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Errata: Classical Nucleation Theory in Multicomponent Systems

- **Page XIV:** add entry to the list of symbols
 R_{av} , average growth rate of a cluster, unit $1/s$
- **Pages XVIII-XX:** add five entries to the list of famous scientists
 - Benoît Paul Émile Clapeyron (1799–1864), French engineer and physicist, is on one of the founders of thermodynamics. He worked on improvements of theory of heat engines and the idea of a reversible process. He also studied perfect gases, phase equilibrium and the statistics of beams. He spent ten years teaching in Saint Petersburg, and supervised the construction of a railway line from Paris to Versailles and Saint-Germain. (Wikipedia 2007a)
 - Rudolf Julius Emanuel Clausius (1822–1888) was a German physicist and mathematician, who studied optical effects in the atmosphere of the earth in his Ph. D. work. His most famous work is the introduction of the concept of entropy in 1850, and he is one of the founders of thermodynamics. He also contributed to the kinetic theory of gases. He served in the ambulance corps in the Franco-Prussian war in 1870, and was awarded the Iron Cross. He raised six children as a single parent after the death of his wife in childbirth. (Wikipedia 2007c)
 - Ernst Ising (1900–1998) was a German physicist. His doctoral thesis in 1924 at University of Hamburg studied a problem suggested by his teacher, Wilhelm Lenz. He investigated the special case of a linear chain of magnetic moments, which are only able to take two positions, "up" and "down," and which are coupled by interactions between nearest neighbors. Mainly through following studies by Rudolf Peierls, Hendrik Kramers, Gregory Wannier and Lars Onsager the model proved to be successful explaining phase transitions between ferromagnetic and paramagnetic states. Jewish Ising was barred from teaching and researching during the Nazi regime, but he survived World War II and taught afterwards in the United States, never published again. He was a professor of physics at Bradley University 1948-1976. Each year, about 800 papers are published that use the model to address problems in such diverse fields as neural networks, protein folding, biological membranes and social behavior (Wikipedia 2009)
 - Richard Chace Tolman (1881–1948) was an American mathematical physicist and physical chemist. He was an expert in statistical mechanics, and studied thermodynamics in relativistic systems and cosmology. He introduced the concept of relativistic mass. During the Second World War he worked as a scientific advisor in the Manhattan Project to develop the first nuclear bomb, and after the war he acted as an advisor to the United Nations Atomic Energy Commission. (Wikipedia 2007b)
 - Thomas Young (1773–1829) was an English physician and physicist studying a wide variety of subjects. He was one of the developers of the wave theory of light when explaining the interference patterns of his double-slit experiment. He studied elasticity giving his name to Young's

modulus, a concept used already before his work. Young founded the theory of capillarity based on the concept of surface tension. In 1807 he was the first to use the term “energy” in its modern sense. He can be considered the founder of physiological optics, proposing the mechanism of trichromatic color vision and studying how the eye accommodates itself to objects at various distances by changing the curvature of the lens. He studied blood circulation, classification of diseases, consumption, and proposed Young’s rule for determining the drug dosage for a child. Young also studied languages, comparing the grammar and vocabulary of 400 languages, and was one of the pioneers in trying to decipher hieroglyphs and introduced the term “Indo-European languages”. He also developed the Young temperament, a method of tuning musical instruments. (Wikipedia 2007d)

- **Page 10:** The last equation should read

$$U = S \left(\frac{\partial U}{\partial(\lambda S)} \right)_{\lambda V, \lambda N_i, \lambda A} + V \left(\frac{\partial U}{\partial(\lambda V)} \right)_{\lambda S, \lambda N_i, \lambda A} + \sum_i N_i \left(\frac{\partial U}{\partial(\lambda N_i)} \right)_{\lambda V, \lambda N_{j \neq i}, \lambda S, \lambda A} + A \left(\frac{\partial U}{\partial(\lambda A)} \right)_{\lambda V, \lambda N_i, \lambda S}$$

[N_i should be inside the summation]

- **Page 11:** Equation (1.6) should read

$$U = S \left(\frac{\partial U}{\partial S} \right)_{V, N_i, A} + V \left(\frac{\partial U}{\partial V} \right)_{S, N_i, A} + \sum_i N_i \left(\frac{\partial U}{\partial N_i} \right)_{V, N_{j \neq i}, S, A} + A \left(\frac{\partial U}{\partial A} \right)_{V, N_i, S}. \quad (1.6)$$

[N_i should be inside the summation]

- **Page 13:** Equation should read

$$d\mu_1 N_1 + d\mu_2 N_2 = 0.$$

[2 is should be a subscript of N in the second term]

- **Page 17:** Equation (2.1) should read

$$dS_{\text{tot}} = \left(\frac{1}{T_g} - \frac{1}{T_s} \right) dU_g + \left(\frac{1}{T_l} - \frac{1}{T_s} \right) dU_l + \left(\frac{P_g}{T_g} - \frac{P_l}{T_l} \right) dV_g - \sum_i \left(\frac{\mu_{i,g}}{T_g} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,g} - \sum_i \left(\frac{\mu_{i,l}}{T_l} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,l}. \quad (2.1)$$

