4. Cosmology I: Homogeneous Isotropic World Models

We will now begin to consider the Universe as a whole. Individual objects such as galaxies and stars will no longer be the subject of discussion, but instead we will turn our attention to the space and time in which these objects are embedded. These considerations will then lead to a world model, the model of our cosmos.

This chapter will deal with aspects of homogeneous cosmology. As we will see, the Universe can, to first approximation, be considered as being homogeneous. At first sight this fact obviously seems to contradict observations because the world around us is highly inhomogeneous and structured. Thus the assumption of homogeneity is certainly not valid on small scales. But observations are compatible with the assumption that the Universe is homogeneous when averaged over large spatial scales. Aspects of inhomogeneous cosmology, and thus the formation and evolution of structures in the Universe, will be considered later in Chap. 7.

4.1 Introduction and Fundamental Observations

Cosmology is a very special science indeed. To be able to appreciate its peculiar role we should recall the typical way of establishing knowledge in natural sciences. It normally starts with the observation of some regular patterns, for instance the observation that the height h a stone falls through is related quadratically to the time t it takes to fall, $h = (g/2)t^2$. This relation is then also found for other objects and observed at different places on Earth. Therefore, this relation is formulated as the "law" of free fall. The constant of proportionality $g/2$ in this law is always the same. This law of physics is tested by its prediction of how an object falls, and wherever this prediction is tested it is confirmed – disregarding the resistance of air in this simple example, of course.

Relations become physical laws if the predictions they make are confirmed again and again; the validity of such a law is considered more secure the more diverse the tests have been. The law of free fall was tested only on the surface of the Earth and it is only valid there with this constant of proportionality.[1] In contrast to this, Newton's law of gravity contains the law of free fall as a special case, but it also describes the free fall on the surface of the Moon, and the motion of planets around the Sun. If only a single stone were available, we would not know whether the law of free fall is a property of this particular stone or whether it is valid more generally.

In some ways, cosmology corresponds to the latter example: we have only one single Universe available for observation. Relations that are found in our cosmos cannot be verified in other universes. Thus it is not possible to consider any property of our Universe as "typical" – we have no statistics on which we could base a statement like this. Despite this special situation, enormous progress has been made in understanding our Universe, as we will describe here and in subsequent chapters.

Cosmological observations are difficult in general, simply because the majority of the Universe (and with it most of the sources it contains) is very far away from us. Distant sources are very dim. This explains why our knowledge of the Universe runs in parallel with the development of large telescopes and sensitive detectors. Much of today's knowledge of the distant Universe became available only with the new generation of optical telescopes of the 8-m class, as well as new and powerful telescopes in other wavelength regimes.

The most important aspect of cosmological observations is the finite speed of light. We observe a source at distance D in an evolutionary state at which it was $\Delta t = (D/c)$ younger than today. Thus we can observe the current state of the Universe only very locally. Another consequence of this effect, however, is of even greater importance: due to the finite speed of light, it is possible to look back into the past. At a distance of 10 billion light years we observe galaxies in an evolutionary state when the Universe had only a third of its current age. Although we cannot observe the past of our own Milky Way, we can study that of other galaxies. If we are able to identify among them the ones that will form objects similar to our Galaxy in the course of cos-

[1] Strictly speaking, the constant of proportionality g depends slightly on the location.

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mic evolution, we will be able to learn a great deal about the typical evolutionary history of such spirals.

The finite speed of light in a Euclidean space, in which we are located at the origin $\boldsymbol{r} = 0$ today ($t = t_0$), implies that we can only observe points in spacetime for which $|\boldsymbol{r}| = c(t_0 - t)$; an arbitrary point $(\boldsymbol{r}, t)$ in spacetime is not observable. The set of points in spacetime which satisfy the relation $|\boldsymbol{r}| = c(t_0 - t)$ is called our *backward light cone*.

The fact that our astronomical observations are restricted to sources which are located on our backward light cone implies that our possibilities to observe the Universe are fundamentally limited. If somewhere in spacetime there would be a highly unusual event, we will not be able to observe it unless it happens to lie on our backward light cone. Only if the Universe has an essentially "simple" structure we will be able to understand it, by combining astronomical observations with theoretical modeling. Luckily, our Universe seems to be basically simple in this sense.

