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Introduction

Waves are everywhere around us. We rely on light and sound to sense our immediate surroundings. Radio waves and microwaves are indispensable means of communication. Water waves are responsible for the ocean's perpetually dynamic image. Quantum waves associated with electrons and atoms, while not directly visible, are important in maintaining the structure and stability of solids. With such a ubiquitous presence, wave phenomena naturally occupy a central position in our study of the physical world. Indeed, for waves in simple systems and ordered structures, an extensive literature already exists. However, for the more difficult problem of waves in disordered media, i.e., multiply scattered waves, a coherent (but by no means complete) understanding has only recently emerged, and from what is already known the picture is very different from that we normally associate with waves. In particular, the possibility that a wave can become localized in a random medium is especially intriguing because localization involves a change in the basic wave character. A localized wave has no spatial periodicity or possibility for transport and thus requires a new theoretical framework for its description and understanding. The purposes of this volume are to delineate the main features of this emerging picture of wave behavior in disordered media and to introduce the theoretical techniques for describing these features. Mesoscopic phenomena, which are the natural manifestations of wave scattering and interference effects, are also treated. A brief sketch below of the prominent random-wave characteristics serves as both an introduction to the subject and a map to what follows.

1.1 Relevant Length Scales

In an infinite, uniform medium, a (plane) wave may be characterized by a frequency and a direction of propagation. In contrast, a wave cannot propagate freely in a disordered medium because of the many scatterings it encounters. There are two types of scattering. One type, inelastic scattering, alters both the wave frequency and the propagation direction. Another type, elastic scattering, preserves the frequency but alters the propagation direction. This book is concerned mainly with the effect of elastic scattering. Accordingly, the term "incoherent" is defined to mean waves having different propagation directions but the same frequency.

The consequence of multiple elastic scatterings may be described in accordance with the scale of observation. There are two obvious yardsticks in the problem. One is the average size R of the inhomogeneous scatterers. If the density of scatterers is not too low, then the interscatterer separation is also on the same order as R. Another yardstick is the wavelength λ. The ratio between R and λ is an important parameter in determining the average distance of coherent propagation between two scatterings. That distance, usually called the mean free path, is the relevant length scale for separating the different regimes of wave phenomena. When $\lambda \gg R$, the scattering is weak and the mean free path is large ($\gg R$) for classical waves, i.e., electromagnetic and elastic waves. In addition, the scatterers and their placement geometry are beyond the resolution limit of the wave. Therefore, on the local scale of one to two mean free paths or less, a disordered medium appears as a *homogeneous effective medium* to the probing wave. In fact, since all matter is discrete at the atomic level, our everyday understanding of a uniform homogeneous medium reflects this effective medium concept. The same effective medium characterization holds for the quantum wave at the local scale. However, on the scale of many mean free paths, the effective medium can no longer be a valid description; even if locally the scattering is weak, over long distances the scattering effect accumulates and a wave can still be significantly randomized. When that happens, the result – *diffusive transport* – is similar to that of a classical particle undergoing random Brownian motion. The same result holds for the case of $\lambda \lesssim R$, except that the onset of diffusive transport occurs at a scale comparable to R, and there is no longer a valid effective medium because the local microstructure can now be clearly resolved by the wave.

1.2 Diffusive Transport

The fact that wave transport can be diffusive has a prominent example in our everyday experience of heat conduction. From statistical mechanics, it is well known that heat in electrically insulating solids is carried by randomly scattered (mostly short-wavelength) elastic waves, called phonons. Since heat conduction in solids is known to be governed by the diffusion equation even in the absence of inelastic scattering (Sheng and Zhou 1991), one immediately concludes that randomly scattered elastic waves transport diffusively.