[minus signs in front of both of the summation marks]

- **Page 19:** Equation (2.5) should read

$$\begin{aligned}
 d\mathcal{S}_{\text{tot}} &= \left(\frac{1}{T_g} - \frac{1}{T_l} \right) dU_g + \left(\frac{1}{T_l} - \frac{1}{T_s} \right) dU_l \\
 &\quad + \left(\frac{P_g}{T_g} - \frac{P_l}{T_l} \right) dV_g - \sum_i \left(\frac{\mu_{i,g}}{T_g} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,g} \\
 &\quad - \sum_i \left(\frac{\mu_{i,l}}{T_l} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,l} - \frac{\sigma}{T_s} dA \\
 &= 0 \text{ for equilibrium.}
 \end{aligned} \tag{2.5}$$

[minus signs in front of both of the summation marks]

- **Page 20:** The first equation should read

$$\begin{aligned}
 d\mathcal{S}_{\text{tot}} &= \left(\frac{1}{T_g} - \frac{1}{T_l} \right) dU_g + \left(\frac{1}{T_l} - \frac{1}{T_s} \right) dU_l \\
 &\quad + \sum_i \left(\frac{\mu_{i,g}}{T_g} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,g} \\
 &\quad + \sum_i \left(\frac{\mu_{i,l}}{T_l} - \frac{\mu_{i,s}}{T_s} \right) dN_{i,l} \\
 &\quad - \left(\frac{P_g}{T_g} - \frac{P_l}{T_l} + \frac{2\sigma}{T_s r} \right) dV_l \\
 &= 0 \text{ for equilibrium.}
 \end{aligned}$$

[minus sign in front of the dV_l term]

- **Page 26:** Fourth line after Table 2.1
 “...and activity coefficients $\Gamma_i(x_{i,l}, T)$ are normally less or equal to one, ...” should be replaced by
 “...and as stated above the activity coefficients $\Gamma_i(x_{i,l}, T)$ can also be less or equal to one, ...”
- **Page 26:** In Table 2.2. and its caption “nominator” should be replaced by “numerator”
- **Page 31:** The first equation should read

$$\frac{2\sigma}{r} = \frac{-\Delta\mu_i}{v_{i,l}}$$

[there should be a minus sign in front of $\Delta\mu_i$]

- **Page 31:** Equation (2.24) should read

$$r = \frac{2\sigma v_{i,l}}{-\Delta\mu_i} = \frac{2\sigma v_{i,l}(x_{i,l})}{kT \ln(\mathcal{A}_{i,g}/\mathcal{A}_{i,l})} \tag{2.24}$$

[there should be a minus sign in front of $\Delta\mu_i$]

- **Page 31:** The last equation should, for clarity, be in the form

$$-\Delta\mu_i = kT \ln \frac{P_{i,g}}{P_{i,e}(x_{i,l})} = \frac{2\sigma v_{i,l}}{r}$$

- **Page 44:** First equation should read

$$dQ = dU - P_0 dV_0 + \sum \mu_{i,0} dN_{i,0}$$

[subscript ₀ added to dV]

- **Page 46:** Third line after equation (3.3) delete confusing sentence “For those systems F is constant in equilibrium.”, and second line after equation (3.4) delete confusing sentence “For those systems G is constant in equilibrium.”
- **Page 49:** The same Maxwell equation $\left(\frac{\partial P}{\partial N_i}\right)_{T,V,N_{j \neq i}} = -\left(\frac{\partial \mu_i}{\partial V}\right)_{T,N_i}$ appears twice under Maxwell equations derived from the Helmholtz free energy. The correct set of equations is
Helmholtz free energy F

$$\left(\frac{\partial S}{\partial V}\right)_{T,N_i} = \left(\frac{\partial P}{\partial T}\right)_{V,N_i} \quad \left(\frac{\partial S}{\partial N_i}\right)_{T,V,N_{j \neq i}} = -\left(\frac{\partial \mu_i}{\partial T}\right)_{V,N_i}$$

$$\left(\frac{\partial P}{\partial N_i}\right)_{T,V,N_{j \neq i}} = -\left(\frac{\partial \mu_i}{\partial V}\right)_{T,N_i}$$
- **Page 49:** In the last Maxwell equation derived from the grand potential on the right hand side of the equation v should be replaced by V in the denominator $\left(\frac{\partial P}{\partial \mu_i}\right)_{T,V} = \left(\frac{\partial N_i}{\partial V}\right)_{T,\mu_i}$
- **Page 56:** $\rho_{1,g}$ and $\rho_{2,g}$ should be swapped in Figure 3.6
- **Page 58:** The first equation should read

$$0 = \left[\frac{dU^*}{dV_l^*}\right] = -P_l^* - P_g^0 \left[\frac{dV_g^*}{dV_l^*}\right] + \left[\frac{d\sigma^*}{dV_l^*}\right] A^* + \sigma^* \left[\frac{dA^*}{dV_l^*}\right]$$

