
Contents

1	Introduction	1
1.1	Application of Mathematics	1
1.2	The Quality of the Mathematical Treatment	2
1.2.1	The Quality of the Model	2
1.2.2	The Quality of the Solving Procedure	5
1.2.3	The Quality of the Data	6
1.3	On Model Harmony	7
1.4	On Information Balance	8
2	Mathematical Representation of Simple Data and Connections	11
2.1	Some Elementary Procedures of Data Analysis	11
2.1.1	Data and Their Representation	11
2.1.2	Simple Procedures of Data Analysis	13
2.2	Representation of Functional Relationships Basing on Data	17
2.2.1	Relationships and Data	18
2.2.2	Interpolation	19
2.3	Local Approximation	21
2.3.1	Approximation	22
2.3.2	Other Approximation Principles	28
2.3.3	Empirical Smoothing	30
2.4	Global Approximation	31
2.4.1	Approximating Functional Relationships	32
2.4.2	Approximation with Locally Variable Setups	33
2.4.3	Approximation in Differential and Integral Equations	37
2.5	Approximate Optimization of Empirical Functions	37
3	Specification and Use of Observation Fuzziness	39
3.1	Specification by Intervals	39
3.1.1	Simple Error Propagation	39
3.1.2	Basic Ideas of Interval Mathematics	40

3.2	Specification by Fuzzy Sets	43
3.2.1	The Idea of a Fuzzy Set	44
3.2.2	Specification of Fuzzy Sets	46
3.2.3	Operations with Fuzzy Sets	52
3.2.4	Connections Via Functions and of Fuzzy Numbers	54
3.2.5	Fuzzy Relations	59
4	Specification and Use of Uncertain Variability	63
4.1	Chance and Probability	63
4.1.1	Model Ideas for the Notion Chance	63
4.1.2	Probability	65
4.1.3	Random Variables and Their Distributions	69
4.1.4	Asymptotic Statements	76
4.2	Probabilistic Inference	77
4.2.1	Samples	78
4.2.2	Parameter Estimation	81
4.2.3	Testing of Hypotheses	85
4.2.4	Problems with Imprecise Data	87
4.3	Bayesian Theory	89
4.3.1	Bayesian Inference	90
4.3.2	Hierarchical Inference and Robustness	92
4.3.3	Numerical Problems	94
5	Specification of Vagueness of Statements on Sets	97
5.1	Fuzzy Measures	97
5.1.1	The Idea of a Fuzzy Measure	97
5.1.2	Forms and Properties of Fuzzy Measures	99
5.2	Simple Inference with Fuzzy Measures	101
5.2.1	Specification of Partial Ignorance	102
5.2.2	Possibilistic Inference	105
5.3	Probability and Fuzziness	107
5.3.1	Probability of Fuzzy Events	108
5.3.2	Random Fuzzy Sets	111
6	Methods from Qualitative Data Analysis	113
6.1	Crisp Classification of Crisp Data	113
6.1.1	The Problem of Cluster Analysis	113
6.1.2	Mathematical Formulation of the Problem	115
6.1.3	Some Procedures for Crisp Cluster Partition	117
6.1.4	Clustering with a Mathematical-Statistical Background	120
6.1.5	Basic Ideas of Neural Networks	122
6.2	Fuzzy Classification of Crisp Data	129
6.2.1	Fuzzy Cluster	129
6.2.2	Procedures of Fuzzy Cluster Analysis	130

6.3	Fuzzy Classification of Fuzzy Data	133
6.3.1	Fuzzy Similarity of Fuzzy Data	133
6.3.2	The Use of The Concept for Classification	137
7	Evaluation of Functional Relationships	141
7.1	Statistical Regression Analysis	142
7.1.1	Model Assumptions with Random Dependent Variables	142
7.1.2	The Problem of Estimation	144
7.1.3	Discussion of the Model Assumptions	147
7.1.4	Further A-Priori Knowledge and Assumptions	151
7.1.5	Random Influence in All Variables	153
7.1.6	Local Regression in a Random Field	154
7.2	Fuzzy Evaluation of Functional Relationships	159
7.2.1	Crisp Data Analysis as a Starting Point	160
7.2.2	Explorative Evaluation of Functional Relationships	162
7.2.3	Evaluation with Additional Assumptions	169
7.2.4	Inference with Fuzzy Parameter Values	170
8	Outlook and Conclusions	173
	References	177
	Index	183



<http://www.springer.com/978-3-540-28457-4>

Mathematics of Uncertainty
Ideas, Methods, Application Problems
Bandemer, H.
2006, XIV, 190 p., Hardcover
ISBN: 978-3-540-28457-4