Despite such clear-cut examples, however, diffusive transport for waves still raises some important questions concerning basic principles. A basic property of the wave equation is that of causality, which means that the speed at which a wave can carry information is always finite. For the diffusion equation, however, causality is not valid because the speed of propagation for a disturbance can be infinite. In addition, directly related to causality is the phase information of a wave, which is completely absent in diffusion. Just from this simple consideration it is already clear that the diffusion description of multiply scattered waves cannot be completely accurate since, whatever its appearance, the multiply scattered wave is still a solution of the wave equation (albeit with random coefficients) and therefore must satisfy the basic properties of its solution. Thus for wave diffusion there should be some deviation from classical diffusion where the wave character asserts itself. In the past decades, this expectation was borne out by the experimental demonstration (Tsang and Ishimaru 1984; van Albada and Lagendijk 1985; Wolf and Maret 1985) of the *coherent backscattering effect*, or the weak-localization effect as it is sometimes called, which represents not only a deviation from classical diffusion, but also the precursor to wave localization.

1.3 Coherent Backscattering and the Approach to Localization

The coherent backscattering effect means what the name implies: After a wave is multiply scattered many times, its phase coherence is preserved in the direction opposite to its incident direction (backscattering direction), but not in other directions. The reason for this behavior is fully explained in a later chapter, but we may appreciate some of its consequences here. By preserving the coherence in the backscattering direction, the probability of backscattering is enhanced through constructive interference. This leads to a decrease in the diffusion constant from its classical value, because whatever the direction of the wave, the increased backscattering tends to drag it back as if the wave medium were more "viscous" than it should be classically.

From elementary kinetic theory, the diffusion constant may be expressed as $D = (1/3)vl$, where v is the velocity of the diffusing "particle" and l is its elastic collision mean free path. When D decreases, either l or v, or both, may be the cause. For wave diffusion, the mean free path may be measured as the distance a plane wave can penetrate into a scattering medium before it loses its phase front. The velocity v, on the other hand, is a problem because the usual wave speed, whether the phase velocity or the group velocity, is defined for wave states that have well-defined wave vectors. Definitions of phase velocity requires a constant-phase surface perpendicular to the wave vector, and the definition of the group velocity – the wave vector derivative of frequency – requires both the wave vector and the dispersion relation between frequency and wave vector to be well defined. However, in a strongly scattering medium a wave vector cannot possibly describe a multiply scattered wave state because such a state has neither a unique direction of propagation nor a unique spatial periodicity for defining a wavelength. What, then, is the v in the wave diffusion constant?

The v relevant to the wave diffusion is the average speed at which the wave *energy* is transported locally. In the presence of strong scattering, especially resonant scattering, this transport velocity can be smaller than the free-space phase velocity for classical waves (van Albada et al. 1991), but it is always identical to the group velocity for the quantum wave associated with free electrons. This striking difference between the classical and the quantum waves has its origin in the different dispersion relations for the two kinds of waves and the manner in which the randomness enters the two types of wave equations. Therefore, through increasingly strong scattering the mean free path must decrease for the quantum case but both the transport speed and the mean free path can decrease for the classical waves.

The downward renormalization of the diffusion constant and the coherent backscattering effect itself have several important features. First, the decrease in the diffusion constant is proportional to the scattering strength. If the scattering is weak, the coherent backscattering effect is correspondingly weak so that it can be ignored in general. But if the backscattering is strong enough, the decrease in the diffusion constant can make it vanish, thus leading to wave *localization*.

Second, the coherent backscattering effect is fully operative only when the system is time-reversal invariant, meaning that macroscopically there should be no preferred direction of time established, for example, by a uniform average velocity of the scatterers or by the presence of a magnetic field. In the presence of effects that break time-reversal invariance, the correction to the wave diffusion constant is diminished.

Third, the coherent backscattering is fully effective only when all scatterings are elastic. In the presence of inelastic scattering, which is inevitable in real materials having various dissipation mechanisms, the coherent backscattering effect is again diminished. However, the manner in which this occurs is directly coupled to the next feature; that is, the magnitude of the coherent backscattering effect, manifested in the amount of (negative) correction to the diffusion constant, is a monotonically increasing function of the physical sample size. Where the sample size is infinite, the de facto "sample size" is set by the inelastic scattering rate, because over a distance where there are several inelastic scatterings, the coherent backscattering can no longer be operative, and that distance essentially becomes the limiting "sample size." Since electron inelastic scattering is generally temperature dependent, the combination of the last two features offers a means by which to observe the coherent backscattering effect (indirectly) in electronic systems through the temperature dependence of conductivity in disordered materials.

