) take the more modern approach of using counting process and martingale theory to analyze these models. Their methods yield extensions of Whitaker and Samaniego's results to the whole line and provide a useful framework for further work on the minimal repair model.

The (p,q) rule and $(p(t),q(t))$ rule for imperfect maintenance seem practical and realistic. It makes imperfect maintenance be somewhere between perfect and minimal ones. The *degree* to which the operating conditions of an item is restored by maintenance can be measured by p or $p(t)$. Especially, in the $(p(t),q(t))$ rule, the *degree* to which the operating condition of an item is restored by maintenance is related to its age t. Thus, the $(p(t),q(t))$ rule seems more realistic but mathematical modeling of imperfect maintenance by using it will be more complicated. The two rules can be expected to be powerful in future imperfect maintenance modeling. In fact, both rules have received much attention and have been used in some imperfect repair models, as shown in the subsequent chapters.

Makis and Jardine (1992) consider a general treatment method for imperfect maintenance and model imperfect repair at failure in a way that repair returns a system to the "as good as new" state with probability $p(n,t)$ or to the "as bad as old" state with probability $q(n,t)$, or with probability $s(n,t)=1-p(n,t)-q(n,t)$ the repair is unsuccessful, the system is scrapped and replaced by a new one, where t is the age of the system and n is the number of failures since replacement. We will refer to this treatment method as $(p(n,t),q(n,t),s(n,t))$ rule later.

2.1.1.3 Modeling Method 3 - Improvement Factor Method

Malik (1979) introduces the concept of improvement factor in the maintenance scheduling problem. He believes that maintenance changes the system time of the failure rate curve to some newer time but not all the way to zero (not new), as shown in Figure 2.1 This treatment method for imperfect maintenance also makes the failure rate after PM lie between 'as good as new' and 'as bad as old'. The degree of improvement in failure rate is called improvement factor. Malik (1979) assumes that since systems need more frequent maintenance with increased age the successive PM intervals are decreasing in order to keep the system failure rate at or below a stated level (sequential PM policy), and proposes an algorithm to determine these successive PM intervals. Lie and Chun (1986) present a general expression to determine these PM intervals. Malik (1979) relies on an expert judgment to estimate the improvement factor, while Lie and Chun (1986) give a set

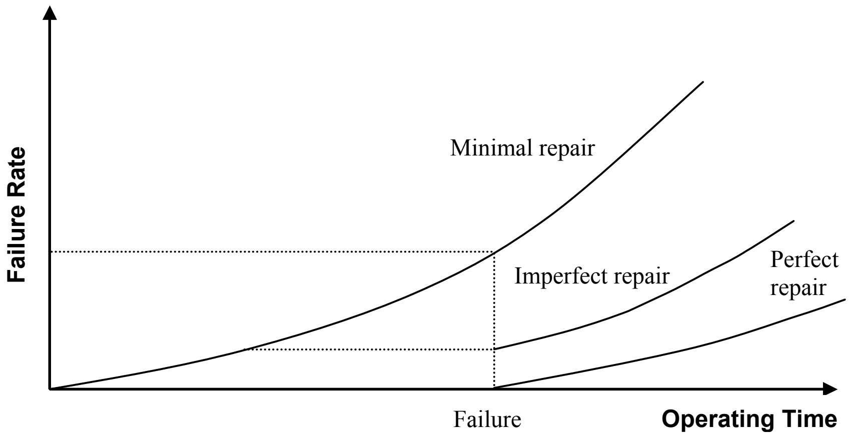

Figure 2.1. Minimal, perfect, imperfect repair *vs*. failure rate changes

of curves as a function of maintenance cost and the age of the system for the improvement factor.

Using the improvement factor and assuming finite planning horizon, Jayabalan and Chaudhuri (1992b) introduce a branching algorithm to minimize the average total cost for a maintenance scheduling model with assured reliability and they (1992c) discuss optimal maintenance policy for a system with increased mean down time and assured failure rate. It is worthwhile to note that using fuzzy set theory and improvement factor, Suresh and Chaudhuri (1994) establish a PM scheduling procedure to assure an acceptable reliability level or tolerable failure rate assuming finite planning horizon. They regard the starting condition, ending condition, operating condition, and type of maintenance of a system as fuzzy sets. Improvement factor is used to find out the starting condition of the system after maintenance.

Chan and Shaw (1993) think that failure rate is reduced after each PM and this reduction of failure rate depends on the item age and the number of PMs. Chan and Shaw propose two types of failure-rate reduction: (1) failure-rate with fixed reduction. After each PM, the failure rate is reduced such that all jump-downs of the failure rate are the same; (2) failure rate with proportional reduction. After PM, the failure rate is reduced such that each jump-down is proportional to the current failure rate. They obtain cycle-availability for single unit system and discuss the design scheme to maximize the probability of achieving a specified stochastic-cycle availability with respect to the duration of the operating interval between PMs .