[$_l$ added to dV^* in the last two terms]

- **Page 58:** The last equation should read

$$dU = TdS - PdV + \sigma dA + \sum \mu_i dN_i$$

[$Ad\sigma$ replaced by σdA and S deleted after dN_i]

- **Page 62:** First line of equation (3.15) $(\mu_{i,g}^0 - \mu_{i,l}(P_0))$ should be $(\mu_{i,l}(P_0) - \mu_{i,g}^0)$
- **Page 62:** Second line of equation (3.15) and the equation below equation (3.15) $\Delta\mu_{i,l}$ should be $\Delta\mu_i$
- **Page 66:** Equation (3.22) should read

$$\Delta\varphi = -N_d \left(kT_0 \ln \frac{P_g}{P_e(T_0)} - v_l(P_g - P'_g) \right) + N_d^{2/3} \sigma (36\pi)^{1/3} v_l^{2/3}, \quad (3.22)$$

[add $-N_d$ and brackets around “ $kT_0 \ln P_g/P_e(T_0) - v_l(P_g - P'_g)$ ”, change sign of $v_l(P_g - P'_g)$ to be consistent when brackets added]

- **Page 67:** Equation (3.23) should read

$$E = -\varepsilon \sum_{\langle lm \rangle} s_l s_m - kTb \sum_l s_l, \quad (3.23)$$

[minus sign in front of the second term]

- **Page 67:** second line after equation (3.23) “ h ” should be replaced by “ b ”
- **Page 69:** middle of page, subscript i missing from $\Delta\mu$ at the last line of the three-line equation, and just after that equation as well

$$\begin{aligned} \mu_{i,s} - \mu_{i,g}^0 &= \mu_{i,l}(P_l, x_{i,l}) - \mu_{i,g}^0 \\ &= \mu_{i,l}(P_g, x_{i,l}) + v_{i,l}(P_l - P_g) - \mu_{i,g}^0 \\ &= \Delta\mu_i + v_{i,l}(P_l - P_g) \end{aligned}$$

with the definition (2.14) $\Delta\mu_i \equiv \mu_{i,l}(P_g, x_{i,l}) - \mu_{i,g}^0$.

- **Page 72:** The last equation should read

$$r = \frac{2\sigma v_{1,l}}{-\Delta\mu_1}$$

[there should be a minus sign in front of $\Delta\mu_i$]

- **Page 84:** Part b) of Problem 4.1. can only be solved after studying Chapter 5.1
- **Page 86:** Beginning of third paragraph
“Clusters of size $\{N_{i,d}\}$ loose one molecule of type k with a rate...” should be replaced by
“Clusters of size $\{N_{i,d}\}$ loose single molecules of type k with a rate...”
- **Page 86:** Point 1. should read
“ $+\sum_k \beta_k(\{N_{j,d}, N_{k,d} - 1\}) \cdot C(\{N_{j,d}, N_{k,d} - 1\})$.
A monomer of type k hits a cluster $\{N_{j,d}, N_{k,d} - 1\}$ which is otherwise like $\{N_{j,d}, N_{k,d}\}$, but one molecule of type k is missing”
[remove d in front of the summation mark and change subscript i to j in
“...hits a cluster $\{N_{j,d}, N_{k,d} - 1\}$...”]
- **Page 86:** Point 2. should read
“The clusters themselves grow by a monomer of any type k :
“ $-\sum_k \beta_k(\{N_{j,d}, N_{k,d}\}) \cdot C(\{N_{j,d}, N_{k,d}\})$.”
[remove d in front of the summation mark]
- **Page 86:** Point 3. should read
“Larger ones break up:
“ $+\sum_k \gamma_k(\{N_{j,d}, N_{k,d} + 1\}) \cdot C(\{N_{j,d}, N_{k,d} + 1\})$.
A monomer of type k leaves a cluster $\{N_{j,d}, N_{k,d} + 1\}$ which is otherwise like $\{N_{j,d}, N_{k,d}\}$, except that it has one more molecule of type k .”
[remove d in front of the summation mark and change subscript i to j in
“...otherwise like $\{N_{j,d}, N_{k,d}\}$...”]