1.4 Sample Size Dependence

From the point of view of classical physics, the sample size dependence of the coherent backscattering effect is revolutionary because it makes the

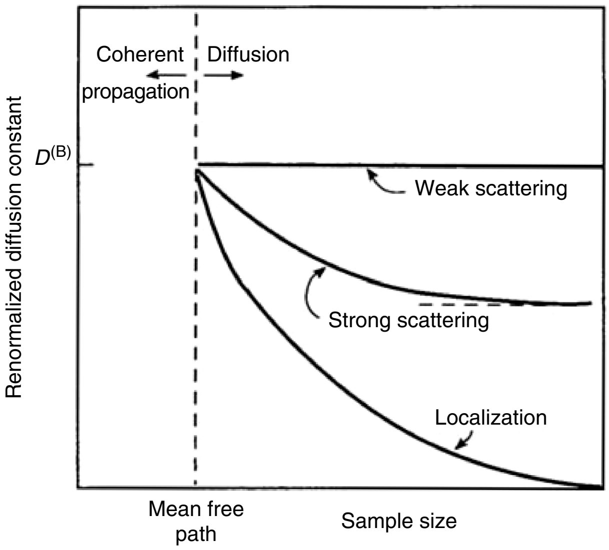

Fig. 1.1. A schematic of the variation of the renormalized diffusion constant with sample size, i.e., the scale of observation. The sample size must be larger than the mean free path before diffusion can be observed. The three cases of weak scattering, strong scattering, and localization are shown. $D^{(B)}$ denotes the classical Boltzmann value of the diffusion constant

renormalized diffusion constant no longer intensive, as would generally be expected, since the diffusion constant belongs to the same class of intensive quantities as density, electrical conductivity, and temperature. Figure 1.1 shows the renormalized diffusion constant schematically as a function of sample size for three cases.

In the weak scattering limit, the diffusion constant is independent of the sample size, as expected classically. When the scattering is strong the diffusion constant is renormalized downward as a function of the sample size, with an asymptotic value that can be significantly less than its classical value (which is nonetheless still observable at small sample size). When the asymptotic value of the renormalized diffusion constant vanishes, then by definition a localized state is created. Therefore, a pulse injected into a strongly scattering medium would evolve initially as in a uniform medium, then quickly make a transition into diffusive transport, accompanied by a gradual slowdown of diffusion over time. Localization occurs when the overall diffusion is stopped.

Two comments can be made concerning the sample size dependence of the coherent backscattering effect. First, on the most elementary level, the backscattering aspect of the effect is responsible for its sample size dependence, because larger samples are more opaque than smaller samples. Thus one can expect more backscattering and less transmission for the larger samples. The coherence part of the effect then enhances the backscattering intensity from what is expected from ordinary diffusion. Through refinement, this simple viewpoint can be made quantitative, as will be seen in Chap. 8. Second, the size dependence is necessary if one looks ahead to localization. When a wave is trapped by randomness, there appears a new length scale – the localization length – which naturally introduces sample size dependence into the physical transport property of the system. Let us expand on this point below by using the quantum waves associated with the electronic system as an example.