In Doyen and Gaudoin (2004), the (conditional) failure intensity before the first repair is a continuous function of time. The repair effect is characterized by the change induced on the failure intensity before and after failure. Repair effect is expressed by a reduction of failure intensity. Several cases are studied, which take into account the possibility of a Markovian memory property.

This kind of study method for imperfect maintenance is in terms of failure rate and seems useful and practical in engineering where it can be used as a general treatment method for imperfect maintenance or even *worse* maintenance. Later on we call this treatment method *Improvement Factor Method.*

Besides, Canfield (1986) assumes that PM at time t restores the failure rate function to its shape at $t-\tau$, while the level remains unchanged where τ is less than or equal to the PM intervention interval.

2.1.1.4 Modeling Method 4 - Virtual Age Method

Kijima *et al.* (1988) develop an imperfect repair model by using the idea of the virtual age process of a repairable system. If the system has the virtual age $V_{n-1}=y$ immediately after the $(n-1)^{\text{th}}$ repair, the n^{th} failure-time X_n is assumed to have the distribution function

$$\Pr\{X_n \le x \mid V_{n-1}=y\}=\frac{F(x+y)-F(y)}{1-F(y)}$$

where $F(x)$ is the distribution function of the time to failure of a new system. Let a be the degree of the n^{th} repair where $0 \le a \le 1$. They construct such a repair model: the n^{th} repair cannot remove the damage incurred before the $(n\text{-}1)^{\text{th}}$ repair. It reduces the additional age X_n to aX_n. Accordingly, the virtual age after the n^{th} repair becomes:

$$V_n = V_{n-1}+aX_n$$

Obviously, $a=0$ corresponds to a perfect repair while $a=1$ to a minimal repair. Later Kijima (1989) extends the above model to the case that a is a random variable taking a value between 0 and 1 and proposes another imperfect repair model:

$$V_n = A_n(V_{n-1}+X_n)$$

where A_n is a random variable taking a value between 0 and 1 for $n=1,2,3,...$ For the extreme values 0 and 1, $A_n=1$ means a minimal repair and $A_n=0$ a perfect repair. Comparing this treatment method with Brown and Proschan's, we can see that if A_n is independently and identically distributed (*i.i.d.*) taking the two extreme values 0 and 1 they are the same. Therefore, the second treatment method by Kijima (1989) is general. He derives various monotonicity properties associated with the above two models.

In Doyen and Gaudoin (2004), repair effect is expressed by a reduction of the system virtual age. Several cases are studied, which take into account the possibility of a Markovian memory property.

This treatment method will be referred to as the *Virtual Age Method* later on.

It is worth mentioning that Uematsu and Nishida (1987) consider a more general model including the above two models by Kijima (1989) as special cases and obtain some elementary properties of the associated failure process. Let T_n

denote the time interval between the (n-1)th failure and the n^{th} one, and X_n denote the degree of repair. After performing the n^{th} repair, the age of the system becomes $q(t_1,...,t_n;x_1,...x_n)$ given that $T_i = t_i$ and $X_i = x_i$ ($i = 1,2,...n$) where T_i and X_i are random variables. On the other hand, $q(t_1,...,t_n;x_1,...x_{n-1})$ represents the age of the system as just before the n^{th} failure. The starting epoch of an interval is subject to the influence of all previous failure history, *i.e.*, the n^{th} interval is statistically dependent on $T_1 = t_1$,...., $T_{n-1} = t_{n-1}$, $X_1 = x_1$,..., $T_{n-1} = t_{n-1}$. For example, if $q(t_1,...,t_n;x_1,...x_n) = \sum_{j=1}^{n} \sum_{i=j}^{n} x_i t_j$, then $X_i = 0$ ($X_i = 1$) represents that perfect repair (minimal repair) performs at the i^{th} failure.

2.1.1.5 Modeling Method 5 - Shock Model

It is well-known that the time to failure of a unit can be represented as a first passage time to a threshold for an appropriate stochastic process that describes the levels of damage. Consider a unit which is subject to shocks occurring randomly in time. At time $t = 0$, the damage level of the unit is assumed to be 0. Upon occurrence of a shock, the unit suffers a non-negative random damage. Each damage, at the time of its occurrence, adds to the current damage level of the unit, and between shocks, the damage level stays constant. The unit fails when its accumulated damage first exceeds a specified level. To keep the unit in an acceptable operating condition, some PM is performed (Kijima and Nakagawa 1991).

