

# Thermodynamics as a Universal Science

In this book we are concerned with engineering and assume a unified and universal point of view. For this we make no distinction between ordinary heat theory and thermodynamics.

In other words, we introduce entropy immediately as thermal charge and state that the transported power equals absolute temperature multiplied by entropy current, in an analogy with electrical engineering, where the power is voltage times electric charge flow or current.

So for us, entropy is not a complex integral but simply a thermal charge; the relation to the integral is valid only in special cases as detailed in section 1.3. In his book, Fuchs (1996) adopts the same point of view and offers many applications.

We include chemistry in our book, where chemical potential is a tension-like variable and molar flow a current-like variable.

Therefore we like to call it chemical tension to bring out the analogy with electric tension or voltage. Thus entropy itself, as accumulated entropy flow, is the analog of electric charge.

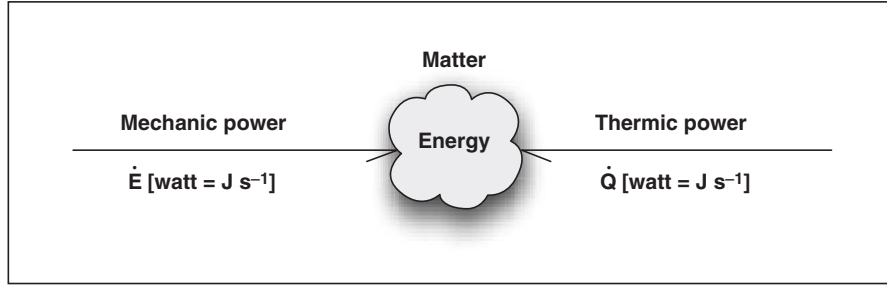
Only Fuchs has dropped the first six letters from his book on thermodynamics and calls it “The Dynamics of Heat”<sup>1</sup>.

## 1.1 Transmission of thermal power

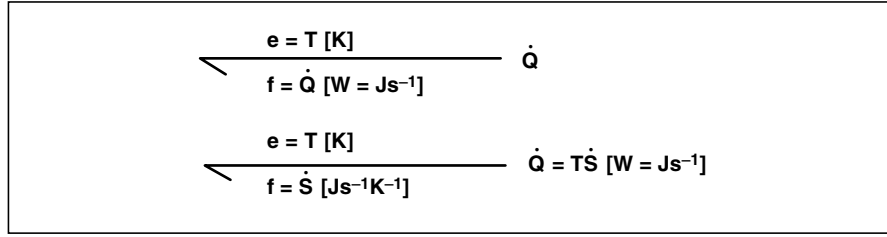
A good part of our book is concerned with the transmission of entropy and heat in thermal systems, mainly because we are preparing for the simulation

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<sup>1</sup> For people used to conventional dimensional analysis, we could substitute  
 $1 \text{ J} = 1 \text{ kg s}^2 \text{ m}^{-1}$ ;  $1 \text{ W} = 1 \text{ kg s}^2 \text{ m}^{-2}$ .



**Fig. 1.1.** Matter, energy and power



**Fig. 1.2.** Connections in BG's: bottom, true with temperature and entropy flow; top, pseudo with temperature and heat flow. Here the product of effort and flow has no physical significance.

of such systems. Here it should be observed that entropy flow and, with it, thermal power can be transmitted by three different processes:

- Thermal conduction in non-moving matter;
- Convection by moving fluids in pipes;
- Thermal radiation as in space by the sun.

These power transmission mechanisms are well known to the thermal engineer and each have different laws as we explain later. Conduction is preponderant in electricity but is only used occasionally in thermodynamics, such as through house walls (sec 1.2). More important is the convection of thermal power by a moving fluid, to which we give most of our attention (sec 3.1). Thermal radiation is important only in principle since, in practice, it is important for solar energy (sec 1.7, with remarks on the thermal balance of the earth in sec 5.5.4).

Limiting ourselves here to thermal conduction, we have simply:

$$\dot{Q} = T\dot{S} \quad (1.1)$$

This equation is fundamental and often referred to as Carnot's. It is a true Bondgraph equation where effort times flow represents a power, the thermal energy flow or heat flow; it is illustrated in Fig 1.2 at bottom. On the other

hand, entropy convection in flowing fluids is qualitatively heat flow divided by absolute temperature as detailed in section 2.4. The more frequent case of heat flow or enthalpy flow in fluid pipes will be examined in section 3.1.

We have already mentioned heat conduction through walls. For this there are two kinds of BG: the true BG with temperature and entropy flow (Fig 1.2 bottom) and the pseudo-BG with temperature and heat flow (Fig 1.2 top); both are used in practice as we show in section 2.4. This is perfectly possible, but the advantages of a true BG are lost.

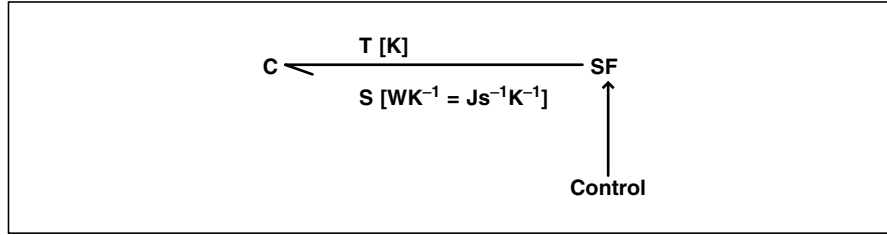
### Accumulation equation

The accumulation of internal energy in a piece of matter, solid, liquid or gas, is given traditionally as:

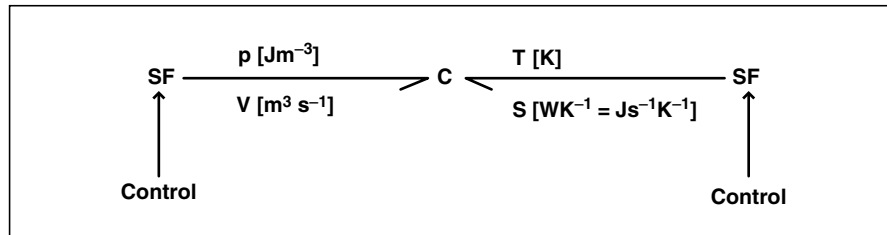
$$\frac{dU}{dt} = T \dot{S} \quad (1.2)$$

It means that the accumulation of thermal power is equal to the (absolute) temperature times the entropy flow. This is the internal energy, conventionally called  $U$  in thermodynamics. The equation would be represented by the true BG of Fig 1.3 or Fig 1.4.