If one measures the electrical conductivity of a sample in which the electrons are localized – a disordered insulator – the usual measurement at finite temperatures is expected to yield only the conductivity of the electrons thermally activated from their localized states to some higher-energy mobile states able to carry electrical current. Now imagine a thought experiment where the temperature is lowered to absolute zero so that all electrons are in their localized states. The electrical conductivity of a bulk insulating sample at zero degree is usually regarded as zero – or too small to be measured. Nevertheless, if the sample dimension is small enough so that it becomes comparable to the localization length – a possibility which is increasingly becoming a technological reality nowadays – then even an insulator can conduct some electricity. Conduction can occur because localization prevents the wave, in this case the quantum wave associated with an electron, only from moving outside the spatial domain defined by the localization length, but the electron can still be mobile inside the domain of localization. If the edge of the localized domain is not abrupt but has an exponential tail, then the ability of a localized electron to conduct electricity would decrease exponentially as a function of the sample size when its linear dimension increases beyond the localization length. The result is the sample size dependence of the transport characteristics expected from localized states. If one did not know about the coherent backscattering effect but wanted to invent a mechanism for localization, such a mechanism would need to incorporate some kind of sample size dependence as necessitated by its desired result. From this perspective, the sample size dependence is an essential and necessary attribute for a localization mechanism.

1.5 Localization and Scaling

Since localization is a major theme of this book, a brief digression on the development of the localization concept would be helpful. In the early days of solid-state physics, the recognition that electronic states in a periodic lattice form energy bands was a breakthrough that clarified a basic question about why some materials are electrically conducting and some are insulating. In the simplest version of the band picture, electrical conductors have a half-filled energy band whereas an insulator has filled bands. Mott took the conductor picture a step further and proposed that if the lattice constant of a half-filled band conductor can be continuously increased, then at some point the conductor becomes an insulator. The rationale behind this hypothesis is that when the atoms are separated far enough, they will behave as individual neutral atoms instead of as a metal in which the conduction electrons can pass freely from one atomic site to the next. The basic physics of the so-called Mott transition is the Coulomb interaction between the electrons (Mott 1949, 1974). It does not involve disorder. In contrast, Anderson proposed in 1958 that electronic diffusion can vanish in a sufficiently random potential, in the absence of any electron–electron interaction (Anderson 1958). This proposal, together with Mott's previous works, formed the theoretical basis for the study of the metal–insulator transition in doped semiconductors. However, it was not until the late 1970s and the early 1980s that the Anderson localization was linked to the coherent backscattering effect and explained on that basis. At about the same time, the study of Anderson localization was extended to classical waves, which offer an advantage over the disordered electronic systems where the Mott mechanism and the Anderson mechanism are inseparable: In classical waves, the Anderson localization may be studied alone, without the additional complication of wave–wave interaction. In this volume, the term "localization" denotes the phenomenon only in the sense of the Anderson mechanism.

Although the coherent backscattering effect was important to the understanding of wave multiple scattering phenomena and localization, an overview of the localization phenomenon was actually first achieved through the different perspective of a phenomenological theory, the *scaling theory of localization* (Abrahams et al. 1979). The scaling theory is a scheme for interpolating between an extended wave state and a localized one. The term "scaling" is popular in physics nowadays. In the present context, it has the following connotations. First, scaling implies that the conclusions of the theory are independent of the many details of the physical model. For example, it is often the case that electron multiple scattering and localization are studied in the context of a lattice model of the solid atomic lattice. A scaling theory of localization would imply that the conclusions of the theory are independent of the type of lattice, be it simple cubic, body-centered cubic, or whatever. The conclusions are also independent of the type of random scatterings, the statistics of the randomness, etc. Therefore, scaling implies broad applicability: Because the theory depends on few essential physical quantities, one hopes to obtain a better overall picture of the phenomenon without being encumbered by details. Herein lies the attraction of a scaling theory.