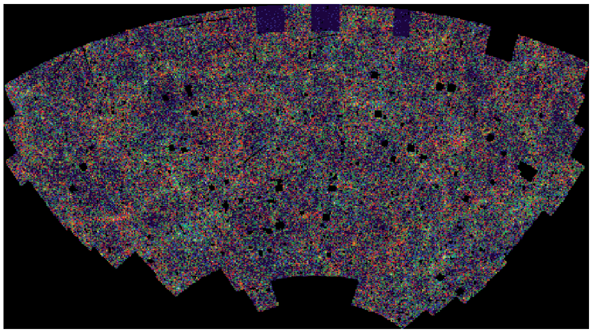

Fig. 4.1. The APM survey: galaxy distribution in a $\sim 100 \times 50\,\mathrm{degree}^2$ field around the South Galactic Pole. The intensities of the pixels are scaled with the number of galaxies per pixel, i.e., the projected galaxy number density on the sphere. The "holes" are regions around bright stars, globular clusters, etc., that have not been surveyed

4.1.1 Fundamental Cosmological Observations

We will begin with a short list of key observations that have proven to be of particular importance for cosmology. Using these observational facts we will then be able to draw a number of immediate conclusions; other observations will be explained later in the context of a cosmological model.

1. The sky is dark at night (Olbers' paradox).
2. Averaged over large angular scales, faint galaxies (e.g., those with $R > 20$) are uniformly distributed on the sky (see Fig. 4.1).
3. With the exception of a very few very nearby galaxies (e.g., Andromeda = M31), a redshift is observed in the spectra of galaxies – most galaxies are moving away from us, and their escape velocity increases linearly with distance (Hubble law; see Fig. 1.10).
4. In nearly all cosmic objects (e.g., gas nebulae, main-sequence stars), the mass fraction of helium is 25–30%.
5. The oldest star clusters in our Galaxy have an age of $\sim 12\,\mathrm{Gyr} = 12 \times 10^9\,\mathrm{yr}$ (see Fig. 4.2).
6. A microwave radiation (cosmic microwave background radiation, CMB) is observed, reaching us from all directions. This radiation is isotropic except for very small, but immensely important, fluctuations with relative amplitude $\sim 10^{-5}$.
7. The spectrum of the CMB corresponds, within the very small error bars that were obtained by the measurements with COBE, to that of a perfect blackbody, i.e., a Planck radiation of a temperature of $T_0 = 2.728 \pm 0.004\,\mathrm{K}$ – see Fig. 4.3.
8. The number counts of radio sources at high Galactic latitude does *not* follow the simple law $N(> S) \propto S^{-3/2}$ (see Fig. 4.4).

4.1.2 Simple Conclusions

We will next draw a number of simple conclusions from the observational facts listed above. These will then serve as a motivation and guideline for developing the cosmological model. We will start with the assumption of an infinite, Euclidean, static Universe, and show that these assumptions are in direct contradiction to observations (1) and (8).

Olbers' Paradox (1): We can show that the night sky would be bright in such a universe – uncomfortably bright, in fact. Let n_* be the mean number density of stars, constant in space and time according to the assumptions, and let R_* be their mean radius. A spherical