The associated energies: free energy  $F$ , enthalpy  $H$  and free enthalpy  $G$  will be described in section 1.3.



**Fig. 1.3.** Connection of a one-port-C and a flow source in a true BG. The vertical arrow indicates that the source can be tuned by a control system.



**Fig. 1.4.** Connections in a true BG: temperature and entropy flow at right, pressure and volume flow at left.

Note that we could integrate equation 1.2 over a time interval  $dt$  and contract it, which leads to

$$dU = T\dot{S}dt = T ds \quad (1.3)$$

This is the usual form in thermodynamics, but we prefer to concentrate on entropy flow and accumulation of energy as shown in equation 1.2. In other words, there are reasons for using the flows of entropy and power, not the differentials; this leads to easier physical interpretation and greater simplicity, as we shall see throughout the text.

Hence entropy is a kind of featureless thermal charge, necessarily connected with the flow of thermal power. Some people call entropy a gray paste to stress that it has no particular feature and can ooze everywhere with thermal effects; we discuss its properties later with the laws of thermodynamics (section 1.3.4). We shall also see when it is better to concentrate on entropy flow and when on heat flow (section 2.4); both exist and are needed for different problems.

To reiterate, entropy is a thermal charge, somewhat akin to an electric charge but having a quite different physical nature and properties. It is like a substance, not conserved but rather semi-conserved. This means that it can never be destroyed, but is generated by all kinds of frictions, usually called irreversibilities (Chap 2). Contrary to electric charge, entropy is not a source of an electric field and is much simpler in this respect. An interesting historical perspective, relating also to the old term “calorics”, can be found in Falk 1985.

In this chapter we will concentrate on frictionless processes and elements, whilst friction with entropy generation will be dealt with in chapter 2.

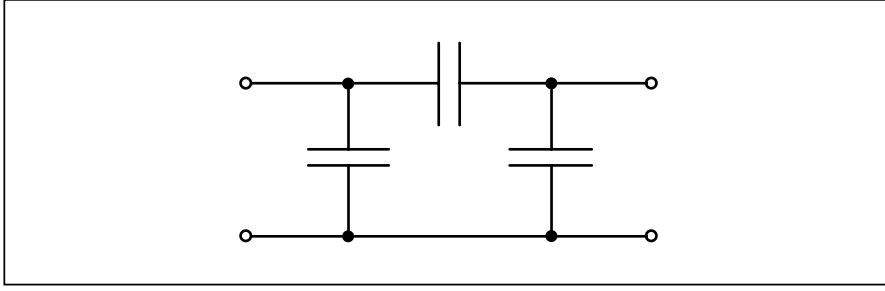
In thermodynamics there can be other variables involved. For example, in a body of ideal gas, the volume can change, as we shall show later in Fig 1.8. Often the enclosed matter is a fluid such as an ideal gas. It only needs to be well stirred to have the same temperature everywhere. Alternatively, it could be a solid but then the processes must be slow enough to allow temperatures to equalize. Apart from this, thermodynamics is equally applicable to fluids and solids.

Multiport-C's are of such great importance for thermodynamics that we shall devote the next section to them, before stating the laws of thermodynamics in section 1.3.

## 1.2 Examples of oneport-C's and multiport-C's

### Oneports

The simplest C-element has just one port and can represent a common capacitor. In the linear case, for the charge  $q$ , it has the equation



**Fig. 1.5.** Electrical example of a multiport-C. It is simply a capacitor network.

$$q = \int i \, dt; \quad u = \frac{1}{C}q \quad (1.4)$$

### Twoports and multiports

We can extend this to electric twoport-C's and multiport-C's. As an example of a multiport-C, Fig.1.5 shows a capacitor network of three capacitors. Here the coupling capacitor between the two capacitors on either side is important; without it the network would decompose into two separate oneport capacitors. Other capacitor networks can also be represented by multiport-C's, as long as they consist only of capacitors. In electrical circuit theory there are equivalencies between the Pi and X networks<sup>2</sup>.

A linear multiport-C can be represented by the following equation

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ C_{i,j} \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_{11}} & \frac{1}{C_{12}} \\ \frac{1}{C_{21}} & \frac{1}{C_{21}} \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad (1.5)$$

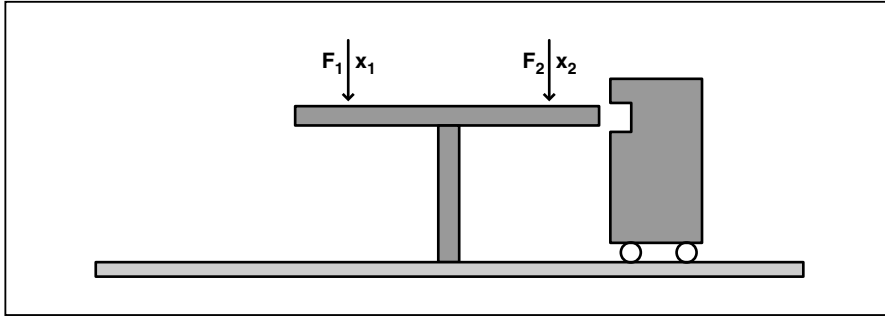
By Maxwell reciprocity, the coupling terms  $C_{12}$  and  $C_{21}$  in the above equation are equal.

Maxwell himself discovered the reciprocity on a mechanical example, and it is still known today as Maxwell's displacement theorem in elastic structures.

### Mechanical example of a table with leg fixed to the ground

One mechanical example is a table with one leg (or a T-beam) with its stem securely fixed to the ground so that it cannot move, as shown in fig 1.6. Each flange has a point of application for the forces  $F_1$  and  $F_2$  which produce

<sup>2</sup> If the capacitors have leakage resistance or if there is resistance of wires, the arrangement ceases to behave like a multiport-C.



