

Preface

This book may be considered as the continuation of the monographs [Tri β] and [Tri γ] with the same title. It deals with the theory of function spaces of type B_{pq}^s and F_{pq}^s as it stands at the beginning of this century. These two scales of spaces cover many well-known spaces of functions and distributions such as Hölder-Zygmund spaces, (fractional and classical) Sobolev spaces, Besov spaces and Hardy spaces.

On the one hand this book is essentially self-contained. On the other hand we concentrate principally on those developments in recent times which are related to the nowadays numerous applications of function spaces to some neighboring areas such as numerics, signal processing and fractal analysis, to mention only a few of them.

Chapter 1 in [Tri γ] is a self-contained historically-oriented survey of the function spaces considered and their roots up to the beginning of the 1990s entitled

How to measure smoothness.

Chapter 1 of the present book has the same heading indicating continuity. As far as the history is concerned we will now be very brief, restricting ourselves to the essentials needed to make this book self-contained and readable. We complement [Tri γ], Chapter 1, by a few points omitted there. But otherwise we jump to the 1990s, describing more recent developments. Some of them will be treated later on in detail. In other words, [Tri γ], Chapter 1, and Chapter 1 of the present book complement each other, providing a sufficiently comprehensive picture of the theory of the spaces B_{pq}^s and F_{pq}^s and their roots from the beginning up to our time. But quite obviously as far as very recent topics are concerned we are somewhat selective, emphasizing those developments which are near to our own interests.

This book has 9 chapters. Chapter 1 is the indicated self-contained survey.

Chapters 2 and 3 deal with building blocks in (isotropic) spaces of type B_{pq}^s and F_{pq}^s in \mathbb{R}^n , especially with (non-smooth) atoms (Chapter 2) and with wavelet bases and wavelet frames (Chapter 3). We discuss some consequences: pointwise multiplier assertions, positivity properties and local smoothness problems.

In recent times there is a growing interest in function spaces in (bounded) Lipschitz domains in \mathbb{R}^n . Here we split our presentation, collecting some old and a few new results in the introductory Section 1.11 and returning to this subject in greater detail in Chapter 4.

Wavelet representations of anisotropic function spaces and of weighted function spaces on \mathbb{R}^n will be treated in Chapters 5 and 6, respectively.

Chapter 7 might be considered as the direct continuation of our studies in [Tri δ] and [Tri ϵ] about fractal quantities of measures and spectral assertions of fractal elliptic operators.

Finally in Chapters 8 and 9 we develop a new theory for function spaces on quasi-metric spaces and on sets.

Formulas are numbered within the nine chapters. Furthermore, within each of these chapters all definitions, theorems, propositions, corollaries, remarks and examples are jointly and consecutively numbered. Chapter n is divided in subsections $n.k$, which occasionally are subdivided in subsubsections $n.k.l$. But when quoted we refer simply to Section $n.k$ or Section $n.k.l$ instead of Subsection $n.k$ or Subsubsection $n.k.l$, respectively.

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Hans Triebel



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Triebel, H.

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