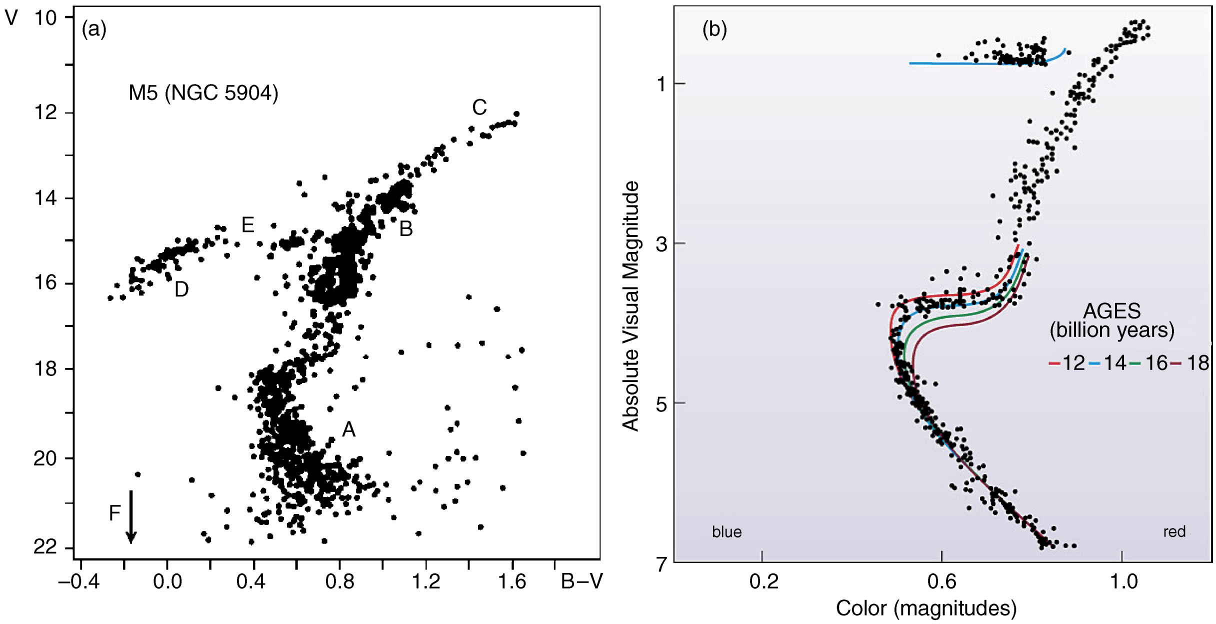

Fig. 4.2. **(a)** Color–magnitude diagram of the globular cluster M 5. The different sections in this diagram are labeled. A: main sequence; B: red giant branch; C: point of helium flash; D: horizontal branch; E: Schwarzschild gap in the horizontal branch; F: white dwarfs, below the arrow. At the point where the main sequence turns over to the red giant branch (called the "turn-off point"), stars have a mass corresponding to a main-sequence lifetime which is equal to the age of the globular cluster (see Appendix B.3). Therefore, the age of the cluster can be determined from the position of the turn-off point by comparing it with models of stellar evolution. **(b)** Isochrones, i.e., curves connecting the stellar evolutionary position in the color–magnitude diagram of stars of equal age, are plotted for different ages and compared to the stars of the globular cluster 47 Tucanae. Such analyses reveal that the oldest globular clusters in our Milky Way are about 13 billion years old, where different authors obtain slightly differing results – details of stellar evolution may play a role here. The age thus obtained also depends on the distance of the cluster. A revision of these distances by the HIPPARCOS satellite led to a decrease of the estimated ages by about 2 billion years

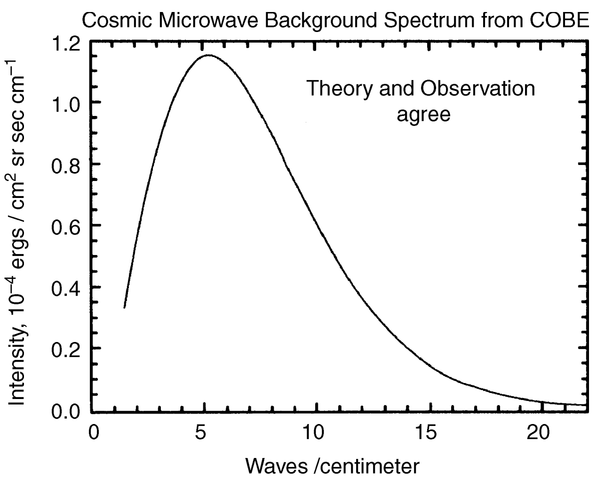

Fig. 4.3. CMB spectrum, plotted as intensity vs. frequency, measured in waves per centimeter. The solid line shows the expected spectrum of a blackbody of temperature $T = 2.728$ K. The error bars of the data, observed by the FIRAS instrument on-board COBE, are so small that the data points with error bars cannot be distinguished from the theoretical curve