Second, scaling here means literally changing the scale, or physical size, of the sample under consideration. The scaling theory of localization considers how a quantity, defined as the dimensionless conductance γ, varies under a sample size change. For an electronic system, γ is simply the ordinary conductance Γ divided by the quantum unit of conductance, e^2/h, where e denotes the electronic charge and h is Planck's constant. For classical waves, γ may be expressed alternatively as the ratio of two energy scales, which are defined and motivated in Chap. 8. How the dimensionless conductance γ varies with sample size is very simple for electrical conductors (samples in which the wave states are extended). How γ varies with sample size for disordered insulators, in which the wave states are all localized, can also be inferred from the earlier discussion on the sample size dependence of the coherent backscattering effect. To these known facts the scaling theory adds the following assumption: The rule governing the variation of γ with sample size can depend on only one parameter, which is the value of γ itself. With this seemingly innocent proposition, the scaling theory reaches some startling conclusions, the most striking of which is the dependence of the localization phenomenon on the *spatial dimensionality* of a sample. In particular, the scaling theory tells us that regardless of how weak the randomness, all waves are localized in 1D or 2D samples of infinite extent. For 3D samples, the scaling theory predicts the possibility of coexistence for extended and localized wave states, where the extended states can exist in one or several frequency regimes and the localized states can exist in the other. The frequency that separates a localized regime from a neighboring extended regime is called the “mobility edge.” In the jargon of the physics community, spatial dimension two is called the “marginal dimension” for localization. In order to understand this prediction, let us first clarify the meaning of spatial dimensionality as applied to physical samples, as well as the conditions under which the predictions apply.

1.6 Spatial Dimensionality in Localization and Diffusion

All physical objects must have finite cross-sections or thicknesses. Therefore, there cannot be true 2D or 1D samples of vanishing thickness or cross-section. Spatial dimensionality means that if wave propagation and scattering are allowed only in two (backward and forward) directions defined by a line, the sample is 1D; if propagation and scattering are allowed only in directions defined by a surface, the sample is 2D; and if they are allowed in all 3D space, the sample is 3D. The restriction on the direction(s) of wave propagation and scattering can be achieved by making the thickness, or cross-sectional dimension of a sample smaller than, or comparable to, the wavelength. For example, if a wave is confined inside a sample whose thickness is smaller than the wavelength (but larger than half the wavelength), then the excitation in the thickness direction must be a standing wave. In addition, all other standing wave states are higher in frequency, with frequency increments large enough to make them inaccessible (e.g., through thermal excitation). Therefore, all scattering and propagation are confined to the planar directions, and the sample may be described as 2D. Another possibility of observing 2D waves is found in interfacial excitations, where the wave amplitude decays exponentially away from the interface and the wave can propagate only along the interface, as the name implies. Similar reasoning applies to the description of a 1D sample.

On the basis of the scaling theory predictions, should all thin and wirelike objects be good electrical and thermal insulators? The answer depends on the conditions under which the measurements are made. The conclusions of the scaling theory are meant to apply only to samples at absolute zero temperature, so that finite-temperature effects, such as the activation of charge carriers and the inelastic scattering (that essentially limits the sample size as described before), are all absent. Therefore, the predicted 1D and 2D localization effects should not be ordinary, everyday experience. However, they should apply under laboratory conditions where the temperature of a sample is lowered to close to absolute zero and the sample dimensions are controlled to within the tolerance of being qualified as 1D or 2D.

A particularly intriguing prediction of the scaling theory is the inevitability of wave localization in 1D and 2D samples even at the weak scattering limit. To better understand this limiting case, let us first construct and analyze an "apparent" dilemma. Consider a 1D sample of finite extent L. The heterogeneties in the sample are characterized by a scale R which is assumed to be smaller than the wavelength λ. To be more specific, the wave is a classical pulse with a gentle envelope whose width is many wavelengths so that the additional frequency components are negligible. The question is: What happens when $L \to \infty$ and $\lambda \to \infty$? If $\lambda \to \infty$ first, the previous discussion on effective medium shows that the 1D random medium should always appear homogeneous to the probing wave, with no net scattering in the static limit of $\lambda/R \to \infty$. This would remain true for every finite L as $L \to \infty$. Therefore, the end result of λ, $L \to \infty$ should be an infinite (homogeneous) effective medium where there is no localization. Now consider the reverse order of taking the limits. By letting $L \to \infty$ first, one can immediately invoke the prediction of the scaling theory – that all waves are localized in 1D samples of infinite extent – and this conclusion remains true for every λ as $\lambda \to \infty$. If a unique physical state is assumed for λ, $L \to \infty$, then the two contradicting conclusions yield a dilemma. The above description applies to a 1D sample, but similar reasoning leads to the same dilemma in 2D samples.