**Fig. 1.6.** Mechanical example of a multiport-C: a table with a central leg fixed in the ground and two points of application of forces  $F_1$  and  $F_2$ , together with the displacements  $x_1$  and  $x_2$ . On the right, under  $x_2$  there is a block that can be applied and removed alternately.

the displacements  $x_1$  and  $x_2$ . The stem and flanges have comparable elastic stiffness. There is also a rigid support which, when applied, prevents movement of one flange.

We now carry out the following experiments on the T-beam. With the support removed, we measure a certain elasticity under  $F_1$ . The flange with  $F_2$  will move depending on the relative elasticity of the stem and flanges. If the stem is infinitely stiff, flange  $F_2$  would not move at all.

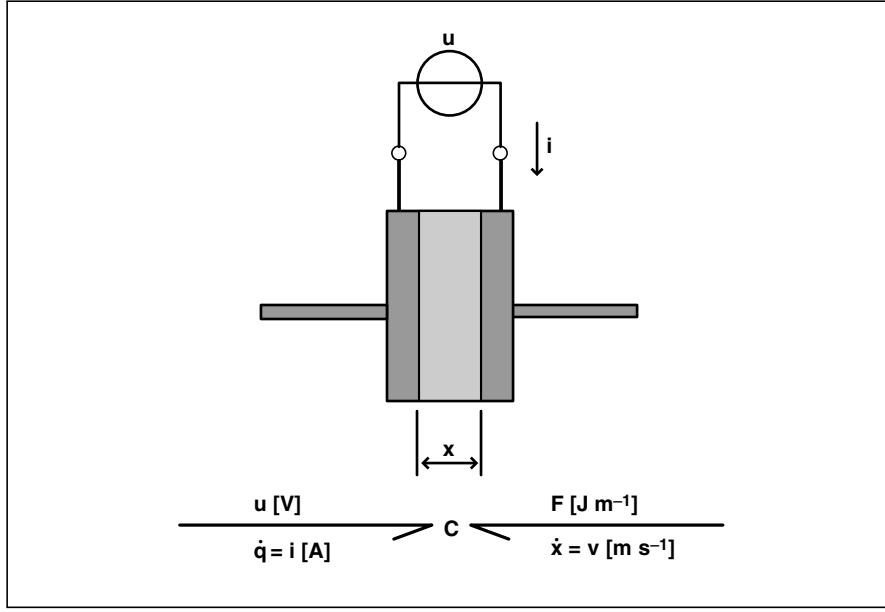
When we apply the support under flange 2, all movement is prevented. The stiffness at flange 1 becomes higher, and flange 2 exerts a force on the support that varies with the force on flange 1. In terms of energies, with an infinitely stiff support no energy will emerge at point 2. With the support removed and replaced by a constant force, energy can emerge at this point. The T-beam is less stiff and energy emerges as a constant force with a finite value.

The whole situation is similar to that in thermodynamics: the applied support is equivalent to constant volume and the reciprocal of entropy capacity is smaller. With the support removed we have less stiffness which corresponds to constant pressure and more entropy capacity. So the difference between  $c_v$  and  $c_p$ , heat capacities at constant volume and constant pressure, is really a property of a multiport-C, entropy being the equivalent of the displacement.

An interesting and frequently asked question is: what happens if we apply so much force that the flanges go to a plastic deformation which they retain on unloading? Answer: the T-beam ceases to behave like a multiport-C.

### Moving plates capacitor as an electromechanical example

Fig 1.7 shows a section through an electric capacitor where the distance between the plates can be varied. There is a force of attraction between the



**Fig. 1.7.** Electromechanical example of a multiport-C. Here the electric bond has voltage and current as effort and flow, the mechanical one attraction force of the plates and displacement.

plates and it is, thus, a multiport-C with voltage and charge at the electric side and force and displacement at the mechanical side.

An interesting aspect is that this is an interdisciplinary twoport-C. Capacity  $C$  and voltage  $u$  are

$$C = \frac{\varepsilon A}{x}; \quad q = Cu; \quad \varepsilon = \varepsilon_0 \varepsilon_r$$

where  $A$  = area,  $x$  = plate separation and  $\varepsilon$  the induction constant. For the voltage and the attractive force we obtain

$$u = \frac{qx}{\varepsilon}; \quad F = \frac{q^2}{2\varepsilon A} \quad (1.6)$$

Note that here voltage and force are given correctly in terms of plate separation and charge.

The increments of voltage and force become

$$\begin{bmatrix} du \\ dF \end{bmatrix} = \begin{bmatrix} \frac{x}{\varepsilon A} & \frac{2q}{\varepsilon A} \\ \frac{2q}{\varepsilon A} & 0 \end{bmatrix} * \begin{bmatrix} dq \\ dx \end{bmatrix} \quad (1.7)$$

Note that Maxwell reciprocity is also true in this nonlinear multiport-C.

One can construct for amusement an electrical Carnot cycle by using a capacitor of this type with movable plates for electromechanical energy conversion: a low electric charge is applied for separating the plates and a high charge for bringing them together. Using Carnot's invention (1824), one can interpose two phases with a constant charge and variable plate separation to adapt the voltage between the phases (see section 2.3). This is not very practical but can serve to illustrate the use of multiport-C.

### 1.3 Thermal multiport-C and the laws of thermodynamics

#### 1.3.1 General

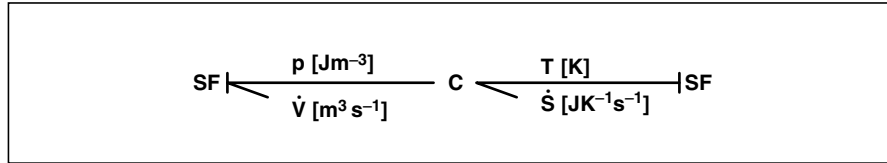
After familiarizing ourselves with multiport-C in different disciplines, most of the thermodynamics is simply contained in the properties of a twoport-C with temperature and entropy at the thermal side and pressure and volume at the hydraulic side. This is shown in Fig 1.8. So we have for the accumulation of power in a twoport

$$\frac{dU}{dt} = T\dot{S} - p\dot{V} \quad (1.8)$$

Here energy is contained in the multiport-C, which can be driven from flow sources. The multiport-C simply shows the accumulation of thermal energy through a thermal bond on the right, and the volume or fluid bond on the left. In the fluid bond, power is given as pressure time volume flow. Multiport C-elements appear frequently in physics and some examples are given in the last section. Note that there exist also multiport I-elements and R-elements used elsewhere to study other fields of physics.