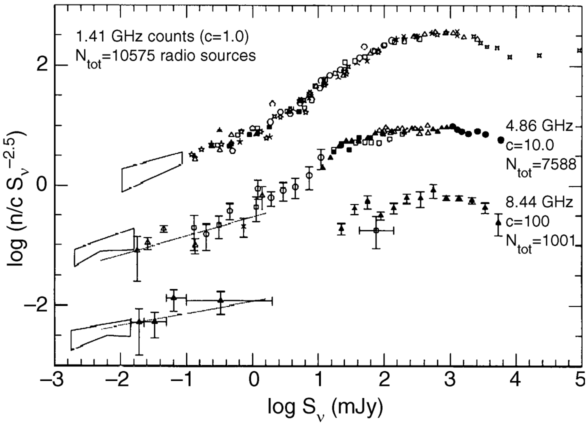

Fig. 4.4. Number counts of radio sources as a function of flux, normalized by the Euclidean expectation $N(S) \propto S^{-5/2}$, corresponding to the integrated counts $N(>S) \propto S^{-3/2}$. Counts are displayed for three different frequencies; they clearly deviate from the Euclidean expectation

shell of radius r and thickness $\mathrm{d}r$ around us contains $n_* \,\mathrm{d}V = 4\pi r^2 \,\mathrm{d}r\, n_*$ stars. Each of these stars subtends a solid angle of $\pi R_*^2/r^2$ on our sky, so the stars in the shell cover a total solid angle of

$$\mathrm{d}\omega = 4\pi r^2 \,\mathrm{d}r\, n_* \,\frac{R_*^2 \pi}{r^2} = 4\pi^2\, n_*\, R_*^2 \,\mathrm{d}r\ . \tag{4.1}$$

We see that this solid angle is independent of the radius r of the spherical shell because the solid angle of a single star $\propto r^{-2}$ just compensates the volume of the shell $\propto r^2$. To compute the total solid angle of all stars in a static Euclidean universe, (4.1) has to be integrated over all distances r, but the integral

$$\omega = \int_0^\infty \mathrm{d}r\, \frac{\mathrm{d}\omega}{\mathrm{d}r} = 4\pi^2\, n_*\, R_*^2 \int_0^\infty \mathrm{d}r$$

diverges. Formally, this means that the stars cover an infinite solid angle, which of course makes no sense physically. The reason for this divergence is that we disregarded the effect of overlapping stellar disks on the sphere. However, these considerations demonstrate that the sky would be completely filled with stellar disks, i.e., from any direction, along any line-of-sight, light from a star would reach us. Since the specific intensity I_ν is independent of distance – the surface brightness of the Sun as observed from Earth is the same as seen by an observer who is much closer to the Solar surface – the sky would have a temperature of $\sim 10^4$ K; fortunately, this is not the case!

Source Counts (8): Consider now a population of sources with a luminosity function that is constant in space and time, i.e., let $n(>L)$ be the spatial number density of sources with luminosity larger than L. A spherical shell of radius r and thickness $\mathrm{d}r$ around us contains $4\pi r^2 \,\mathrm{d}r\, n(>L)$ sources with luminosity larger than L. Because the observed flux S is related to the luminosity via $L = 4\pi\, r^2\, S$, the number of sources with flux $> S$ in this spherical shell is given as $\mathrm{d}N(>S) = 4\pi r^2 \,\mathrm{d}r\, n(>4\pi\, r^2\, S)$, and the total number of sources with flux $> S$ results from integration over the radii of the spherical shells,

$$N(>S) = \int_0^\infty \mathrm{d}r\, 4\pi\, r^2\, n(>4\pi\, r^2\, S)\ .$$

Changing the integration variable to $L = 4\pi\, r^2\, S$, or $r = \sqrt{L/(4\pi S)}$, with $\mathrm{d}r = \mathrm{d}L/(2\sqrt{4\pi L S})$, yields

$$\begin{aligned} N(>S) &= \int_0^\infty \frac{\mathrm{d}L}{2\sqrt{4\pi L S}}\, \frac{L}{4\pi S}\, n(>L) \\ &= \frac{1}{16\pi^{3/2}}\, S^{-3/2} \int_0^\infty \mathrm{d}L\, \sqrt{L}\, n(>L)\ . \end{aligned} \tag{4.2}$$

From this result we deduce that the source counts in such a universe is $N(>S) \propto S^{-3/2}$, independent of the luminosity function. This is in contradiction to the observations.

From these two contradictions – Olbers' paradox and the non-Euclidean source counts – we conclude that at least one of the assumptions must be wrong. Our Universe cannot be all three of Euclidean, infinite, and static. The Hubble flow, i.e., the redshift of galaxies, indicates that the assumption of a static Universe is wrong.