What can be the resolution of this dilemma? The answer is that both conclusions are correct, because even as L, $\lambda \to \infty$, the ratio L/λ still needs to be adjusted. There can indeed be different physical states depending on whether that ratio approaches 0 or ∞, which are the two possibilities probed by taking the limits in two different orders. But what does this imply physically for wave localization? If $L \to \infty$ first, then as $\lambda \to \infty$, locally (inside the wave packet) the effective medium should be an increasingly good approximation, because scattering by classical waves diminishes as $\lambda/R \to \infty$. As the wave packet travels through the medium, the scattering is small at every instant of time. Therefore, if localization were to occur as predicted, the small scatterings at successive times *must accumulate* so that the net effect is large enough to localize. In other words, even if $R/\lambda = \varepsilon$ is small, yet over large travel distances, measured in terms of $1/\varepsilon$ (or even larger in 2D), there is still an order one effect, i.e., localization. If the travel distance is limited by the sample size L, then the localization limit can never be reached and one always obtains the effective medium limit instead. Therefore, the following physical picture emerges from an analysis of this apparent dilemma: In the weak scattering limit, a localized wave can exhibit propagating behavior *locally*; through increasing sample size and the effect of accumulated scatterings, the wave transport character is altered progressively from propagating to diffusion to localized. Of course, one recognizes the coherent backscattering effect in this scattering accumulation process, especially its sample size dependence. But why should this accumulation process be especially effective only in 1D and 2D? What is so magical about the spatial dimensionality two? Some insight into this question may be obtained through the special character of diffusion and its interaction with the coherent backscattering effect.

The usual way to describe diffusion is through the net distance r traveled by a random walker at the end of a time period t. The diffusion relation is described by $r^2 \propto t$, independent of the spatial dimensionality where the random walker executes its "walks." That is, in time t the random walker covers an "area" proportional to r^2. If the random walker is limited to moving on a line or a flat surface, the trace of its path would appear dense, but if the random walker is a flying particle that can freely traverse the 3D space, then the trace of its path would appear to be flimsy, because an area does not go a long way toward covering a volume. While this description of diffusion may be somewhat simplistic, it can be made rigorous by adding qualifying conditions. Let us consider the path traversed by a random walker from $t = 0$ onward. As $t \to \infty$ in $d = 1, 2$ the random walker will visit the infinitesimal neighborhood of any given point with probability one, i.e., with certainty. In $d = 3$, however, the probability is infinitesimal that it would visit any given infinitesimal neighborhood. One way to appreciate that difference is by calculating the probability, at a time $t_m > 0$ onward, that a random walker will return to the neighborhood of the origin, the point where the walker started its motion at $t = 0$. According to the solution of the diffusion equation, the probability density for the walker at distance r from the origin at time t is given by $P(r, t) = (4\pi Dt)^{-d/2} \exp(-r^2/4\pi Dt)$, where d denotes the spatial dimensionality of the random walk and D is the diffusion constant. The desired probability is therefore given by

$$\lim_{T\to\infty} \int_{t_m}^{T} P(0, t)\mathrm{d}t = \lim_{T\to\infty} \int_{t_m}^{T} \frac{\mathrm{d}t}{(4\pi Dt)^{d/2}}.$$

For $d = 1, 2$ the integral diverges as $T \to \infty$, independent of t_m, implying that the random walker will certainly return to the neighborhood of the origin. This is intuitively plausible from the fact that the path of a random walker covers an "area." For $d = 3$, on the other hand, the integral is proportional to $1/\sqrt{t_m}$, so that as t_m increase, the probability decreases toward zero. Thus, in terms of this probability, there is a qualitative difference between diffusion in one and two dimensions and diffusion in three dimensions. If the probability of returning to the origin is heuristically equated with backscattering, then this special property of diffusion translates into more effective coherent backscattering in one and two dimensions, leading to localization. From this viewpoint, the marginal dimension of localization is a direct result of the exponent two in the diffusion relation $r^2 \propto t$.