The above accumulation equation 1.8 is shown in Fig 1.8 as a Bondgraph with two flow sources for thermal and fluid energy. It can be integrated (compare equation 1.3) into

$$dU = TdS - pdV; \quad dQ = T dS \quad (1.9)$$



**Fig. 1.8.** Similar to Fig 1.2, but with two bonds and two sources in a true BG.



In many texts this is called a TdS equation. Here similar remarks apply: the power formulation in 1.8 is easier to interpret than the TdS formulation, the differentials in 2<sup>3</sup>.

Here energy is contained in the multiport-C, which can be driven from two flow sources as shown in Fig 1.8. To continue, an accumulation of thermal energy occurs through a thermal bond on the right and the volume or fluid bond on the left.

It follows that temperature and pressure can be expressed as derivatives:

$$T = \frac{\partial U(S, V)}{\partial S}; \quad p = -\frac{\partial U(S, V)}{\partial V} \quad (1.10)$$

Here we have S and V as independent variables. Usually a Legendre transformation is introduced at that point to obtain temperature and pressure as independent variables. This is fine but rather obscures the physical content of the argumentation, therefore we shall not do so. It is also known that the Maxwell reciprocity is determined from the cross derivatives of equation 1.10.

### 1.3.2 Control engineering block diagrams

Here it is helpful to use a tool from control engineering: block diagrams.

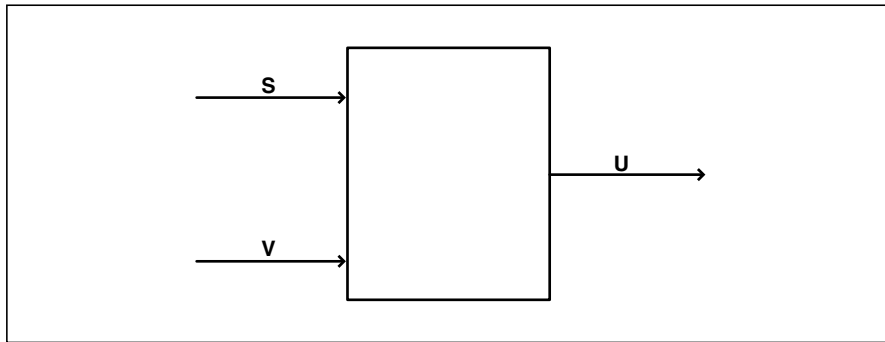
Our equation 1.8 can be represented by the block diagram of Fig 1.9. It shows the internal energy U as a function of entropy and volume in general terms.

In fig 1.10 we go on to the increments. Here the general block of Fig 1.9 splits up into two blocks with a point of addition. As stated, this is for the increments dS and dV. In the blocks, we write the gains T and V as given by equation 1.9. Therefore it can be said that Fig 1.10 is a pictorial representation of this equation.

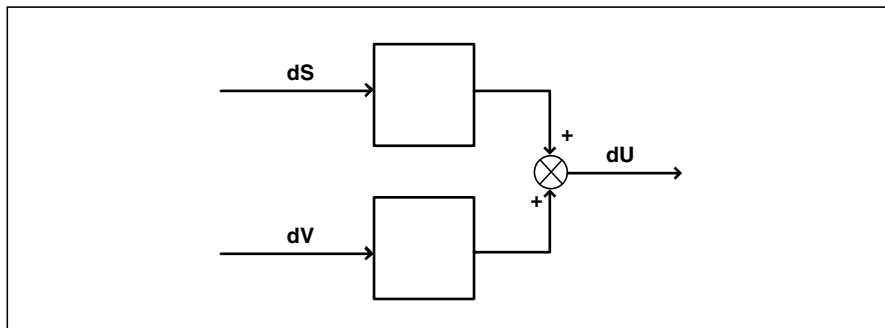
One application of these block diagrams is to express entropy as a function of temperature and pressure, instead of temperature and volume. This is explained by Fig 1.11. Here we have one block with entropy as a function of temperature and volume, and we place before it a block with volume as a function of pressure. It must not be forgotten that volume also depends on temperature, as shown by the vertical connection on the left. We shall continue this development in the next section 1.4.

The next Fig 1.12 shows a BG with the multiport-C subject to pressure and entropy, as is usually the case.

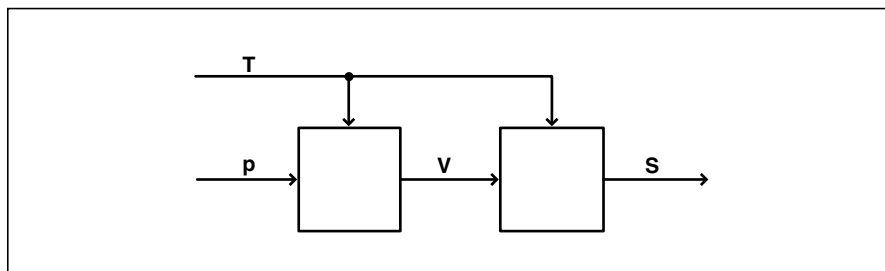
<sup>3</sup> One author (P. Grassmann) uses an asterisk instead of the point above the extensive variable to show clearly the difference between flows (entering and leaving) and the time derivative of the stored energy in accumulation equations like 1.



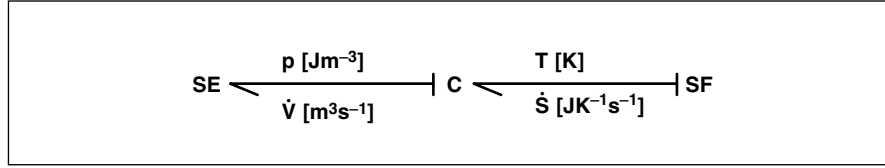
**Fig. 1.9.** Internal energy as a function of entropy and volume.



