
Contents

Volume I Basic Theory

1	Generalized Differentiation in Banach Spaces	3
1.1	Generalized Normals to Nonconvex Sets	4
1.1.1	Basic Definitions and Some Properties	4
1.1.2	Tangential Approximations	12
1.1.3	Calculus of Generalized Normals	18
1.1.4	Sequential Normal Compactness of Sets	27
1.1.5	Variational Descriptions and Minimality	33
1.2	Coderivatives of Set-Valued Mappings	39
1.2.1	Basic Definitions and Representations	40
1.2.2	Lipschitzian Properties	47
1.2.3	Metric Regularity and Covering	56
1.2.4	Calculus of Coderivatives in Banach Spaces	70
1.2.5	Sequential Normal Compactness of Mappings	75
1.3	Subdifferentials of Nonsmooth Functions	81
1.3.1	Basic Definitions and Relationships	82
1.3.2	Fréchet-Like ε -Subgradients and Limiting Representations	87
1.3.3	Subdifferentiation of Distance Functions	97
1.3.4	Subdifferential Calculus in Banach Spaces	112
1.3.5	Second-Order Subdifferentials	121
1.4	Commentary to Chap. 1	132
2	Extremal Principle in Variational Analysis	171
2.1	Set Extremality and Nonconvex Separation	172
2.1.1	Extremal Systems of Sets	172
2.1.2	Versions of the Extremal Principle and Supporting Properties	174
2.1.3	Extremal Principle in Finite Dimensions	178
2.2	Extremal Principle in Asplund Spaces	180

2.2.1	Approximate Extremal Principle in Smooth Banach Spaces	180
2.2.2	Separable Reduction	183
2.2.3	Extremal Characterizations of Asplund Spaces	195
2.3	Relations with Variational Principles	203
2.3.1	Ekeland Variational Principle	204
2.3.2	Subdifferential Variational Principles	206
2.3.3	Smooth Variational Principles	210
2.4	Representations and Characterizations in Asplund Spaces	214
2.4.1	Subgradients, Normals, and Coderivatives in Asplund Spaces	214
2.4.2	Representations of Singular Subgradients and Horizontal Normals to Graphs and Epigraphs	223
2.5	Versions of Extremal Principle in Banach Spaces	230
2.5.1	Axiomatic Normal and Subdifferential Structures	231
2.5.2	Specific Normal and Subdifferential Structures	235
2.5.3	Abstract Versions of Extremal Principle	245
2.6	Commentary to Chap. 2	249
3	Full Calculus in Asplund Spaces	261
3.1	Calculus Rules for Normals and Coderivatives	261
3.1.1	Calculus of Normal Cones	262
3.1.2	Calculus of Coderivatives	274
3.1.3	Strictly Lipschitzian Behavior and Coderivative Scalarization	287
3.2	Subdifferential Calculus and Related Topics	296
3.2.1	Calculus Rules for Basic and Singular Subgradients	296
3.2.2	Approximate Mean Value Theorem with Some Applications	308
3.2.3	Connections with Other Subdifferentials	317
3.2.4	Graphical Regularity of Lipschitzian Mappings	327
3.2.5	Second-Order Subdifferential Calculus	335
3.3	SNC Calculus for Sets and Mappings	341
3.3.1	Sequential Normal Compactness of Set Intersections and Inverse Images	341
3.3.2	Sequential Normal Compactness for Sums and Related Operations with Maps	349
3.3.3	Sequential Normal Compactness for Compositions of Maps	354
3.4	Commentary to Chap. 3	361
4	Characterizations of Well-Posedness and Sensitivity Analysis	377
4.1	Neighborhood Criteria and Exact Bounds	378
4.1.1	Neighborhood Characterizations of Covering	378

4.1.2	Neighborhood Characterizations of Metric Regularity and Lipschitzian Behavior	382
4.2	Pointbased Characterizations	384
4.2.1	Lipschitzian Properties via Normal and Mixed Coderivatives	385
4.2.2	Pointbased Characterizations of Covering and Metric Regularity	394
4.2.3	Metric Regularity under Perturbations	399
4.3	Sensitivity Analysis for Constraint Systems	406
4.3.1	Coderivatives of Parametric Constraint Systems	407
4.3.2	Lipschitzian Stability of Constraint Systems	414
4.4	Sensitivity Analysis for Variational Systems	421
4.4.1	Coderivatives of Parametric Variational Systems	422
4.4.2	Coderivative Analysis of Lipschitzian Stability	436
4.4.3	Lipschitzian Stability under Canonical Perturbations ..	450
4.5	Commentary to Chap. 4	462