- **Page 86:** Point 4. should read
 “The clusters themselves break up, a molecule of type k leaves:
 $-\sum_k \gamma_k(\{N_{j,d}, N_{k,d}\}) \cdot C(\{N_{j,d}, N_{k,d}\})$.”
 [remove d in front of the summation mark]
- **Page 86:** Equation (5.2) should read

$$\begin{aligned} \frac{dC(\{N_{j,d}\})}{dt} = & \sum_k \left[\beta_k(\{N_{j,d}, N_{k,d} - 1\}) \cdot C(\{N_{j,d}, N_{k,d} - 1\}) \right. \\ & - \beta_k(\{N_{j,d}, N_{k,d}\}) \cdot C(\{N_{j,d}, N_{k,d}\}) \\ & - \gamma_k(\{N_{j,d}, N_{k,d}\}) \cdot C(\{N_{j,d}\}) \\ & \left. + \gamma_k(\{N_{j,d}, N_{k,d} + 1\}) \cdot C(\{N_{j,d}, N_{k,d} + 1\}) \right]. \end{aligned} \quad (5.2)$$

[remove d in front of the summation mark and for clarity, remove $N_{k,d}$ from the left hand side of the equation, insert subscript k to β in the second line of the equation]

- **Page 88:** Equation (5.4) should read

$$\frac{dC(\{N_{j,d}\})}{dt} = \sum_k I_k(\{N_{j,d}, N_{k,d} - 1\}) - I_k(\{N_{j,d}, N_{k,d}\}). \quad (5.4)$$

[remove d in front of the summation mark, change subscript k to j in $I_k(\{N_{j,d}, N_{k,d} - 1\})$, change subscript i to j in $-I_k(\{N_{j,d}, N_{k,d}\})$, and for clarity, remove $N_{k,d}$ from the left hand side of the equation]

- **Page 88:** Equation (5.6) should read

$$\gamma_k(\{N_{j,d}, N_{k,d} + 1\}) = \beta_k^e(\{N_{j,d}, N_{k,d}\}) \frac{C^e(\{N_{j,d}, N_{k,d}\})}{C^e(\{N_{j,d}, N_{k,d} + 1\})}. \quad (5.6)$$

[+1 should not be in the subscript of $N_{k,d}$ in the denominator on the right hand side of the equation]

- **Page 88:**
 NOTE 1: first line replace “condensation” with “evaporation”
 NOTE 2: first line replace “equilibrium constants” with “evaporation coefficients”
- **Page 90:** In the text after the third equation line replace “nominator” with “numerator”
- **Page 91:** Add after equation (5.10):
 “Equation (5.8) together with the result (5.10) can be used to calculate the cluster size distribution in the nucleating vapour”

$$C(\mathcal{N}) = C^e(\mathcal{N})S^{\mathcal{N}} \left[1 - I \sum_{n=1}^{\mathcal{N}-1} \left(\frac{1}{\beta(n)C^e(n)S^n} \right) \right].$$

- **Page 92:** The last equation of the page should read

$$I^{-1} = \int_1^\infty dn \frac{1}{\beta(n)C^e(n)S^n} = \int_1^\infty dn \frac{1}{\beta(n)} \frac{1}{F^e} \exp\left(\frac{\Delta\varphi(n)}{kT_0}\right).$$

[dn added to the last form of the integral]

- **Pages 92-93:** Text starting with the last sentence of page 92, “Inside the integral...”, and ending just before the second equation of page 93 should be replaced with:

“Condensation coefficient $\beta(n)$, although a function of n , changes only a little in this narrow region, and we do not compromise much in accuracy if we use a constant value of $\beta(n^*)$ when performing the integral. For the normalization factor F^e we use (see p. 83)...”

- **Page 93:** Equation for I^{-1} should read

$$\begin{aligned} I^{-1} &= \frac{1}{\beta(n^*)C^e(1)} \int_1^\infty \exp\left(\frac{\Delta\varphi^*}{kT_0} + \frac{1-W^*}{2} \frac{(n-n^*)^2}{kT_0}\right) dn \\ &= \frac{\exp(\frac{\Delta\varphi^*}{kT_0})}{\beta(n^*)C^e(1)} \int_{-\infty}^\infty \exp\left(\frac{-W^*}{2kT_0} x^2\right) dx, \end{aligned}$$

[superscript ² of n replaced with superscript ^{*} on the first line of the equation].

- **Page 95:** Equation (5.16) should read

$$r^* = \frac{2\sigma v_l}{-\Delta\mu} = \frac{2\sigma v_l}{kT \ln S}. \quad (5.16)$$

[superscript ^{*} added to r]

- **Page 95:** Equation for $\beta(n^*)$ should read

$$\beta(n^*) = SP_e^p(T_0) \sqrt{\frac{6}{kT_0}} \left(\frac{3}{4\pi}\right)^{1/6} \left(\frac{1}{n^* m_1} + \frac{1}{m_1}\right)^{1/2} (V^{*1/3} + v_l^{1/3})^2,$$

[replace v^* by V^*]

- **Page 97:** The first equation of the page should read

$$\frac{dC(i,j)}{dt} = I_1(i-1,j) - I_1(i,j) + I_2(i,j-1) - I_2(i,j) \approx -\frac{\partial I_1}{\partial i} - \frac{\partial I_2}{\partial j},$$

[replace $i-1$ by i on the left hand side of the equation]