**Fig. 1.10.** Block diagram for the increments of the variables.



**Fig. 1.11.** General block diagram for entropy as a function of temperature and volume, preceded by a block giving volume as a function of pressure and temperature.



**Fig. 1.12.** Matter under constant pressure shown as a multiport-C and a SE. Here free energy is not conserved.

In this case, on changing the setting of the entropy/temperature source, part of the energy supplied goes into the volume/pressure source on the left. Hence the energy coming from the source on the right is not conserved. This is “elementary, my dear Watson” (quoted freely from Sherlock Homes by Arthur C. Doyle).

In many textbooks the energy emerging on the right of figure 1.12 is called free energy, but its conservation principles are never given: it is not conserved!

**Box:** The Bondgrapher sees immediately that Fig 1.12, with two effort sources on a multiport-C, leads to derivative causality. Hence the BG is incomputable. However, it is useful to explain the principles. If computation is required, we could apply coupling resistors, see appendix A1.

Note in particular the power orientations in Fig 1.12 and in the above equation 1.8. Power is taken positive as it flows toward the multiport-C in the thermal branch, but negative in the fluid branch of the multiport-C. This is the unfortunate convention in thermodynamics, and results in the minus signs of equations 1.8 and 1.9: this sign convention is widely used in thermodynamics, with unpleasant minus signs resulting. It would be better to make the signs all positive, but we shall adhere to convention.

Note also that matter in such a twoport-C has two DOF (degrees of freedom),  $S$  and  $V$  in this case. The associated energies mentioned at the end of this section also have two DOF.

### 1.3.3 Entropy after Clausius

Entropy was introduced by R. Clausius about 1875, by setting the volume constant, stating  $dQ = TdS$  and transposing

$$S = \int \frac{dQ}{T} \quad (1.11)$$

Now this equation is true at constant volume, or when, in a Multiport-C, no energy enters by or leaves from the volume bond. However, to accept it as a definition of entropy after Clausius is unnecessarily limiting.

To reiterate, entropy is a universal concept appearing in all thermal phenomena, or as already mentioned, a featureless gray paste. Sometimes, to add insult to injury, we add to  $dQ$  the suffix *rev*, to indicate that the heat is applied reversibly: we shall discuss its meaning in section 2.3. Other authors use a small circle around the integral sign to show that  $dQ$  is not an exact differential. For us this is a nightmare!

### 1.3.4 Principles or laws of thermodynamics

These principles are in all books on thermodynamics, but we shall look at them here from our own point of view. Firstly we should mention that in BG there is a distinction between power conservation and energy conservation as follows:

- Power conservation: all power is conserved in each instance and therefore energy is also conserved. Examples are junctions, transformers and gyrators.
- Energy conservation: power and thus energy can enter an element but then stays inside and somehow changes its state. The same energy must come out to return the system to rest. Hence energy is conserved in the long run. Examples are accumulators, or in general all the C and I-elements of BGs. This is true also for multiport-C and multiport-I.

Then we come to the laws of thermodynamics, called principles of thermodynamics in French and German.

First law: power and therefore energy is universally conserved; hence it is also referred to as the law of conservation.

Second law: There exists a variable entropy which is never destroyed but is generated by all kinds of friction: it is semi-conserved. Entropy is necessarily associated with heat flow, thus it is also referred to as the law of evolution.

These two laws are sufficient to study thermal effects and their interaction with fluid and other effects. However, generally one adds two more laws:

Third law: At zero absolute temperature, the entropy content of all bodies is zero (Nernst, Germany circa 1910). There is one important proviso: entropy content can be frozen in a body on rapid cooling, important for glass manufacture. In that case, the entropy content is higher also at absolute zero and entropy is calculated only from a certain datum temperature and pressure: usually a 273 K temperature and a 0.1 MPa (atmospheric) pressure.

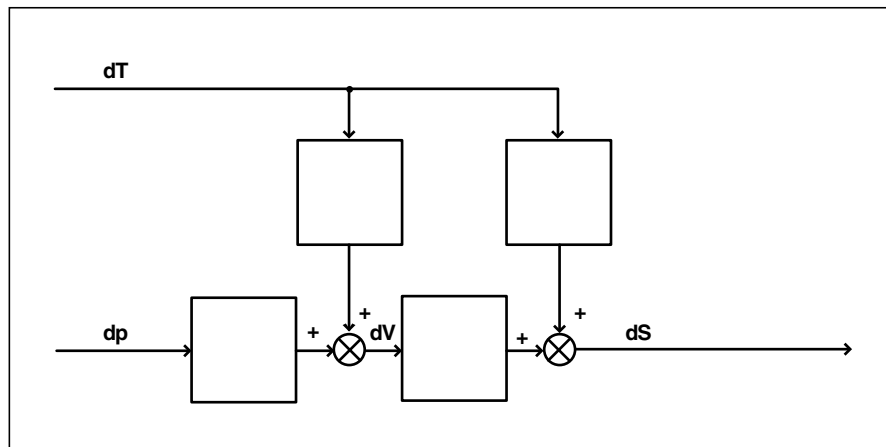
Zero'th law: If a body A has a temperature  $T$  and the temperature of another body B is also equal to  $T$ , then the temperatures of A and B are equal. This seems obvious but it needs to be stated to define temperature properly.

We do not describe here the associated energies such as (Helmholtz) free energy  $F$ , enthalpy  $H$  and free enthalpy (Gibbs free energy)  $G$ . These are obtained by a Legendre transformation [THOMA 2000]. We would merely say that  $U(S, V)$  is energy in terms of entropy and volume, quite a different animal from  $F(T, V)$ , internal energy in terms of temperature and volume. The same holds for the other associated energies  $H$  and  $G$ , except that the associated energies are taken as functions of the different variables  $H(S, p)$  and  $G(T, p)$ : the Legendre transformation is often used but does not clarify the issues.

## 1.4 Thermodynamics in pictures

Thermodynamics is often regarded as a very abstract subject, but it can also be shown in pictures, in the form of schematics, BGs and the well known block diagrams of control engineering.