The **age of globular clusters (5)** requires that the Universe is at least 12 Gyr old because it cannot be younger than the oldest objects it contains. Interestingly, the age estimates for globular clusters yield values which are very close to the *Hubble time* $H_0^{-1} = 9.78\, h^{-1}$ Gyr. This similarity suggests that the Hubble expansion may be directly linked to the evolution of the Universe.

The apparently isotropic **distribution of galaxies (2)**, when averaged over large scales, and the **CMB isotropy (6)** suggest that the Universe around us is isotropic on large angular scales. Therefore we will first consider a world model that describes the Universe around us as isotropic. If we assume, in addition, that our place in the cosmos is not privileged over any other place, then the assumption of isotropy around us implies that the Universe appears isotropic as seen from any other place. The homogeneity of the Universe follows immediately from the isotropy around every location, as explained in Fig. 4.5. The combined assumption of homogeneity and isotropy of the Universe is also known as the *cosmological principle*. We will see that a world model based on the cosmological principle in fact provides an excellent description of numerous observational facts.

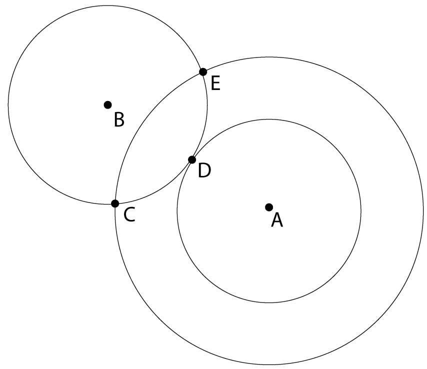

Fig. 4.5. Homogeneity follows from the isotropy around two points. If the Universe is isotropic around observer B, the densities at C, D, and E are equal. Drawing spheres of different radii around observer A, it is seen that the region within the spherical shell around A has to be homogeneous. By varying the radius of the shell, we can conclude the whole Universe must be homogeneous

However, homogeneity is in principle unobservable because observations of distant objects show those at an earlier epoch. If the Universe evolves in time, as the aforementioned observations suggest, evolutionary effects cannot directly be separated from spatial variations.

The assumption of homogeneity of course breaks down on small scales. We observe structures in the Universe, like galaxies and clusters of galaxies, and even accumulations of clusters of galaxies, so-called superclusters. Structures have been found in redshift surveys that extend over $\sim 100\,h^{-1}$ Mpc. However, we have no indication of the existence of structures in the Universe with scales $\gg 100$ Mpc. This length-scale can be compared to a characteristic length of the Universe, which is obtained from the Hubble constant. If H_0^{-1} specifies the characteristic age of the Universe, then light will travel a distance c/H_0 in this time. With this, one obtains the *Hubble radius* as a characteristic length-scale of the Universe (or more precisely, of the observable Universe),

$$\boxed{R_{\rm H} := \frac{c}{H_0} = 2997\,h^{-1}\,{\rm Mpc}\ :\ \text{Hubble length}}\ . \tag{4.3}$$

The Hubble volume $\sim R_{\rm H}^3$ can contain a very large number of structures of size $\sim 100\,h^{-1}$ Mpc, so that it still makes sense to assume an on-average homogeneous Universe. In this homogeneous Universe we then have density fluctuations that are identified with the observed large-scale structures; these will be discussed in detail in Chap. 7. To a first approximation we can neglect these density perturbations in a description of the Universe as a whole. We will therefore consider next world models that are based on the cosmological principle, i.e., in which the Universe looks the same for all observers (or, in other words, if observed from any point).

Homogeneous and isotropic world models are the simplest cosmological solutions of the equations of General Relativity (GR). We will examine how far such simple models are compatible with observations. As we shall see, the application of the cosmological principle results in the observational facts which were mentioned in Sect. 4.1.1.

4.2 An Expanding Universe

Gravitation is the fundamental force in the Universe. Only gravitational forces and electromagnetic forces can act over large distance. Since cosmic matter is electrically neutral on average, electromagnetic forces do not play any significant role on large scales, so that gravity has to be considered as the driving force in the http://www.springer.com/978-3-540-33174-2

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