Kijima and Nakagawa (1991) propose a cumulative damage shock model with imperfect periodic PM. The PM is imperfect in the sense that each PM reduces the damage level by $100(1-b)\%$, $0 \le b \le 1$, of total damage. Note that if $b = 1$ the PM is minimal and if $b = 0$ the PM coincides with a perfect PM. This research approach is similar to the one in treatment method 1. They derive a sufficient condition for the time to failure to have an IFR distribution and discuss the problem of finding the number of PMs that minimizes the expected maintenance cost rate.

Kijima and Nakagawa (1992) establish a cumulative damage shock model with a sequential PM policy assuming that PM is imperfect. They model imperfect PM in a way that the amount of damage after the k^{th} PM becomes $b_k Y_k$ when it was Y_k before PM, *i.e.*, the k^{th} PM reduces the amount Y_k of damage to $b_k Y_k$ where b_k is called the improvement factor. They assume that a system is subject to shocks occurring according to a Poisson Process and, upon occurrence of shocks, it suffers a non-negative random damage which is additive. Each shock causes a system failure with probability $p(z)$ when the total damage is z at the shock. In this model, PM is done at fixed intervals x_k for $k = 1,2,3,...,N$ because more frequent maintenance is needed with age, and the N^{th} PM is perfect. If the system fails between PMs it undergoes only minimal repair. They derive the expected maintenance cost rate until replacement assuming that $p(z)$ is an exponential function and damage is independently and identically distributed and discuss the optimal replacement policies.

Finkelstein (1997) investigates the performance of a repairable system subject to shocks: each shock with a probability that depends on a virtual age of the effected system causes a breakdown that ends the process of system functioning. In Finkelstein (1997), various models of repair, ranging from minimal till perfect repair are studied. It is assumed that shocks occur according to the non-homogeneous Poisson process or a renewal process with identically distributed cycles. The probability of a system functioning without breakdowns for the mentioned models is derived.

This study approach for imperfect maintenance will be called *Shock Model* later in this book.

2.1.1.6 Modeling Method 6 - Quasi-renewal Process

Wang and Pham (1996b) treat imperfect repair in a way that, upon each repair, the lifetime of a system will be reduced to a fraction α of its immediately previous one where $0 < \alpha < 1$ and all lifetimes are independent, *i.e.*, the lifetime decreases with the number of repairs. In Wang and Pham (1996b), the successive lifetimes are defined to constitute a decreasing quasi-renewal process, whose details are given in Chapter 4. Assuming that the *pdf* of the lifetime of a system which has been subject to $(n-1)$ repairs, X_n, is $f_n(x)$ for $n = 1,2,3,...$, Wang and Pham (1996b) study this quasi-renewal process and prove that:

a. If $f_1(x)$ belongs to IFR, DFR, IFRA, DFRA, and NBU (for definitions see Chapter 4 Appendix), then $f_n(x)$ is in the same category, $\forall n, n = 2,3,...$.

b. The shape parameter of X_n are the same for $n = 1,2,3,...$ for a quasi-renewal process if X_1 follows the Gamma, Weibull or Lognormal distribution.

The second result means that after "renewal" the shape parameters of the interarrival time will not change. In reliability theory, the shape parameters of a lifetime of an item tend to relate to its failure mechanism. Usually, if a product possesses the same failure mechanism then its lifetimes will have the same shape parameters at different application conditions. Because most maintenance does not change the failure mechanism we can expect that the lifetime of a system will have the same shape parameters. Thus, in this sense, the quasi-renewal process will be plausible to model the imperfect maintenance process.

Wang and Pham (1996c) further assume that repair time is non-negligible, not as in most imperfect maintenance models, and upon repair the next repair time becomes a multiple β of its current one where $\beta > 1$ and all repair times are independent, *i.e.*, the time to repair increases with the number of repairs. In Wang and Pham (1996b), the successive repair times are defined to form an increasing quasi-renewal process. This method modeling imperfect maintenance will be referred to as the *(α, β) rule* or *quasi-renewal process* method. Yang and Lin (2005) apply the quasi-renewal process in age and block PM.

In investigating the optimal replacement problem, Lam (1988) uses the fixed life reduction idea after repair, referred to as the geometric process. Lam (1988, 1996) studies the geometric process by means of the ordinary renewal process. In

Chapter 4, the quasi-renewal process is introduced from defining the quasi-renewal function.

2.1.1.7 Modeling Method 7 – Multiple (p,q) Rule

Shaked and Shanthikumar (1986) introduce the multivariate imperfect repair concept. They consider a system whose components have dependent lifetimes and are subject to imperfect repairs respectively until they are replaced. For each component the repair is imperfect according to the (p, q) rule, *i.e.*, at failure the repair is perfect with probability p and minimal with probability q. Assume that n components of the system start to function at the same time 0, and no more than one component can fail at a time. They establish the joint distribution of the times to next failure of the functioning devices after a minimal repair or perfect repair, and derive the joint density of the resulting lifetimes of the components and other probabilistic quantities of interest, from which the distribution of the lifetime of the system can be obtained. Sheu and Griffith (1992) further extend this work. Later we will call this treatment method the *multiple* (p,q) *rule*.