As an example, let us start from Fig 1.13 above and go on to incremental variables. The block with the two entries  $dT$  and  $dp$ , producing  $dV$ , splits into two simple blocks with an addition point. The derivatives can be written as gains into the various blocks. We emphasise that this block diagram is a consequence of our accumulation equation and independent of the nature of the enclosed body, whether an ideal gas or not.



**Fig. 1.13.** Incremental block diagram for entropy as a function of volume and temperature, with a block for volume as a function of pressure and temperature. This last connection is easily forgotten.

As an application, we calculate the specific heat at constant volume and constant pressure. The heat flow under constant volume is

$$dQ = T dS = c_v dT$$

Which gives

$$c_v = T \frac{\partial S(V, T)}{\partial T} = T S_{-v}^{/T} \quad (1.12)$$

where  $c_v$  is the specific heat at constant volume. Note that the shortened form of the derivative is very practical and that we shall use it for our calculations, but not elsewhere. See box for explanation.

**Box:** Note on derivatives. In any multiport-C there appear many derivatives with certain parameters held constant. Because of this we have used the following simplified notation. This notation is very compact and can be entered in a block diagram.

We use a vertical stroke in front of the superscript to give derivation, followed by the variable to which it is applied. As a subscript we have a horizontal stroke followed by the variable that is kept constant. An example is

$$p_{-v}^{/t} = T \frac{\partial p(T, V)}{\partial T}$$

which is the increase in pressure when volume is kept constant. In the case of the rails in railway lines, this is about 3 MPa/K, which can lead to train derailment when the weather is hot. We shall use this compact notation only in the present section 1.4.

Returning to specific heat at constant pressure  $c_p$ , we support Fig 1.9 with the following mathematical development. We make entropy a function of temperature and volume, where volume itself is a function of pressure and temperature.

This was shown in Fig 1.8 in general terms and in Fig 1.9 in terms of increments  $dT$ ,  $dS$  and so on. According to the rules of differentiation, the blocks with two entries split into an addition point and two simple blocks.

Now at constant volume we have  $dV = 0$  and obtain equation 1.12. At constant pressure, on the other hand, we have  $dp = 0$  and get equation 1.13

$$dS_{-p} = S_{-v}^{/t} dT + V_{-p}^{/T} S_{-T}^{/v} dT \quad (1.13)$$

In other words, here the volume  $V$  changes also through the vertical branch of fig 1.11 and this increases  $S$ . The next step is to invoke Maxwell reciprocity

in the form

$$S_{-T}^{/v} dT = p_{-v}^{/T}$$

which gives, for the increases at constant pressure

$$S_{-p}^{/T} = S_{-v}^{/T} + V_{-p}^{/T} p_{-v}^{/T} \quad (1.14)$$

Now, referring to unit mass we have.

$$c_p = c_v + \frac{T}{m} V_{-p}^{/T} p_{-v}^{/T} \quad (1.15)$$

using equation 1.3 and

$$c_v = T S_{-p}^{/v} \quad (1.16)$$

Equation 1.15 is the desired relation between the specific heat values.

## 1.5 Case of an ideal gas

We now turn our attention to the case of an ideal gas. According to Falk, this is a general law of nature that is applicable to all matter in very low concentration, i.e. at high temperature and low density. See also Fuchs 1996, page 172,

An ideal gas has the equations

$$\begin{aligned} V(p, T) &= m \frac{RT}{p} \\ p(V, T) &= m \frac{RT}{V} \end{aligned} \quad (1.17)$$

Inserting them in equation 1.13 we have

$$c_p = c_v + \frac{R^2 T}{pV}; \quad c_p = c_v + R \quad (1.18)$$

The latter form is obtained by again inserting equation 1.17, and results in the well-known formula of equation 1.18.

Summarizing the procedure, we have considered that, at constant pressure, part of the thermal energy comes out in the fluid bond. Therefore the heat capacity is larger. To repeat, this variable stiffness or specific entropy, depending on what happens at the other bond, is a property of all multiport-Cs.

Many such relations can be established between the flows (displacements) and the integrals of flows of a multiport-C. In particular, there can be three

or more bonds in a multiport-C, all of which have Maxwell reciprocity. The only important aspect is that frictions and irreversibilities are negligible. This is said to be obtained by running the process infinitely slowly, but we shall discuss the meaning of that statement in section 2.3.

## 1.6 Equilibrium in thermodynamics and electricity

To explain further the notion of equilibrium, which is often abused in thermodynamics, we illustrate it with an example: electric equilibrium. Referring to Fig 1.14, the electric voltages decrease until they become equal, whereupon the current vanishes. Note also that the voltage and current can flow in either direction, but dissipation is always positive. As a formula it can be expressed as

$$\dot{Q} = R \frac{i^2}{2} \quad (1.19)$$

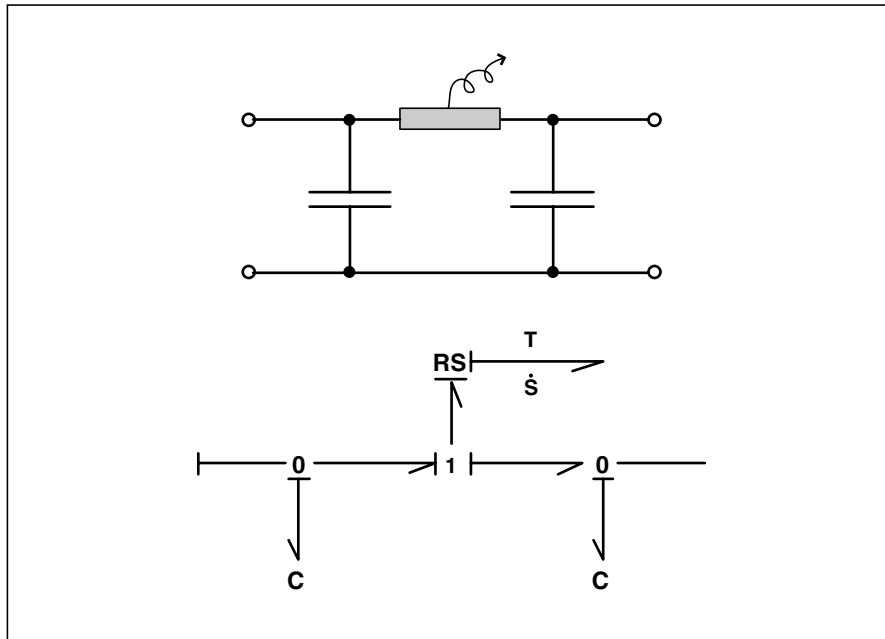
This formula is valid for both positive and negative currents and always gives a positive dissipation.

