

# Preface

This book is intended to be a self-contained introduction to analytic foundations of a level set method for various surface evolution equations including curvature flow equations. These equations are important in various fields including material sciences, image processing and differential geometry. The goal of this book is to introduce a generalized notion of solutions allowing singularities and solve the initial-value problem globally-in-time in a generalized sense. Various equivalent definitions of solutions are studied. Several new results on equivalence are also presented.

We present here a rather complete introduction to the theory of viscosity solutions which is a key tool for the level set method. Also a self-contained explanation is given for general surface evolution equations of the second order. Although most of the results in this book are more or less known, they are scattered in several references, sometimes without proof. This book presents these results in a synthetic way with full proofs. However, the references are not exhaustive at all.

The book is suitable for applied researchers who would like to know the detail of the theory as well as its flavour. No familiarity with differential geometry and the theory of viscosity solutions is required. The prerequisites are calculus, linear algebra and some familiarity with semicontinuous functions. This book is also suitable for upper level undergraduate students who are interested in the field.

I am grateful to Professor Herbert Amann for inviting me to write this book which is based on my Lipschitz lectures in Bonn 1997. I am also grateful to its audience for their interest. The first version of the book was included in Series of Lipschitz Lecture Notes as volume 44 (2002). It was also included in Hokkaido University Technical Report Series in Mathematics as volume 71 (2002). However, since then the author has been fully occupied with the Center of Excellence Programme ‘Mathematics of Nonlinear Structures via Singularities’ (Hokkaido University) and with editing the 4-th version of the Encyclopedic Dictionary of Mathematics. Moreover, the author moved to Tokyo from Sapporo in the middle of 2004. So it has taken a rather long time to complete the present version of this book. After the first version appeared, the field continued to grow and many new articles have been published. They could not all be included without significant

expansion of the text. Although some effort was made to cite them, the reference list is not intended to be exhaustive.

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