2.1.1.8 Others

Nakagawa (1979b) models imperfect PM in a way that, in the steady-state, PM reduces the failure rate of an item to a fraction of its value just before PM and during operation of the item the failure rate climbs back up. He believes that the portion by which the failure rate is reduced is a function of some resource consumed in PM and a parameter. That is, after PM the failure rate of the unit becomes $\lambda(t) = g(c_1,\theta)\cdot\lambda(t+T)$ where the fraction reduction of failure rate $g(c_1,\theta)$ lies between 0 and 1, T is the time interval length between PMs, c_1 is the amount of resource consumed in PM, and θ is a parameter. This treatment method is different from the improvement factor method in that, for improvement factor method, maintenance makes the system younger in terms of its age, *i.e.*, its age becomes younger after maintenance.

Nakagawa (1986, 1988) uses two other methods to deal with imperfect PM for two sequential PM policies: (1) The failure rate after PM k becomes $a_k h(t)$ given that it was $h(t)$ in the previous period where $a_k \geq 1$. That is, the failure rate increases with the number of PMs; (2) The age, after PM k, reduces to $b_k t$ when it was t before PM where $0 \leq b_k < 1$. That is, PM reduces the age. Obviously, the second method is similar to the improvement factor method. Besides, in investigating periodic PM models, Nakagawa (1980) treats imperfect PM in that the age of the unit becomes x units of time younger by each PM and further suggests that x is in proportion to the PM cost where x is less than or equal to the PM interval. We will call it the *x Rule* later.

Nguyen and Murthy (1981) model imperfect PM in a way that, after PM, the unit has a different (worse) failure time distribution than after CM. Yak (1984) assumes that maintenance may result in its failure (the worst maintenance) in modeling the MTTF and the availability of a system.

Some typical work on imperfect maintenance are summarized in Table 2.1 by modeling methods. From this table we can see that the (p,q) rule and $(p(t),q(t))$

rule are popular in treating imperfect maintenance. This is partly because these two rules make imperfect maintenance modeling mathematically tractable, as demonstrated in Chapters 6 – 8.

Table 2.1. Summary of treatment methods for imperfect maintenance

Modeling method	References
(p,q) Rule	Chan and Downs (78), Helvic (80), Nakagawa (79, 80, 87), Brown and Proschan (82, 83), Fontenot and Proschan (84), Lie and Chun (86), Yun and Bai (87), Bhattacharjee (87), Rangan and Grace (89), Sheu and Liou (92), Srivastava and Wu (93), Wang and Pham (96a,b,c, 97b), Lim *et al.* (98), Pham and Wang (00), Cha and Kim (01), Kvam *et al.* (02), Li and Shaked (03)
$(p(t),q(t))$ Rule	Beichelt (80, 81), Block *et al.* (85, 88), Abdel-Hameed (87a), Whitaker and Samaniego (89), Sheu (91a, 92, 93), Makis and Jardine (91), Iyer (92), Hollander *et al.* (92), Sheu and Kuo (94), Sheu *et al.* (95), Wang *et al.* (01), Wang and Pham (99, 03)
Improvement factor	Malik (79), Canfield (86), Lie and Chun (86), Jayabalan and Chaudhuri (92a, b,c, 95), Chan and Shaw (93), Suresh and Chaudhuri (94), Doyen and Gaudoin (04)
Virtual age	Uematsu and Nishida (87), Kijima (88, 89), Makis and Jardine (93), Liu *et al.* (95), Gasmi *et al.* (03), Doyen and Gaudoin (04)
Shock model	Bhattacharjee (87), Kijima and Nakagawa (91,92), Sheu and Liou (92c), Finkelstein (97)
(α,β) Rule or quasi-renewal process	Lam (88, 96), Wang and Pham (96a,b,c, 97b, 99, 06), Pham and Wang (00, 01), Yang and Lin (05), Wu and Clements-Croome (05), Bai and Pham (06a)
Multiple (p,q) rule	Shaked and Shanthikumar (86), Sheu and Griffith (92)
Others	Nakagawa (79b, 80, 86, 88), Subramanian and Natarajan (80), Nguyen and Murthy (81), Yak (84), Yun and Bai (88), Dias (90), Subramanian and Natarajan (90), Zheng and Fard (91), Jack (91), Chun (92), Dagpunar and Jack (94)

The following further work on imperfect maintenance is necessary:

- Study optimal maintenance policy for multicomponent systems because previous work on imperfect maintenance was focused on one-unit systems.
- Construct statistical estimation methods for parameters of various imperfect maintenance models.
- Develop more and better methods for treating imperfect maintenance.
- Study more realistic imperfect maintenance models, for example, including non-negligible repair time, finite horizon.
- Use the reliability measures as the optimality criteria for maintenance policies instead of cost rates, or combine both, as stated in Section 1.5.1.