- **Page 99:** Equation for $\beta_i^0(im, j)$ should read

$$\beta_i^0(i, j) = P_{i,e}^p(T_0) \sqrt{\frac{6}{kT_0}} \left(\frac{3}{4\pi}\right)^{1/6} \left(\frac{1}{m(i, j)} + \frac{1}{m_1}\right)^{1/2} (V(i, j)^{1/3} + v_1^{1/3})^2.$$

[replace $v(i, j)$ by $V(i, j)$, replace $_o$ by $_0$ as a subscript of T]

- **Page 99:** Third equation from the bottom of the page and the text just before it should read

“By replacing i with $i - 1$ we get

$$C^e(i - 1, j) = F^e \left(\frac{\mathcal{A}_{1,g}^e}{\mathcal{A}_{1,l}(i - 1, j)} \right)^{i-1} \left(\frac{\mathcal{A}_{2,g}^e}{\mathcal{A}_{2,l}(i - 1, j)} \right)^j \cdot \exp \left(\frac{-A(i - 1, j)\sigma(i - 1, j)}{kT_0} \right).$$

”

[$i - 1$ instead of $i + 1$ everywhere, note that in there should be $i - 1$ instead of i in $(\mathcal{A}_{2,g}^e/\mathcal{A}_{2,l}(i - 1, j))^j$]

- **Page 99:** Last equation of the page should read

$$\gamma_1(i, j) = \mathcal{A}_{1,g}^e \beta_1^0(i - 1, j) \frac{F^e [\mathcal{A}_{1,g}^e]^{i-1}}{F^e [\mathcal{A}_{1,g}^e]^i} \frac{[\mathcal{A}_{1,l}(i, j)]^i}{[\mathcal{A}_{1,l}(i - 1, l)]^{i-1}} \frac{[\mathcal{A}_{2,g}^e]^j}{[\mathcal{A}_{2,g}^e]^j} \frac{[\mathcal{A}_{2,l}(i, j)]^j}{[\mathcal{A}_{2,l}(i - 1, j)]^j} \cdot \exp \left(\frac{A(i, j)\sigma(i, j) - A(i - 1, j)\sigma(i - 1, j)}{kT_0} \right),$$

[i replaced by $(i - 1)$ and $(i + 1)$ replaced by i , and $(\mathcal{A}_{2,l}(i, j)/\mathcal{A}_{2,l}(i - 1, j))^j$ added to the first line of the equation. For clarity, also $[\mathcal{A}_{2,g}^e]^j/[\mathcal{A}_{2,g}^e]^j$ and $[\mathcal{A}_{1,g}^e]^{i-1}/[\mathcal{A}_{1,g}^e]^i$ added, and $\mathcal{A}_{1,g}^e$ deleted from the denominator]

- **Page 100:** In the middle of the page, first of the four lines of text replace “nominator” with “numerator”, and the last of the four lines of text replace “liquids” with “liquid”
- **Page 100:** Fourth equation line from the bottom of the page should read

$$= F^e \exp \left(\frac{-A(i, j)\sigma(i, j) + ikT_0 \ln \frac{\mathcal{A}_{1,g}^e}{\mathcal{A}_{1,l}(i, j)} + jkT_0 \ln \frac{\mathcal{A}_{2,g}^e}{\mathcal{A}_{2,l}(i, j)}}{kT_0} \right)$$

[closing bracket added to $\sigma(i, j)$ and “ln” added to the terms with ikT_0 and jkT_0 , closing bracket removed after the term with jkT_0 from the numerator]

- **Page 101:** The first equation should read

$$\begin{aligned}
I_1(i, j) &= \beta_1(i, j) C^e(i, j) \left(\frac{\mathcal{A}_{1,g}}{\mathcal{A}_{1,g}^e} \right)^i \left(\frac{\mathcal{A}_{2,g}}{\mathcal{A}_{2,g}^e} \right)^j \\
&\quad \cdot \left(-\frac{\partial}{\partial i} \left(\frac{C(i, j)}{C^e(i, j) \left(\frac{\mathcal{A}_{1,g}}{\mathcal{A}_{1,g}^e} \right)^i \left(\frac{\mathcal{A}_{2,g}}{\mathcal{A}_{2,g}^e} \right)^j} \right) \right) \\
&= \beta_1(i, j) F^e \exp \left(\frac{-\Delta\varphi(i, j)}{kT_0} \right) \cdot \left(-\frac{\partial}{\partial i} \left(\frac{C(i, j)}{F^e \exp \left(\frac{-\Delta\varphi(i, j)}{kT_0} \right)} \right) \right),
\end{aligned}$$

$[\mathcal{A}_{1,l}^e$ and $\mathcal{A}_{1,l}^e$ replaced by $\mathcal{A}_{1,g}^e$ and $\mathcal{A}_{1,g}^e$, respectively, on the first and second lines]