See also section 3.3 for thermodynamic equilibrium. Note likewise that  $R$  must be positive.

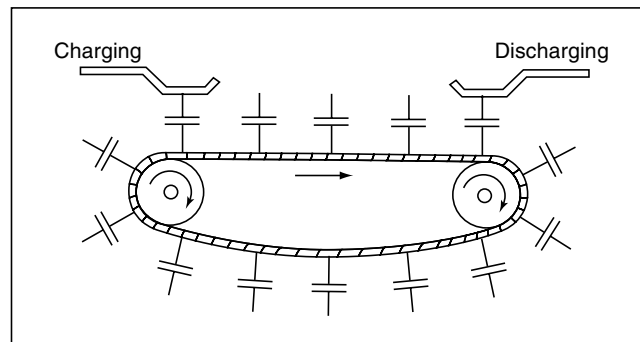
**Box:** In the terminology of Falk 1980, there is a flow of carriers (extensive variable), here carriers of electric charge, and a load (intensive variable) with a load factor, here the voltage. The division between the flow of carriers and their load factor also applies, whether one uses BGs or not, to all other disciplines, such as force (load factor) and speed (flow of charge carrier) in rectilinear mechanics or torque (load factor) and rotation frequency (load carrier) in rotary mechanics. In BG terms, power is effort as load factor multiplied by flow as flow of carriers. This is one reason why BGs fit so admirably well in our system of physical variables.

Remember that the resistor produces heat as long as a current flows between the two capacitors, and the heat flow is always the product of entropy current and absolute temperature. The BG below of Fig 1.14 shows this by a RS-element, which is a BG symbol representing irreversible power. In general, the RS-field is reversible on the electrical side where the electric current can reverse, but irreversible on the thermal side. This means that dissipation must always be positive. This is nicely illustrated by a linear resistor where current can reverse but dissipation not. The limiting case is reached when no current flows or no dissipation exists. The whole concept is also a good introduction to irreversibility in nature (see chapter 2).



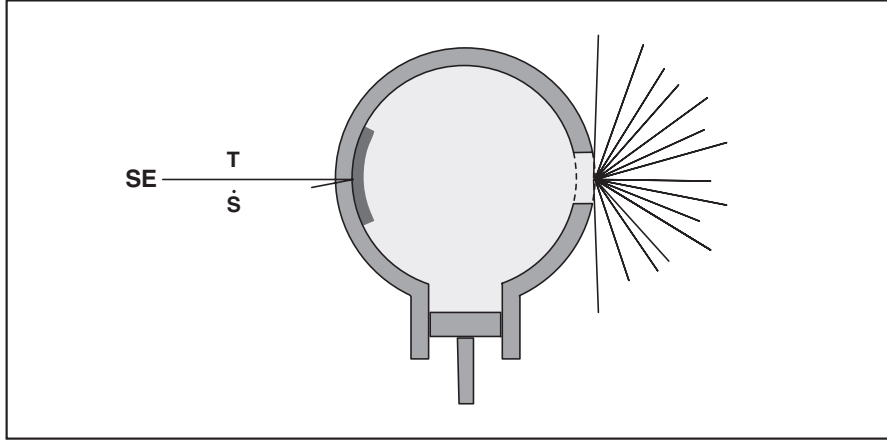


**Fig. 1.14.** Example of electric equilibrium with the voltages of both capacitors equal. If they are not, a current will flow through the resistor with consequent dissipation which appears as heat flow. Above circuit, below BG.



**Fig. 1.15.** Conveyor belt as an example of convection of electricity.

Let us mention at this point that convection of thermal energy is very important in thermodynamics, as we shall explain in section 3.1. Convection can also be conceived for electricity as shown in Fig 1.15. This is not a very practical scheme, although it bears some resemblance to the van de Graff high voltage generator, as used formerly in nuclear physics. We have a conveyor



**Fig. 1.16.** Cavity whose volume can be varied by the piston below, and which is subject to radiation pressure.

belt with capacitors between a charging and a discharging station and also the necessary capacitor return. So the charging station fills the capacitors to a certain voltage  $u$  and takes it back at the discharging station. We define the current  $i$  as full charge multiplied by the transmission frequency. Then the transmitted power is

$$\dot{E} = \frac{1}{2} i u \quad (1.20)$$

Contrary to conduction, there arises a factor one half for power, which demonstrates that such a reduction factor exists in electricity as well. Also, by suitably controlling the sources, the conveyor line can be made reversible and sparks on the contacts avoided. This means that there is no voltage drop between the charging and discharging wires and the capacitors.

## 1.7 Thermal radiation

We come now to the third way of transmitting thermal energy: by radiation. The most prominent example is solar power. Anticipating appendix A1, we denote the sun as a black body radiator of 6000 K with a power density of 1 400 Watt per  $\text{m}^2$  approximately<sup>4</sup>.

To fix ideas, Fig 1.16 shows a black cavity with a small hole and reflecting walls. The hole is small enough not to disturb the radiation field inside, but

<sup>4</sup> The power of the sun is very large, but only 1.4 kW meets each square meter of the higher atmosphere and only about 1.0 kW comes down per square meter of the earth.

can allow radiation to pass from the inside into space, as shown at right by the rays in all directions. Also it can admit radiation from the outside, i.e. the sun.

Two special features of this cavity are:

Inside at left there is a heat conductor leading to an effort source that supplies as much entropy as necessary to keep the temperature constant.

In the lower part of the cavity there is a cylinder with a piston that can vary the volume of the cavity. Naturally the radiation exerts pressure on the piston.