2.1.2 Typical Imperfect Maintenance Models by Maintenance Policies

2.1.2.1 Age-dependent PM Policy

In the age-dependent PM model, a unit is preventively maintained at predetermined age T, or repaired at failure, whichever comes first. For this policy there are various imperfect maintenance models according to the conditions that either or both of PM and CM is imperfect. The research under the age-dependent PM policy and its extensions is summarized in Table 2.2.

Table 2.2. Imperfect maintenance study under age-dependent PM policy

Study	PM	CM	Treatment method	Optimality criteria	Modeling tool	Planning horizon
Chan and Downs (1978)	Imperfect	Perfect	(p,q) rule	Availability cost rate	Semi-Markov	Infinite
Nakagawa (1979a)	Imperfect	Perfect	(p,q) rule	Cost rate	Renewal theory	Infinite
Beichelt (1980)	Perfect	Imperfect	$(p(t),q(t))$ rule	Cost rate	Renewal theory	Infinite
Fontenot and Proschan (1984)	Perfect	Imperfect	(p,q) rule	Cost rate	Renewal theory	Infinite
Block *et al.* (1988)	Perfect	Imperfect	$(p(t),q(t))$ rule	Cost rate total cost	Renewal theory	Infinite finite
Rangan and Grace (1989)	Perfect	Imperfect	(p,q) rule	Total cost	Renewal theory	Finite
Sheu (1991a)	Perfect	Imperfect	$(p(t),q(t))$ rule	Cost rate (random cost)	Renewal theory	Infinite finite
Sheu and Kuo (1993)	Perfect	Imperfect	$(p(t),q(t))$ rule	Cost rate (random cost)	Renewal theory	Infinite
Sheu *et al.* (1995)	Perfect	Imperfect	$(p(t),q(t))$ rule	Cost rate (random cost)	Renewal theory	Infinite
Wang and Pham (1996a)	Imperfect	Imperfect	(p,q) rule (α,β) rule	Cost rate Availability	Renewal theory	Infinite

One of the pioneer imperfect maintenance models for the age-dependent PM policy is due to Nakagawa (1979a) and NAPS Document No. 03476-A. Nakagawa (1979a) investigates three age-dependent PM models with imperfect PM and perfect or minimal repair at failure using the (p, q) rule. He derives the expected maintenance cost rate and discusses the optimal maintenance policies in terms of PM time interval T.

Using the $(p(t),q(t))$ rule, Block *et al.* (1988) discuss an age-dependent PM policy where CM is imperfect and the cost of the i^{th} minimal repair is a function, $c_i(y)$, of age and number of repairs. Sheu *et al.* (1993) generalized the age-dependent PM policy where if a system fails at age $y < t$, it is subject to perfect repair with $p(y)$, or undergoes minimal repair with probability $q(y) = 1 - p(y)$.

Otherwise, a system is replaced when the first failure after t occurs or the total operating time reaches age T ($0 \le t \le T$), whichever occurs first. They discussed the optimal policy (t^*, T^*) to minimize the expected cost rate. This is a realistic PM model. Sheu *et al.* (1995) further extend this model. They assume that a system has two types of failures when it fails at age z and is replaced at the n^{th} type 1 failure or first type 2 failure or at age T, whichever occurs first. Type 1 failure occurs with probability $p(z)$ and is corrected by minimal repair. Type 2 failure occurs with probability $q(z) = 1 - p(z)$ and is corrected by perfect repair (replacement). Using the $(p(t), q(t))$ rule and random minimal repair costs, they derive the expected cost rate and a numerical example is presented.