- **Page 101:** The second equation should read

$$\begin{aligned}
I_2(i, j) &= \beta_2(i, j) C^e(i, j) \left(\frac{\mathcal{A}_{1,g}}{\mathcal{A}_{1,g}^e} \right)^i \left(\frac{\mathcal{A}_{2,g}}{\mathcal{A}_{2,g}^e} \right)^j \\
&\quad \cdot \left(-\frac{\partial}{\partial j} \left(\frac{C(i, j)}{C^e(i, j) \left(\frac{\mathcal{A}_{1,g}}{\mathcal{A}_{1,g}^e} \right)^i \left(\frac{\mathcal{A}_{2,g}}{\mathcal{A}_{2,g}^e} \right)^j} \right) \right) \\
&= \beta_2(i, j) F^e \exp \left(\frac{-\Delta\varphi(i, j)}{kT_0} \right) \cdot \left(-\frac{\partial}{\partial j} \left(\frac{C(i, j)}{F^e \exp \left(\frac{-\Delta\varphi(i, j)}{kT_0} \right)} \right) \right).
\end{aligned}$$

$[\mathcal{A}_{1,l}^e$ and $\mathcal{A}_{1,l}^e$ replaced by $\mathcal{A}_{1,g}^e$ and $\mathcal{A}_{1,g}^e$, respectively, on the first and second lines]

- **Page 101:** The last equation of the page and the text after it should read

$$\underline{R} = \begin{pmatrix} \beta_1(i, j) & \beta_{12}(i, j) \\ \beta_{12}(i, j) & \beta_2(i, j) \end{pmatrix}.$$

The off-diagonal elements of \underline{R} (β_{12}) are equal to zero in this case...”
[comma removed from subscript of β_{12}]

- **Page 103:** Third equation from the bottom should read

$$\nabla \equiv \nabla_{i,j} = \left(\frac{\partial}{\partial i}, \frac{\partial}{\partial j} \right) = \underline{R}^{*-1/2} \nabla_\eta$$

[Remove overline from ∇_η of the right hand side]

- **Page 106:** After the second equation, for clarity, replace
“ $\det \underline{\Gamma} = \lambda_1 \lambda_2$ ” with
“ $\det \underline{\Gamma} = \det \underline{\Gamma}' = \lambda_1 \lambda_2$ ”

- **Page 106:** The sentence starting the second paragraph should read:
“Now we assume that we have found the desired coordinates $x = \Delta\eta'_1$ and $y = \Delta\eta'_2$...”
[“=” added after y]

- **Page 106:** Second equation line from the bottom of the page should read:

$$\overline{\Delta n} = (i - i^*, j - j^*) = \underline{Q} \cdot \underline{R}^{*1/2} \overline{\Delta\eta}$$

$[\underline{Q} \cdot \underline{R}^{*1/2}$ instead of $\underline{Q}\underline{R}^{*1/2}]$

- **Page 106:**

The correct form of the second line from the bottom is:

$$\overline{\Delta n} = (i - i^*, j - j^*) = \underline{Q}^{-1} \cdot \underline{R}^{*1/2} \overline{\Delta\eta}'$$

[should be \underline{Q}^{-1} instead of \underline{Q} and $\overline{\Delta\eta}'$ instead of $\overline{\Delta\eta}$]

- **Page 107:** In three places (left hand side of the first three equations)

$$\iota'_1 \int_{-\infty}^{\infty} \exp\left(\frac{-|\lambda_1|}{2kT_0} \Delta\eta'^2_1\right) d\Delta\eta'_1$$

should have $\Delta\eta'^2_1$ instead of $\Delta\eta^2_1$ inside the exponential.

- **Page 107:** In the text after the first equation replace “nominator” with “numerator”
- **Page 108:** In two places (just before the middle of the page, last terms of two equations)

$$\exp\left(\frac{-|\lambda_2|}{2kT_0} \Delta\eta'^2_2\right)$$

should have $\Delta\eta'^2_1$ instead of $\Delta\eta^2_1$ inside the exponential.

- **Page 109:** In the first line replace “nominator” with “numerator”
- **Page 111:** Equation (5.30) and the text after the second equation: remove comma in the subscript of β^*_{12}
- **Page 111:** The third and fourth equation of the page should be replaced by

$$\underline{W}^*_{x,y} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} \cos\theta - \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and thus

$$W_{x,y}(1,1) = \frac{W_{11} + 2W_{12}\tan\theta + W_{22}\tan^2\theta}{1 + \tan^2\theta},$$

[“and thus

$W_{x,y}(1,1) =$ ” is added]

- **Page 111:** In the text above equation (5.30) “ $\underline{R}^* \underline{W}^*$ ” should be replaced by “ $\underline{R}^* \cdot \underline{W}^*$ ”
- **Page 113:** Third line after equation (5.33) “direction of the eigenvalue” should be replaced by “direction of the eigenvector”

