Authors' Comments on the Corrected Second Printing

The original printing of the book appeared in 2006. Its very positive reception has led to its being sold out in less than three years. Springer's decision to reprint the book gave us the opportunity to correct minor mathematical and typographical errors in the original printing of the monograph. We would appreciate receiving any suggestions for further improvements and thank all those readers who have pointed out misprints and errors to us.

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Preface

In recent years products based on financial derivatives have become an indispensable tool for risk managers and investors. Insurance products have become part of almost every personal and business portfolio. The management of mutual and pension funds has gained in importance for most individuals. Banks, insurance companies and other corporations are increasingly using financial and insurance instruments for the active management of risk. An increasing range of securities allows risks to be hedged in a way that can be closely tailored to the specific needs of particular investors and companies. The ability to handle efficiently and exploit successfully the opportunities arising from modern quantitative methods is now a key factor that differentiates market participants in both the finance and insurance fields. For these reasons it is important that financial institutions, insurance companies and corporations develop expertise in the area of *quantitative finance*, where many of the associated quantitative methods and technologies emerge.

This book aims to provide an *introduction to quantitative finance*. More precisely, it presents an introduction to the mathematical framework typically used in financial modeling, derivative pricing, portfolio selection and risk management. It offers a unified approach to risk and performance management by using the *benchmark approach*, which is different to the prevailing paradigm and will be described in a systematic and rigorous manner.

This approach uses the *growth optimal portfolio* as numeraire and the real world probability measure as pricing measure. The existence of an equivalent risk neutral probability measure is not required, which is one of the aspects distinguishing the approach in this book from other more conventional texts in the area. It is our experience that many practitioners find the use of the *real world* probability measure attractive for *pricing* because it is natural and pricing can still be carried out even under circumstances when a risk neutral probability measure cannot exist.

We have attempted to write a multi-purpose book that provides information and methods for a wide range of professionals, researchers and graduate students. It is designed for three groups of readers. In the first instance it

should provide useful information to financial analysts and practitioners in the investment, banking and insurance industries. Other professionals at financial software companies, hedge funds, consultants, regulatory authorities and government agencies may significantly benefit from using this book. Secondly, the book aims to introduce those with a reasonable basic mathematical background to the area of quantitative finance. Engineers, computer scientists, numerical analysts, physicists, theoretical chemists, biologists, astrophysicists, statisticians, econometricians, actuaries and other readers should be able to gain access to the field through the book. Thirdly, researchers in financial mathematics will find the later parts of the book interesting and possibly challenging. In particular, the monograph aims to stimulate further developments of the benchmark approach.

The material presented is a self-contained introduction that could be part of a coursework masters or PhD program in quantitative finance. The areas of probability and statistics, stochastic calculus, optimization and numerical methods relevant to finance are all introduced. The book has been designed in a modular way with cross references so that it can also be used as a handbook allowing relevant definitions, formulas and results to be easily looked up.

The monograph is divided into fifteen chapters. The first two chapters summarize fundamental results from probability and statistics which are essential for quantitative finance. Some statistical analysis on the log-return distribution of indices is included at the end of Chap. 2.

The Chaps. 3 and 4 introduce stochastic processes. The stochastic calculus needed for financial modeling using stochastic differential equations is presented in Chaps. 5 to 7. Stochastic differential equations with jumps are introduced from a finance perspective. Some of the material goes beyond what can be found in standard textbooks.

In Chap. 8 basic financial derivatives are introduced from a hedging perspective. European call and put options are priced via the corresponding Black-Scholes partial differential equation. The sensitivities of these option prices to movements in parameter values are studied. Hedge simulations are performed, which illustrate derivative pricing and hedging.

Chapter 9 presents various alternative pricing methodologies. First, the concept of *real world pricing* is introduced. Several other pricing methods are shown to be special cases of real world pricing. These include actuarial pricing, risk neutral pricing and pricing under change of numeraire. The existence of an equivalent risk neutral probability measure is *not* required under the benchmark approach. The chapter concludes by introducing the Girsanov theorem, the Bayes rule and the Feynman-Kac formula.

Chapter 10 develops a unified modeling framework for continuous financial markets under the benchmark approach. It presents a range of new concepts and ideas that do not fit under the presently prevailing approaches. A *diversification theorem* is derived, which shows under some regularity condition that diversified portfolios approximate the growth optimal portfolio. This allows

us to interpret a diversified market index as a proxy for the growth optimal portfolio.

Chapter 11 derives results on portfolio optimization via the maximization of Sharpe ratios. The capital asset pricing model (CAPM), the Markowitz efficient frontier, two fund separation and results on expected utility maximization, utility indifference pricing, derivative pricing and hedging are also presented in this chapter.

The modeling of stochastic volatility of stock market indices under the benchmark approach is discussed in Chap. 12. This analysis includes the pricing of index derivatives under models that do not admit an equivalent risk neutral probability measure. More general volatility models than those permitted under the standard risk neutral approach are covered.

In Chap. 13 it is shown that the discounted growth optimal portfolio follows the dynamics of a time transformed squared Bessel process of dimension four. Making the drift of the discounted growth optimal portfolio a function of time, yields the *minimal market model*. Derivative prices which follow under this parsimonious model appear to be rather realistic. Long term derivatives can be realistically priced. These prices deviate significantly from those obtained under risk neutral pricing because the hypothetical risk neutral measure has after several years a total mass that is significantly less than one. Extensions of the minimal market model with random scaling are considered.

In Chap. 14 models are analyzed that permit jumps to model event risk. Most of the results of previous chapters are generalized to jump diffusion markets. Two market models illustrate differences in derivative pricing under the standard risk neutral and the benchmark approach.

Finally, in Chap. 15 a brief introduction is given from a unifying perspective to basic numerical methods for quantitative finance. This introduction covers scenario simulation, Monte Carlo simulation, tree based methods and finite difference methods. A binomial tree method is developed for the benchmark approach and finite difference methods are explained as numerical methods for systems of coupled ordinary differential equations.

Selected *exercises* at the end of each chapter should enable the reader to further develop skills and test the understanding of the subject. *Solutions* to these exercises are included at the end of the book. The material can be taught at different levels. The first sections in most chapters provide a less technical presentation of the subject. At the end of some sections or chapters (*)-subsections or (*)-sections have been included. These are more technical in nature and are usually not necessary for a first reading.

The formulas are numbered according to the chapter and section where they appear. Assumptions, theorems, lemmas, definitions and corollaries are numbered sequentially in each section. The most common notations are listed at the beginning of the book and an *index of keywords* is given at its end. Some readers may find the *author index* at the end of the book useful.

Substantial work is involved in studying the material presented. This should not be underestimated by the reader. Actively solving exercises is

strongly recommended. The reward for this demanding work will be a sound understanding of essential methods in quantitative finance with an emphasis on the benchmark approach.

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It is greatly appreciated if readers could forward any errors, misprints or suggested improvements to: eckhard.platen@uts.edu.au

The interested reader is likely to find updated information about the benchmark approach, as well as, teaching material related to the book on the webpage of the first author under "Benchmark_Approach":

`http://www.business.uts.edu.au/`
`finance/staff/Eckhard/Benchmark_Approach.html`

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