

Contents

I	Sets, Numbers, and Graphs	1
1	Fundamentals—Concepts and Logic	3
1.1	Propositional Logic	4
1.2	Architecture of Concepts	8
2	Axiomatic Set Theory	15
2.1	The Axioms	17
2.2	Basic Concepts and Results	20
3	Boolean Set Algebra	25
3.1	The Boolean Algebra of Subsets.....	25
4	Functions and Relations	29
4.1	Graphs and Functions	29
4.2	Relations	41
5	Ordinal and Natural Numbers	45
5.1	Ordinal Numbers	45
5.2	Natural Numbers	50
6	Recursion Theorem and Universal Properties	55
6.1	Recursion Theorem.....	56
6.2	Universal Properties	58
6.3	Universal Properties in Relational Database Theory	66
7	Natural Arithmetic	73
7.1	Natural Operations	73
7.2	Euclid and the Normal Forms	76
8	Infinities	79
8.1	The Diagonalization Procedure	79

9	The Classical Number Domains \mathbb{Z}, \mathbb{Q}, \mathbb{R}, and \mathbb{C}	81
9.1	Integers \mathbb{Z}	82
9.2	Rationals \mathbb{Q}	87
9.3	Real Numbers \mathbb{R}	90
9.4	Complex Numbers \mathbb{C}	102
10	Categories of Graphs	107
10.1	Directed and Undirected Graphs	108
10.2	Morphisms of Digraphs and Graphs	114
10.3	Cycles	125
11	Construction of Graphs	129
12	Some Special Graphs	137
12.1	n -ary Trees	137
12.2	Moore Graphs	139
13	Planarity	143
13.1	Euler's Formula for Polyhedra	143
13.2	Kuratowski's Planarity Theorem	147
14	First Advanced Topic	149
14.1	Floating Point Arithmetic	149
14.2	Example for an Addition	154
II	Algebra, Formal Logic, and Linear Geometry	157
15	Monoids, Groups, Rings, and Fields	159
15.1	Monoids	159
15.2	Groups	163
15.3	Rings	171
15.4	Fields	177
16	Primes	181
16.1	Prime Factorization	181
16.2	Roots of Polynomials and Interpolation	186
17	Formal Propositional Logic	191
17.1	Syntactics: The Language of Formal Propositional Logic ..	193
17.2	Semantics: Logical Algebras	196
17.3	Signification: Valuations	200
17.4	Axiomatics	203

Contents	XIII
18 Formal Predicate Logic	209
18.1 Syntactics: First-order Language	211
18.2 Semantics: Σ -Structures	217
18.3 Signification: Models	218
19 Languages, Grammars, and Automata	223
19.1 Languages	224
19.2 Grammars	229
19.3 Automata and Acceptors	243
20 Categories of Matrixes	261
20.1 What Matrixes Are	262
20.2 Standard Operations on Matrixes	265
20.3 Square Matrixes and their Determinant	271
21 Modules and Vector Spaces	279
22 Linear Dependence, Bases, and Dimension	287
22.1 Bases in Vector Spaces	288
22.2 Equations	295
22.3 Affine Homomorphisms	296
23 Algorithms in Linear Algebra	303
23.1 Gauss Elimination	303
23.2 The LUP Decomposition	307
24 Linear Geometry	311
24.1 Euclidean Vector Spaces	311
24.2 Trigonometric Functions from Two-Dimensional Rotations	320
24.3 Gram's Determinant and the Schwarz Inequality	323
25 Eigenvalues, the Vector Product, and Quaternions	327
25.1 Eigenvalues and Rotations	327
25.2 The Vector Product	331
25.3 Quaternions	333
26 Second Advanced Topic	343
26.1 Galois Fields	343
26.2 The Reed-Solomon (RS) Error Correction Code	349
26.3 The Rivest-Shamir-Adleman (RSA) Encryption Algorithm .	353
A Further Reading	357

XIV	Contents
B Bibliography	359
Index	363

Volume II

III Topology and Calculus

Limits and Topology, Differentiability, Inverse and Implicit Functions, Integration, Fubini and Changing Variables, Vector Fields, Fixpoints, Main Theorem of ODEs

IV Selected Higher Subjects

Numerics, Probability and Statistics, Splines, Fourier, Wavelets, Fractals, Neural Nets, Global Coordinates and Manifolds, Categories, Lambda Calculus

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