Preface

When the algebraic formalism of quantum statistical mechanics and quantum field theory gained momentum in the 1960's, it started a very fruitful interplay between mathematical physics and operator algebras. The study of automorphisms and their invariant states became a blooming discipline, and a subject of noncommutative ergodic theory evolved. With the great success of entropy in classical (abelian) ergodic theory it was natural to extend that theory to operator algebras. In some cases, like that of quantum spin lattice systems, that is rather straightforward, since in those models the mean entropy definition in the classical case can be extended to C*-algebras by replacing partitions by local algebras. But in more general cases the C*-algebras generated by finite dimensional C*-algebras can easily be infinite dimensional, so the mean entropy cannot be used as a definition. In order to define dynamical entropy for automorphisms of C*-algebras one has to rewrite the classical definition in a form independent of the join of partitions and use that as the basis for a definition. This was done by Connes and Størmer in 1975 for finite von Neumann algebras, and a useful definition was accomplished, giving in particular the expected entropy for noncommutative Bernoulli shifts [51]. The theory evolved slowly; it took 10 years before Connes extended the definition to normal states of von Neumann algebras [49], and a little later he and Narnhofer and Thirring [50] extended the theory to states of C*-algebras. Several other attempts have been made to define dynamical entropy for C*-algebras, see Notes to Chaps. 3 and 6, the most useful of which being those of Voiculescu [227]. His idea was to consider finite dimensional C*-algebras which approximately contain finite sets of operators instead of the algebras they generate. In particular he obtained a definition of topological entropy which is an extension of topological entropy in the classical case.

In the present book we shall develop the basic theory for the dynamical and topological entropies alluded to above. Then we shall discuss the special situations which have attracted most attention. We start with a chapter on the classical case, mainly for motivation and background. Then we develop in Chap. 2 the necessary theory of relative entropy for states, which is indispensable for noncommutative entropy. In Chap. 3 we give the definition of dynamical entropy and show its main properties, and follow this up in Chap. 5 with a definition, due to Sauvageot and Thouvenot [188], inspired by the classical concept of joinings. Topological entropy is treated in Chap. 6. The rest of the contents of the book depends heavily on the above chapters, while the other chapters are more loosely connected. The book is divided into two parts; the first contains chapters of general nature, while we in the second part consider dynamical systems in more special settings. At this stage it should also be remarked that parts of the theory have been treated in the books by Benatti [13], Ohya and Petz [147] and the survey article by Størmer [211].

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