The main properties of this cavity are that the radiation field inside has many frequencies, is independent of the material of the walls and is proportional to the volume  $V$ . So, with  $u$  energy density and  $U$  total energy

$$U = uV \quad (1.21)$$

Note in particular that the total energy is not conserved but increases with volume. The necessary energy will be supplied from the source at left. On the other hand, radiation density depends on temperature, as given by equation 1.24 below.

The radiation field is often called a photon gas. As is known, photons have no rest mass but a definite momentum  $P$ , as they travel in any direction in a cavity with the speed of light  $c$  (see Falk et al 1983, pages 39, 52).

Yet they do have a certain direction and their momentum is given by this direction.

$$p = \frac{U}{c} \quad (1.22)$$

The radiation field inside will exert a pressure

$$p = \frac{u}{3} \quad (1.23)$$

Note that the dimensions are correct: pressure is energy per volume, thus an energy density, as is  $u$ .

Equation 1.23 is the basis of the following development. It was first derived by Max Plank (circa 1900) from electromagnetic considerations, but for a modern derivation see Fuchs 1996, section 2.6.2.

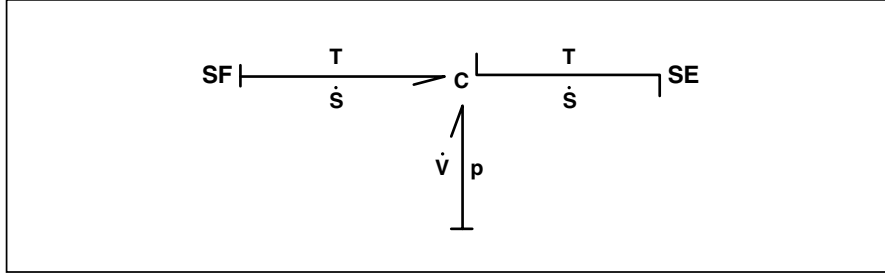
Fig 1.17 shows a BG corresponding to the previous principle drawing.

The cavity becomes a multiport-C, which has a normal bond at left for connection with the flow source, and a bond below to allow for a change in volume. However, on the right there is a pseudo bond for the hole with temperature and heat flow.

Hence Fig 1.17 is a combined true and pseudo BG. The bond for the hole is taken as bicausal<sup>5</sup> rather than as a normal bond without causality, a newer

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<sup>5</sup> Bicausal bonds are often used in other disciplines.



**Fig. 1.17.** BG of radiation cavity with variable volume. The radiation bond is described by a so-called bicausal bond.

development of BG'ing. It indicates that both temperature and heat flow come from the sun.

In fact the energy in the cavity is

$$U = uV = aVT^4 \quad (1.24)$$

with  $a$  = Stephan-Boltzman constant and  $u$  = energy density, and the entropy is (Fuchs 1996, section 2.6.3)

$$S = \frac{4}{3}aVT^3 \quad (1.25)$$

The radiation entering through the hole travels at the speed of light and disperses hemispherically as indicated by the rays in Fig 1.16. Travelling at the speed of light, it carries the above entropy and energy density along.

Hence we have, with  $A$  denoting the area of the hole, for the entropy current

$$\dot{S} = \frac{4}{3}AcaVT^3; \quad \dot{Q} = AcaT^4 \quad (1.26)$$

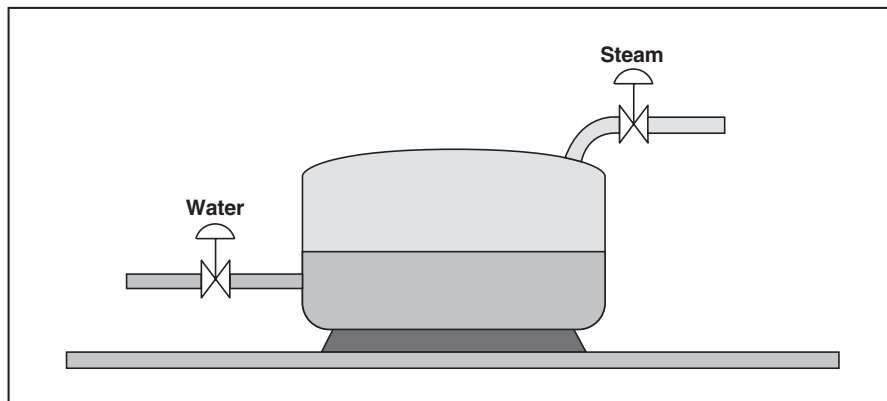
where  $c$  = speed of light and for the heat power

$$\dot{Q} = \frac{3}{4}\dot{S}T; \quad \dot{S} = \frac{4}{3}\frac{\dot{Q}}{T} \quad (1.27)$$

The first of these equations is equivalent to Carnot's equation 1.1 We see that the heat transfer in thermal radiation is only 3/4 of entropy current times temperature: not breathtaking but interesting.

If the volume of the cavity is increased in Fig 1.16 and 1.17, the source must keep the temperature constant by supplying the necessary entropy. This is the essential action of an effort source: it supplies as much flow as is necessary to keep the effort constant.

This energy is 4/3 times higher than heat flow divided by temperature. However 1/3 goes into the piston through radiation pressure, hence 1/1 remains



**Fig. 1.18.** A boiler containing some water heated from below. Here the quantity of steam is not constant but depends on evaporation and pressure.

in the new volume: entropy comes from the source but is conserved in the cavity. So the movement of the piston is entropy-conserving and reversible. All irreversibilities lie in the hole, i.e. in the transition to the pseudo BG.

Here it is very important to think clearly in terms of entropy and thermal energy (heat). Max Plank (1950) wrote in an article that he had success at the beginning of the century with cavity radiation, and later quantum theory, because he concentrated on entropy and not on heat, as his competitors did.

Also interesting is the comparison made between a radiation cavity and a steam boiler by Falk (private communication circa 1985) as shown in Fig 1.18.

The steam boiler with a small quantity of water is heated from below. There is equilibrium between water and steam at a certain pressure as given by the Clausius-Clapeyron equation.

If the heating is increased, some water evaporates, compressing the steam, until there is a new equilibrium between water and steam at a higher pressure. So the mass of steam is not constant or conserved. This is similar to what happens with radiation cavities and also with light, where quantities are not constant but depend on external parameters like temperature.

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