- **Page 117:** Problem 5.6 should read

5.6. a) Calculate explicitly $\overline{\Delta n}^T \underline{W}^* \overline{\Delta n}$
where

$$\overline{\Delta n} = \begin{pmatrix} i - i^* \\ j - j^* \end{pmatrix}$$

$$\underline{W}^* = \begin{pmatrix} \left(\frac{\partial^2 \Delta \varphi}{\partial i^2} \right) & \left(\frac{\partial^2 \Delta \varphi}{\partial i \partial j} \right) \\ \left(\frac{\partial^2 \Delta \varphi}{\partial i \partial j} \right)^* & \left(\frac{\partial^2 \Delta \varphi}{\partial j^2} \right)^* \end{pmatrix}$$

b) Show that $\nabla = ((\underline{A})^T)^{-1} \nabla_\eta$, where

$$\nabla = \left(\frac{\partial}{\partial i}, \frac{\partial}{\partial j} \right)$$

$$\nabla_\eta = \left(\frac{\partial}{\partial \Delta \eta_1}, \frac{\partial}{\partial \Delta \eta_2} \right)$$

$$\overline{\Delta n} = \underline{A} \overline{\Delta \eta}.$$

What is the result if \underline{A} is a real, symmetric matrix?

What if \underline{A} is an orthogonal matrix ($\underline{A}^T = \underline{A}^{-1}$)?

c) Calculate $\left(\underline{R}^{*1/2} \overline{\Delta \eta} \right)^T \underline{R}^{*-1/2}$

when $\underline{R}^{-1/2} \underline{R}^{1/2} = \underline{1}$ and both $\underline{R}^{-1/2}$ and $\underline{R}^{1/2}$ are real, symmetric matrices.

[definition of \underline{W}^* added, indentation and order of definitions changed, and in b) Δ removed from the subscript of ∇_η]

- **Pages 119-120:** For clarity, replace text starting after the second equation on page 119, “Equilibrium conditions are always...”, and finishing before equation (6.2) by
“Critical cluster is in equilibrium with the vapour and thus the Laplace equation (2.8) is valid

$$(P_l^* - P_0) = \frac{2\sigma^*}{r^*},$$

assuming that surface of tension has been chosen as the dividing surface. Using the Laplace equation we see that the second and third terms of $d(\Delta \varphi^*)$ cancel in the following way:

$$(P_0 - P_l^*) dV_l^* + \sigma^* dA^* = \frac{2\sigma^*}{r^*} dV_l^* + \sigma^* dA^* = dV_l^* \left(\frac{-2\sigma^*}{r^*} + \sigma^* \frac{dA^*}{dV_l^*} \right) = 0,$$

since for a spherical cluster $dA^*/dV_l^* = 2/r^*$ (page 20), and we are left with the differential...”

[also subscript l missing from V_l^* in the last form of the first equation on page 120]

- **Page 121:** Footnote should be: “ NOTE: This relation is a close relative to the basic equation $N_i = - \left(\frac{\partial \Omega}{\partial \mu_i} \right)_{T, V, \mu_{j \neq i}}$ ”
[add subscript $j \neq i$ to the μ that is kept constant]
- **Page 122:** Footnote should be: “ NOTE: This is closely linked with the basic relation $\mathcal{S} = - \left(\frac{\partial \Omega}{\partial T} \right)_{V, \mu_i}$ ”
[add subscript i to the μ that is kept constant]
- **Page 126:** The text between the first and the second equations of the page, “When N molecules change phase...”, should read
“When N molecules change phase at equilibrium pressure and temperature stay constant, and thus Gibbs free energy is the relevant thermodynamic potential to use, see page 45. In this case it can be explicitly seen that Gibbs free energy stays constant since chemical potentials in coexisting phases are equal $\mu_g = \mu_l \dots$ ”
- **Page 127:** In the second line of the text after the third equation, add “ T_0 ” to read
“...when P , N and T_0 are constants...”
- **Page 127:** 7th line after equation (6.11) replace “ $\Delta H^{p,e} = H_g - H_l < 0$ ” with “ $\Delta H^{p,e} = H_g - H_l > 0$ ”
- **Page 128:** Middle row of sub-figures in Figure 6.5 minus sign is missing in both of the figures, should be $S = -(\partial G / \partial T)_{P, N}$
- **Page 129:** In the text after equation (6.14) replace “nominator” with “numerator”
- **Page 130:** The last equation of the page should read

$$\left(\frac{\partial \mu_{i,g}^p(P_{i,e}^p)}{\partial T_0} \right) = v_{i,g}^{p,e} \frac{\partial P_{i,e}^p}{\partial T_0} - s_{i,g}^{p,e}.$$

[subscript l replaced by g in $v_{i,g}^{p,e}$]

- **Page 132:** The second equation line should read

$$= -P_l^* V_l^* + A^* \sigma^* + T_0 \mathcal{S}_d^* + \sum N_{i,d}^* \mu_{i,l}^* - \sum N_{i,d}^* \mu_{i,g}^0$$

[minus sign added in front of $P_l^* V_l^*$]