Although the scaling theory gives a global view of what can happen in different spatial dimensions, the details of this explanation need to be filled in with results of calculations based on the mechanisms of diffusion plus coherent backscattering. This proves to be possible, because the correction to the diffusion constant introduced by the coherent backscattering is consistent with the scaling hypothesis. As a result, formulas for the localization length and its frequency dependence, plus explicit forms for the scaling functions, can all be derived in spatial dimensions one and two. In spatial dimension three, however, our knowledge of the subject is still incomplete. It is known that if randomness is introduced into a periodic structure or if the system is discrete, such as an electron in a disordered lattice, then the localized states and mobility edge(s) can exist. This is due to the fact that in the absence of randomness, periodic structures yield frequency bands, and near the band edges the Bragg scattering gives rise to standing waves, i.e., waves with zero group velocity. Such wave states are, in a sense, waiting to be localized as randomness is introduced. For a continuous random system without any long-range periodic correlation, however, quantum wave is known to localize at low energies, whereas classical wave localization is possible only if the scattering is strong enough. One possibility is resonant scattering, e.g., Mie resonance for electromagnetic wave scattering from small particles, which can give a much higher scattering cross-section than usual. But resonant scattering has the disadvantage of associated large absorption, which can mask the localization effect. Hence the most likely systems for classical wave localization are those with large index of refraction contrast between the scatterers and the matrix material in which they are dispersed (Sheng 1986). Indeed, light localization was observed in GaAs powders with an index of refraction value ~ 3.5. (Wiersma et al. 1997)

1.7 Mesoscopic Phenomena

Wave multiple scattering and localization can lead to various physical manifestations. For electronic systems, the direct proportionality relation between the conductivity and the diffusion constant is known as the "Einstein relation." Therefore, the observation of electronic conductivity is equivalent to observing electron diffusion. For classical phonons, heat conduction has already been mentioned as a manifestation of phonon diffusion. In dirty conductors, indirect observation of the coherent backscattering effect is contained in the measurements of anomalous temperature dependence of resistivity and the anomalous magnetoresistance, both in conducting films. However, these indirect measurements suffer from the ambiguity that a competing effect – that of electron – electron interaction–can yield similar results, so there is no way to attribute the measured behavior entirely to the coherent backscattering effect. A more direct observation of the consequences of wave multiple scattering is via measurements on *mesoscopic samples*. Here "mesoscopic" denotes a sample size regime intermediate between the molecular and the bulk. More precisely, it means a sample size whose linear dimension is smaller than a "dephasing length," $\sqrt{D\tau_{\text{in}}}$, where τ_{in} is the inelastic collision time for the wave. In a mesoscopic sample, all collisions are elastic, so the effects of coherent backscattering and interference can be manifest and not be averaged out. For classical waves, the inelastic scattering rate is generally low and temperature insensitive. Wave–wave interaction is also negligible, in contrast to the interaction between electrons. Thus a mesoscopic sample in this case is a bulk sample, and the experiment can be done at room temperature. In fact, the cleanest quantitative demonstration of the coherent backscattering effect came from light scattering from bulk disordered systems. For electronic systems, on the other hand, the inelastic scattering length is generally small and inversely temperature dependent. Therefore, the observation of mesoscopic phenomena in electronic systems generally requires small samples whose linear dimensions are on the order of micrometers or smaller. In addition, the experiments must be performed at low temperatures to ensure that the inelastic scattering rate is sufficiently low. However, with today's microfabrication and cryogenic technologies these requirements are not difficult barriers, and the relentless push for miniaturization in the electronic industry means that the mesoscopic phenomena could very well serve as the basis for tomorrow's quantum devices.

Mesoscopic phenomena are manifest in the measurements of conventional quantities in mesoscopic samples. Electrical conductivity is an example. The definition of mesoscopic conductance is problematic at first sight because electrical conductance is always associated with dissipation, whereas in mesoscopic samples the absence of inelastic scattering means that there is no dissipation. Thus, there is a need to define what one means by conductance in a mesoscopic sample. By analogy with the resistance of a tunnel junction, Landauer has proposed that the conductance of a mesoscopic sample should be proportional to the wave transmission probability (Landauer 1957). In that case, dissipation occurs not inside the sample but in the leads connected to the sample, and through consideration of equilibrium with the leads an expression for the mesoscopic conductance can be derived.