Table 2.3. Imperfect maintenance study under periodic PM policy

Study	PM	CM	Treatment method	Optimality criterion	Modeling tool	Planning horizon
Nakagawa (1979)	Imperfect	Minimal	(p,q) rule	Cost rate	Renewal theory	Infinite
Nakagawa (1980)	Imperfect	Perfect minimal	(p,q) rule x rule	Cost rate	Renewal theory	Infinite
Beichelt (1981a, b)	Perfect	Imperfect	$p(t), q(t)$	Cost rate	Renewal theory	Infinite
Fontenot and Proschan (1984)	Perfect	Imperfect	(p,q) rule	Cost rate	Renewal theory	Infinite
Nakagawa (1986)	Imperfect	Minimal	Different failure rates	Cost rate	Renewal theory	Infinite
Abdel-Hameed (1987a)	Perfect	Imperfect	$p(t), q(t)$	Cost rate	Stochastic process	Infinite
Nakagawa and Yasui (1987)	Imperfect	Perfect	(p,q) rule	Availability	Renewal theory	Infinite
Kijima *et al.* (1988)	Perfect	Imperfect	Virtual age	Cost rate	Renewal theory	Infinite
Kijima and Nakagawa (1991)	Imperfect	Perfect	Shock model	Cost rate	Renewal theory	Infinite
Jack (1991)	Perfect	Imperfect	Others	Total cost	Renewal theory	Finite
Chun (1992)	Imperfect	Minimal	x rule	Total cost	Probability	Finite
Sheu (1992)	Perfect	Imperfect	$p(t), q(t)$	Cost rate	Renewal theory	Infinite
Liu *et al.* (1995)	Imperfect	Minimal	Virtual age	Cost rate	Renewal theory	Infinite
Wang and Pham (1996c)	Imperfect	Imperfect	(p,q) rule (α,β) rule	Cost rate Availability	Quasi-renewal theory	Infinite
Wang and Pham (1999)	Imperfect	Imperfect	$p(t), q(t)$	Cost rate Availability	Renewal theory	Infinite

2.1.2.2 Periodic PM Policy

In the periodic PM policy, a unit is preventively maintained at fixed time intervals and repaired at intervening failures. Liu *et al.* (1995) investigate an extended periodical PM model using the notation of the virtual age. They assume that a unit receives (imperfect) PM every T time unit, intervening failures are subject to minimal repairs and the unit is replaced every fixed number of PMs. Nakagawa (1986) studies a similar model but he assumes that PM is imperfect in the sense that after PM the failure rate will be changed. The research under the periodic PM policy and its extensions are summarized in Table 2.3.

Table 2.4. Imperfect maintenance study under failure limit policy

Study	PM	CM	Measure improved	Optimality criterion	Modeling tool	Planning horizon
Malik (1979)	Imperfect	None	Reliability	Reliability	Probability	Infinite
Canfield (1986)	Imperfect	None	Failure rate	Cost rate	Renewal theory	Infinite
Lie and Chun (1986)	Imperfect	Imperfect	Failure rate	Cost rate	Renewal theory	Infinite
Jayabalan (1992a)	Imperfect	Minimal	Failure rate	Total cost	Probability	Finite
Jayabalan and Chaudhuri (1992c)	Imperfect	Minimal	Age Others	Cost rate	Probability	Infinite
Jayabalan and Chaudhuri (1992d)	Imperfect	None	Age	Total cost	Probability	Finite
Chan and Shaw (1993)	Imperfect	Perfect	Failure rate	Availability	Probability	Infinite
Suresh and Chaudhuri (94)	Imperfect		Reliability and failure rate	Total cost	Probability	Finite
Jayabalan and Chaudhuri (1995)	Imperfect	Minimal	Age	Total cost	Renewal theory	Finite
Monga *et al.* (1996)	Imperfect	Minimal	Reduction (age and failure rate)	Cost rate	Renewal theory	Infinite

2.1.2.3 Failure Limit Policy

This policy assumes that PM is performed only when the failure rate or reliability of a unit reaches a predetermined level. Malik (1979) derives the PM schedule points so that a unit works at or above the minimum acceptable level of reliability. Lie and Chun (1986) formulate a maintenance cost model where PM is performed whenever the unit reaches the predetermined maximum failure rate. Jayabalan and Chaudhuri (1992a) obtain the optimal maintenance policy for a specific period of time given that downtime for installation and for PM are negligible. In other work, Jayabalan and Chaudhuri (1992b) consider downtime for replacement as a nonzero constant. As a unit ages, the successive downtime for PM interventions is expected to consume more time. To incorporate this point, Jayabalan and Chaudhuri (1992b) assume that PM time follows exponential distribution and is increasing with age. Jayabalan and Chaudhuri (1995) present an algorithm to obtain optimal maintenance policies which require less computational time. The research under the failure limit policy and its extensions is summarized in Table 2.4.

2.1.2.4 Sequential PM Policy

When a system is maintained at unequal intervals, the PM policy is known as *Sequential PM policy*. Nakagawa (1986,1988) discusses a sequential PM policy where PM is done at fixed intervals x_k where $x_k \le x_{k-1}$ for $k = 2,3...$ This policy is practical because most units need more frequent maintenance with increased age. This PM policy is different from the failure limit policy in that it controls x_k lengths directly but the failure limit policy controls failure rate, age, reliability, *etc*., directly. In Wu and Clements-Croome (2005), the PM is sequentially executed with τ_n time units after the $(n-1)^{\text{th}}$ PM, where $n = 1,2,...$ Between two adjacent PMs, a CM is carried out immediately on failure. Both PM and CM are imperfect. τ_n is dependent on n and determined through minimizing the maintenance cost rate. The research under the sequential PM policy and its extensions is summarized in Table 2.5.

