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## Part I Green's Functions in Mathematical Physics

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**Part II Green's Functions in One-Body Quantum Problems**


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