Conductance of mesoscopic samples has many unconventional characteristics. Not only does it vary with sample size (due to the coherent backscattering effect), it can also fluctuate wildly from sample to sample, even if the samples were fabricated identically in the same batch. The conductivity can also show large fluctuations upon the application of a varying magnetic field. Moreover, as the sample size increases, these fluctuations do not decrease (as long as inelastic scattering is absent); i.e., the fluctuations are not averaged out as one would expect from additive noise. All these effects are basically due to the fact that even though the waves are multiply scattered, they can still exhibit phase interference and retain some long-range memory.

1.8 Localization vs. Confinement

The term localization has often been used in the literature to denote non-propagating wave states which are not localized in the Anderson sense. In particular, wave confinement, e.g., standing waves formed inside a cavity or by totally reflecting interfaces, should be distinguished from the Anderson wave localization that has just been described. Wave confinement may involve walls made of material that has no wave state at the relevant frequency. Hence waves can only totally reflect from such interfaces, penetrating the interface only by an exponentially decaying evanescent tail. Spectral bandgaps in photonic or phononic crystals or crystalline electronic systems are typical examples which can form effective wave confining cavities or waveguides. Viewed in more general terms, the crucial difference in spectral bandgap confinement and Anderson localization is that a bandgap denotes a frequency regime which is empty of wave states, whereas a localized wave is a nonpropagating wave state. The two mechanisms can interact, nevertheless. A most interesting case is the disordered spectral bandgap systems in which the crystalline periodicity is perturbed by disorder as mentioned before. The sharp edge of the crystalline bandgaps would then be smeared out to form a transition region. In that transition regime there would be spatial regions that are deficit of wave states, so that instead of total confinement, the waves would be restricted in their propagation directions. That is, instead of propagating in straight lines, the waves would be traveling in a labyrinth. Another way of saying the same thing is that the total scattering is increased, leading to the enhancement of localization effect. That is why the bandedge states are easily localized.

1.9 Topics not Covered

The discussion thus far is a brief and qualitative guide to the contents of this volume. At this point it is perhaps also important to point out the relevant subjects that are left out. First is wave dissipation. In this volume, dissipation is treated only as a constraint, or limitation, on the effects due to elastic scattering. No attempt is made to examine either the mechanism of dissipation or the combined effect of both elastic scattering and dissipation. The second neglected topic is nonlinearity, which can couple waves of different frequencies. Since the magnitude of nonlinearity depends on the amplitude of a wave, its neglect means that we will be concerned only with small-amplitude waves, and waves of different frequencies will be treated as independent. The third neglected topic is wave–wave interaction. A well-known example in this regard is the electron–electron interaction through their electrical charges. Here, overlooking wave–wave interaction implies that wave scattering is treated as a "single-body" problem, in contrast to the "many-body" problem where the presence of more than one electron changes the nature of the system. For electrons, interaction effects are important at low densities. At high densities, the Coulomb interaction effect is weakened by screening, resulting in an electron dressed in a cloud of screening charges that can be treated as an independent "quasiparticle." The neglect of electron–electron interaction here implies that whenever the term electron is used in this volume, it denotes a quasi-particle in the high-density limit. For classical waves, on the other hand, the interaction between electromagnetic waves is weak, and the same is true for elastic waves of small amplitudes. Therefore, their neglect is well justified.

The three topics not covered – dissipation, nonlinearity, and interaction – are interesting and rich subjects in themselves. They have been omitted solely to attain the simplicity and coherence made possible by focusing on the subjectively chosen main line of exposition. The author, rather than trying to be inclusive and complete, intends this volume to clarify a few essential points, leaving the readers to further explore this challenging field.

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