Volume II Applications

5	Constrained Optimization and Equilibria	3
5.1	Necessary Conditions in Mathematical Programming	3
5.1.1	Minimization Problems with Geometric Constraints ...	4
5.1.2	Necessary Conditions under Operator Constraints	9
5.1.3	Necessary Conditions under Functional Constraints	22
5.1.4	Suboptimality Conditions for Constrained Problems ...	41
5.2	Mathematical Programs with Equilibrium Constraints	46
5.2.1	Necessary Conditions for Abstract MPECs	47
5.2.2	Variational Systems as Equilibrium Constraints	51
5.2.3	Refined Lower Subdifferential Conditions for MPECs via Exact Penalization	61
5.3	Multiobjective Optimization	69
5.3.1	Optimal Solutions to Multiobjective Problems	70
5.3.2	Generalized Order Optimality	73
5.3.3	Extremal Principle for Set-Valued Mappings	83
5.3.4	Optimality Conditions with Respect to Closed Preferences	92
5.3.5	Multiobjective Optimization with Equilibrium Constraints	99
5.4	Subextremality and Suboptimality at Linear Rate	109
5.4.1	Linear Subextremality of Set Systems	110
5.4.2	Linear Suboptimality in Multiobjective Optimization ..	115
5.4.3	Linear Suboptimality for Minimization Problems	125
5.5	Commentary to Chap. 5	131

6	Optimal Control of Evolution Systems in Banach Spaces . . .	159
6.1	Optimal Control of Discrete-Time and Continuous-time Evolution Inclusions	160
6.1.1	Differential Inclusions and Their Discrete Approximations	160
6.1.2	Bolza Problem for Differential Inclusions and Relaxation Stability	168
6.1.3	Well-Posed Discrete Approximations of the Bolza Problem	175
6.1.4	Necessary Optimality Conditions for Discrete-Time Inclusions	184
6.1.5	Euler-Lagrange Conditions for Relaxed Minimizers . . .	198
6.2	Necessary Optimality Conditions for Differential Inclusions without Relaxation	210
6.2.1	Euler-Lagrange and Maximum Conditions for Intermediate Local Minimizers	211
6.2.2	Discussion and Examples	219
6.3	Maximum Principle for Continuous-Time Systems with Smooth Dynamics	227
6.3.1	Formulation and Discussion of Main Results	228
6.3.2	Maximum Principle for Free-Endpoint Problems	234
6.3.3	Transversality Conditions for Problems with Inequality Constraints	239
6.3.4	Transversality Conditions for Problems with Equality Constraints	244
6.4	Approximate Maximum Principle in Optimal Control	248
6.4.1	Exact and Approximate Maximum Principles for Discrete-Time Control Systems	248
6.4.2	Uniformly Upper Subdifferentiable Functions	254
6.4.3	Approximate Maximum Principle for Free-Endpoint Control Systems	258
6.4.4	Approximate Maximum Principle under Endpoint Constraints: Positive and Negative Statements	268
6.4.5	Approximate Maximum Principle under Endpoint Constraints: Proofs and Applications	276
6.4.6	Control Systems with Delays and of Neutral Type	290
6.5	Commentary to Chap. 6	297
7	Optimal Control of Distributed Systems	335
7.1	Optimization of Differential-Algebraic Inclusions with Delays . .	336
7.1.1	Discrete Approximations of Differential-Algebraic Inclusions	338
7.1.2	Strong Convergence of Discrete Approximations	346

7.1.3	Necessary Optimality Conditions for Difference-Algebraic Systems	352
7.1.4	Euler-Lagrange and Hamiltonian Conditions for Differential-Algebraic Systems	357
7.2	Neumann Boundary Control of Semilinear Constrained Hyperbolic Equations	364
7.2.1	Problem Formulation and Necessary Optimality Conditions for Neumann Boundary Controls	365
7.2.2	Analysis of State and Adjoint Systems in the Neumann Problem	369
7.2.3	Needle-Type Variations and Increment Formula	376
7.2.4	Proof of Necessary Optimality Conditions	380
7.3	Dirichlet Boundary Control of Linear Constrained Hyperbolic Equations	386
7.3.1	Problem Formulation and Main Results for Dirichlet Controls	387
7.3.2	Existence of Dirichlet Optimal Controls	390
7.3.3	Adjoint System in the Dirichlet Problem	391
7.3.4	Proof of Optimality Conditions	395
7.4	Minimax Control of Parabolic Systems with Pointwise State Constraints	398
7.4.1	Problem Formulation and Splitting	400
7.4.2	Properties of Mild Solutions and Minimax Existence Theorem	404
7.4.3	Suboptimality Conditions for Worst Perturbations	410
7.4.4	Suboptimal Controls under Worst Perturbations	422
7.4.5	Necessary Optimality Conditions under State Constraints	427
7.5	Commentary to Chap. 7	439
8	Applications to Economics	461
8.1	Models of Welfare Economics	461
8.1.1	Basic Concepts and Model Description	462
8.1.2	Net Demand Qualification Conditions for Pareto and Weak Pareto Optimal Allocations	465
8.2	Second Welfare Theorem for Nonconvex Economies	468
8.2.1	Approximate Versions of Second Welfare Theorem	469
8.2.2	Exact Versions of Second Welfare Theorem	474
8.3	Nonconvex Economies with Ordered Commodity Spaces	477
8.3.1	Positive Marginal Prices	477
8.3.2	Enhanced Results for Strong Pareto Optimality	479
8.4	Abstract Versions and Further Extensions	484
8.4.1	Abstract Versions of Second Welfare Theorem	484
8.4.2	Public Goods and Restriction on Exchange	490
8.5	Commentary to Chap. 8	492

References 507

List of Statements 573

Glossary of Notation 595

Subject Index 599

Variational Analysis and Generalized Differentiation II
Applications

Mordukhovich, B.S.

2006, XXII, 610 p., Hardcover

ISBN: 978-3-540-25438-6