Table 2.5. Sequential PM policy

Study	PM	CM	Treatment	Optimality criterion	Modeling tool	Planning horizon
Nakagawa (1986)	Imperfect	Minimal	Different failure rates	Cost rate	Renewal theory	Infinite
Nakagawa (1987)	Imperfect	Minimal	Reduction (age and failure rate)	Cost rate	Renewal theory	Infinite
Kijima and Nakagawa (1992)	Imperfect	Minimal	Shock model	Cost rate	Renewal theory	Infinite
Monga *et al.* (1996)	Imperfect	Minimal	Reduction (age and failure rate)	Cost rate	Renewal theory	Infinite
Wu and Clements-Croome (2005)	Imperfect	Imperfect	(α,β) rule	Cost rate	Renewal theory	Infinite

2.1.2.5 Repair Limit Policy

When a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit; otherwise, the unit is replaced. This is called the *Repair Cost Limit Policy* in the literature. Yun and Bai (1987) study the optimal repair cost limit policies under an imperfect maintenance assumption.

The *Repair Time Limit Policy* is proposed by Nakagawa and Osaki (see Nguyen and Murthy 1981) in which a unit is repaired at failure: if the repair is not completed within a specified time *T*, it is replaced by a new one; otherwise the repaired unit is put into operation again where *T* is called the *repair limit time*. Nguyen and Murthy (1981) study the repair time limit replacement policies with imperfect repair in which there are two types of repair – local and central repair. The local repair is imperfect while the central repair is perfect. The optimal policies are derived to minimize the expected cost rate for an infinite time span. The research under the repair limit policy and its extensions is summarized in Table 2.6.

Table 2.6. Imperfect maintenance study under repair limit policy

Study	CM before cost limit	CM after cost limit	Treatment method	Optimality criterion	Modeling tool	Planning horizon
Beichelt (1978, 1981b)	Minimal	Perfect	$(p(t), q(t))$	Cost rate	Renewal theory	Infinite
Nguyen and Murthy (1981)	Imperfect	Perfect	Others	Cost rate	Renewal theory	Infinite
Yun and Bai (1987)	Imperfect	Perfect	(p,q) rule	Cost rate	Renewal theory	Infinite
Yun and Bai (1988)	Minimal	Perfect	Others	Cost rate	Renewal theory	Infinite
Wang and Pham (1996c)	Imperfect	Imperfect	(p,q) rule/ (α,β) rule	Cost rate Availability	Quasi-renewal theory	Infinite

2.1.2.6 Multicomponent Systems

Imperfect maintenance models for multi-unit systems are summarized in Table 2.7. For series system Zhao (1994) establishes a series system availability model in which either minimal repair or perfect repair of all components can be modeled based on Barlow and Proschan's work (1975). He assumes that the repaired component might not be as good as new and its lifetime may follow any distribution which can be different from that of old one after repair and obtain mean limiting availability and mean system down and up time. In this model of series system, repair time is not negligible and thus it is practical. This treatment method for imperfect repair is similar to the one by Subramanian and Natarajan (1980). Besides, Sheu *et al*. have done some work on this problem. The related research is summarized in Table 2.7.

2.1.2.7 Others

Jack (1991) investigates a maintenance policy involving imperfect repairs on failure with replacement upon the N^{th} failure. Dagpunar and Jack (1994) determine the optimal number of imperfect PM during a finite horizon given that the minimal repairs are made at any failures between PMs and the i^{th} PM makes the age of a unit x_i units of time younger (*x rule*). Chun (1992) studies determination of the optimal number of periodic PMs under a finite planning horizon using the *x rule*.

Makis and Jardine (1992) contemplate a replacement policy without PMs in which a unit can be replaced at any time at a cost c_0, and at the n^{th} failure the unit is either replaced at the cost c_0 or undergoes an imperfect repair at a cost $c(n,t)$ where t is the age of the unit. They use the $(p(n,t), q(n,t), s(n,t))$ rule to model imperfect repair. Makis and Jardine (1991, 1993) discuss the optimal replacement policy with imperfect repair at failure: a unit is replaced each time at the first failure after some fixed time using the ($p(t), q(t), s(t)$) rule and the virtual age method, respectively.