- **Page 132:** Third line above equation (6.20), the term should read $\sum N_{i,d}^* u_{i,l}^{p,e}$
[superscript s replaced by e in $u_{i,l}^{p,e}$]
- **Page 132:** The last equation should read

$$J = C^* Z \exp \left(\frac{-\Delta \varphi^*}{kT_0} \right),$$

[minus sign added inside the exponential]

- **Page 133:** The second equation should read

$$\left(\frac{\partial \ln J}{\partial T_0} \right)_{\mathcal{A}_{i,g}} = \frac{\Delta h^{p,e} - kT_0 + \Delta U_{l,p}^*}{kT_0^2}.$$

[power 2 should be removed from $\Delta h^{p,e}$]

- **Page 133:** In Problem 6.1. replace “ $m_1 \gg m^*$ ” by “ $m_1 \ll m^*$ ”
- **Page 141:** Equation (7.11) should read

$$f_V = \frac{1}{4} \left\{ 2 + 3 \left(\frac{1 - X\mathcal{M}}{d_X} \right) - \left(\frac{1 - X\mathcal{M}}{d_X} \right)^3 - X^3 \left[2 - 3 \left(\frac{X - \mathcal{M}}{d_X} \right) + \left(\frac{X - \mathcal{M}}{d_X} \right)^3 \right] \right\}. \quad (7.11)$$

[replace x by X in the subscript of d in the last term of the the first line]

- **Page 142:** The first equation should read

$$\begin{aligned} \Delta \varphi^{\text{het}*} &= \sum_i \Delta \mu_i N_{i,d}^{\text{het}*} + \sigma_{g,l} A_{g,l}^* + (\sigma_{l,\text{sol}} - \sigma_{g,\text{sol}}) A_{l,\text{sol}}^* \\ &= \sum_i \Delta \mu_i N_{i,d}^{\text{het}*} + \sigma_{g,l} A_{g,l}^* - \cos \vartheta \sigma_{g,l} A_{l,\text{sol}}^*, \end{aligned}$$

$[(\sigma_{g,\text{sol}} - \sigma_{gl\text{sol}})]$ replaced by $(\sigma_{l,\text{sol}} - \sigma_{g,\text{sol}})$ on the first line]

- **Page 142:** Equation (7.12) should read

$$\begin{aligned} \Delta \varphi^{\text{het}*} &= - \sum_i N_{i,d}^{\text{het}*} v_{i,l} \frac{2\sigma_{g,l}^*}{r^*} + \sigma_{g,l}^* A_{g,l}^* - \cos \vartheta \sigma_{g,l}^* A_{l,\text{sol}}^* \\ &= \sigma_{g,l}^* \left(\frac{-2V_l^{\text{het}*}}{r^*} + A_{g,l}^* - \cos \vartheta A_{l,\text{sol}}^* \right), \end{aligned} \quad (7.12)$$

[- sign in front of $\cos \vartheta \sigma_{g,l}^* A_{l,\text{sol}}^*$ instead of a + sign]

- **Page 142:**
Just before equation (7.14) $X = r^*/R_p$ should be $X = R_p/r^*$
- **Page 142:** Equation (7.14) should read

$$\begin{aligned} f_\varphi &= \frac{1}{2} \left\{ 1 + \left(\frac{1 - X\mathcal{M}}{d_X} \right)^3 + X^3 \left[2 - 3 \left(\frac{X - \mathcal{M}}{d_X} \right) + \left(\frac{X - \mathcal{M}}{d_X} \right)^3 \right] \right. \\ &\quad \left. + 3\mathcal{M}X^2 \left(\frac{X - \mathcal{M}}{d_X} - 1 \right) \right\}. \end{aligned} \quad (7.14)$$

[“ $1 - 3(\dots$ ” replaced by “ $2 - 3(\dots$ ” in the term proportional to X^3]

- **Page 146:**

Equation(7.20) should read

$$\beta_i^{\text{het}} = C_{i,\text{mon}}^{\text{vapour}} \sqrt{\frac{kT_0}{2\pi m_i}} 2\pi r^{*2} (1 - \cos \psi^*). \quad (7.20)$$

$[C_{i,\text{mon}}^{e,\text{vapour}}$ should be replaced by $C_{i,\text{mon}}^{\text{vapour}}$]

- **Page 146:** The text four lines before equation(7.21) and equation (7.21) should read

... $C_{i,\text{mon}}^{\text{het}} L^* \delta_i = C_{i,\text{mon}}^{\text{het}} \delta_i 2\pi R_p \sin \phi^*$. If $\nu_{i,\text{diff}}$...

$$\beta_i^{\text{het}} = C_{i,\text{mon}}^{\text{het}} \delta_i 2\pi R_p \sin \phi^* \nu_{i,\text{diff}} \exp \left[\frac{-\Delta e_{i,\text{diff}}}{kT_0} \right]. \quad (7.21)$$

$[C_{i,\text{mon}}^{e,\text{het}}$ should be replaced by $C_{i,\text{mon}}^{\text{het}}$]

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