Table 2.7. Multicomponent systems subject to imperfect maintenance

Study	PM	CM	Treatment	Optimality criterion	Modeling tool	Horizon / architecture / policy
Shaked and Shanthikumar (1986)	None	Imperfect	Multiple (p,q) rule	None	Renewal	Infinite / Arbitrary/
Subramanian and Natarajan (1990)	None	Imperfect	Other	Reliability Availability	Stochastic process	Infinite / Two-unit standby/
Zheng and Fard (1991)	Perfect	Imperfect	Other	Cost rate	Probability	Infinite / Arbitrary / Failure limit
Sheu and Griffith (1991b)	None	Imperfect	Multiple $(p(t),q(t))$	None	Renewal theory	Infinite / Arbitrary / Age-dependent
Sheu and Liou (1992c)	Perfect	Imperfect	$(p_1(t),\dots,p_n(t))$	Cost rate	NHPP	Infinite / *k*-out-of-*n* / Age-dependent
Zhao (1994)	None	Imperfect	Other	Availability	Probability	Infinite/ Series
Sheu and Kuo (1994)	Perfect	Imperfect	$(p(t),q(t))$	Cost rate (random cost)	Renewal theory	Infinite / *k*-out-of-*n* / Age-dependent
Wang and Pham (2006)	None	Imperfect	(α,β)	Availability	Quasi-renewal theory	Infinite / Series /
Wang (1997)	Perfect	Imperfect	$(p(t),q(t))$	Availability Cost rate	Renewal theory	Infinite / Arbitrary / Age-dependent
Pham and Wang (2000)	Im-perfect	Imperfect	(p,q) rule	Availability cost rate	Renewal theory	Infinite / *k*-out-of-*n* / Periodic
Wang *et al.* 2001)	Im-perfect	Perfect	$(p(t),q(t))$	Availability Cost rate	Renewal theory	Infinite / Arbitrary / Age-dependent

Block *et al.* (1992) introduce a generalized age replacement policy – repair replacement policy where units are preventively maintained when a certain time has elapsed since their last repair. If the last repair was a perfect repair, this policy is essentially the same as an age replacement policy.

Srivastava and Wu (1993) present an imperfect-inspection model in which failures can only be detected with probability p, and they discuss the estimation of parameter p. Sheppard (1983) and Nicolescu (1985) study imperfect testing which may result in reduction of availability of a unit. Ebrahimi (1985) derives the mean time to keep a failure-free operation with imperfect repair which either restore a unit to "as good as new" or to "as bad as old". Guo and Love (1992), and Love and Guo (1993) contemplate the statistical analysis for imperfect repair models.

Fontenot and Proschan (1984) explore four imperfect maintenance models using the (p,q) rule under various maintenance policies. Helvic (1980) invest-tigates maintenance of the fault-tolerant system using the (p,q) rule. Besides, Abdel-Hameed (1987b, 1995), Makis and Jardine (1992), Murthy(1991), Nguyen and Murthy (1981), Natvig (1990), and Zheng and Fard (1991) also discuss the imperfect repair problem.

2.2 Dependence

There are three kinds of dependencies: Economic Dependence, Correlated Failures and Repairs, and Failure Dependence, as stated in Chapter 1. McCall (1963), and Radner and Jorgenson (1963) address the economic dependence in a system with n components. Ozekici (1988) studies the effects of failure and economic dependencies on periodic replacement policies and provide useful characterizations of the optimal replacement policy.

Goel *et al.* (1992, 1993, 1996) investigate the *correlated failure and repair* for some repairable systems: a two-unit standby system, two unit priority redundant system, warm standby system, two-unit cold standby system, two server two unit cold standby system, *etc*. He uses a bivariate exponential distribution to model the joint distribution of failure and repair times of components. Gupta (1999) discusses profit analysis of a two non-identical unit cold standby system with correlated failure and repair and switchover.

Harris (1968) utilizes a bivariate exponential to describe *correlated failures* of two components and derives the mean time to system failure by using the supplementary variable technique for an arbitrary repair time distribution. Osaki (1970) extends this analysis to obtain the availability of the system by using a variant of a semi-Markov process with some non-regeneration points. Pijinenburg (1993) obtains reliability, mean time to system failure, pointwise and steady-state availability and joint availability and interval reliability using the imbedded renewal process. Shaked and Shanthikumar (1986), and Sheu and Griffith (1992) model failure dependence in a system with n components using the joint distribution of the lifetimes of n components.

Albin *et al.* (1992) investigates the PM policies for the series system with failure dependence and economic dependence using Markov chain and presents a brief summary of previous work on failure dependence. Nakagawa and Murthy (1993) consider a two-unit system with two kinds of failure dependence: when unit 1 fails, (1) unit 2 fails with probability α_j where j represents the j^{th} failure of unit 1 and (2) unit 1 causes damage with distribution $G(z)$ to unit 2, and the damage is cumulative and unit 2 fails once the total damage exceeds some specified level. Nakagawa and Murthy (1993) derive expected maintenance cost rates of the two models assuming that the system is replaced at failure of unit 2 or at the n^{th} failure of unit 1 and discuss the optimum maintenance policies. Murthy and Wilson (1994) consider the parameter estimation problem for failure dependence models.

Pham (1992) studies a high voltage system reliability with dependent failures and treats the failure dependence in the way that the failure of one component causes the failure rate of the other working component to increase.

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