Editorial Introduction

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The present volume is basically formed by contributions of participants of the International Conference "C^*-algebras and elliptic theory" hold in Stefan Banach International Mathematical Center in Będlewo (Poland) in February 2004. The history of this Conference goes back to the idea of Prof. Bogdan Bojarski to strengthen collaboration between mathematicians from Poland and Russia, especially from Moscow, on the base of common scientific interests in the field of noncommutative geometry.

This idea leaded very quickly to the organization of the mentioned Conference which brought together about 60 mathematicians not only from Russia and Poland, but from other leading centers and awarded a support from the European program "Geometric Analysis Research Training Network". The conference started a series of annual conferences in Będlewo and Moscow alternately. Up to the present time three conferences of this series were organized and the forth one is planned to take place in Moscow in 2007 (`http://higeom.math.msu.su/oat2007`).

The contributions are mainly concentrated on applications of C^*-algebraic technic to geometrical and topological problems and appropriately present the main actual problems in this field of noncommutative geometry and topology and indicate principal directions of its development.

To present the volume into perspective let us remind that the notion "noncommutative geometry" was coined out by Alain Connes in 1980's to indicate a new trend in mathematics. A naive look on this trend goes back to the prominent theorem of Gelfand and Naimark, which identifies the category of commutative unital C^*-algebras and the category of compact Hausdorff topological spaces. The passage to noncommutative algebras gives rise to the notion of "noncommutative topological space", which turned out to be fruitful despite the fact that they are not spaces in usual sense. The method and problems of this domain brought together a number of important achievements and open questions from topology, geometry, algebra and functional analysis. A most fruitful interference here is an enriching of the index theory of elliptic operators by the theory of C^*-algebras.

The papers from the present collection reflect some important actual problems and achievements of noncommutative geometry.

Index of elliptic operators: The paper *"Index Theory for Generalized Dirac Operators on Open Manifolds"* by **J. Eichhorn** is devoted to the index theory on open manifolds. In the first part of the paper, a short review of index theory on open manifolds is given. In the second part, a general relative index theorem admitting compact topological perturbations and Sobolev perturbations of all other ingredients is established. **V. Nazaikinskii and B. Sternin** in the paper *"Lefschetz Theory on Manifolds with Singularities"* extend the Lefschetz formula to the case of elliptic operators on the manifolds with singularities using the semiclassical asymptotic method. In the paper *"Pseudodifferential Subspaces and Their Applications in Elliptic Theory"* by **A. Savin and B. Sternin** the method of so called pseudodifferential projectors in the theory of elliptic operators is studied. It is very useful for the study of boundary value problems, computation of the fractional part of the spectral AtiyahPatodiSinger eta invariant and analytic realization of topological K-groups with finite coefficients in terms of elliptic operators. In the paper *"Residues and Index for Bisingular Operators"* **F. Nicola and L. Rodino** consider an algebra of pseudo-differential operators on the product of two manifolds, which contains, in particular, tensor products of usual pseudo-differential operators. For this algebra the existence of trace functionals like Wodzickis residue is discussed and a homological index formula for the elliptic elements is proved. **B. Bojarski** and **A. Weber** in their paper *"Correspondences and Index"* define a certain class of correspondences of polarized representations of C^*-algebras. These correspondences are modeled on the spaces of boundary values of elliptic operators on bordisms between two manifolds. In this situation an index is defined. The additivity of this index is studied in the paper.

Noncommutative aspects of Morse theory: In the paper *"New L2-invariants of Chain Complexes and Applications"* by **V.V. Sharko** homotopy invariants of free cochain complexes and Hilbert complex are studied. These invariants are applied to calculation of exact values of Morse numbers of smooth manifolds. **A. Connes** and **T. Fack** in their paper *"Morse Inequalities for Foliations"* outline an analytical proof of Morse inequalities for measured foliations obtained by them previously and give some applications. The proof is based on the use of a twisted Laplacian.

Riemannian aspects: The paper *"A Riemannian Invariant, Euler Structures and Some Topological Applications"* by **D. Burghelea** and **S. Haller** discusses a numerical invariant associated with a Riemannian metric, a vector field with isolated zeros, and a closed one form which is defined by a geometrically regularized integral. This invariant extends the ChernSimons class from a pair of two Riemannian metrics to a pair of a Riemannian metric and a smooth triangulation. They discuss a generalization of Turaevs Euler structures to manifolds with non-vanishing Euler characteristics and introduce the Poincare dual concept of co-Euler structures. The duality is provided by a geometrically regularized integral and involves the invariant mentioned above. Euler structures have been introduced because they permit to remove the ambiguities in the definition of the Reidemeister torsion. Similarly, co-Euler structures can be used to eliminate the metric dependence of

the RaySinger torsion. The BismutZhang theorem can then be reformulated as a statement comparing two genuine topological invariants. The paper *"Semiclassical Asymptotics and Spectral Gaps for Periodic Magnetic Schrödinger Operators on Covering Manifolds"* by **Yu.A. Kordyukov** is devoted to an exposition of a method to prove the existence of gaps in the spectrum of periodic second-order elliptic partial differential operators, which was suggested by Kordyukov, Mathai and Shubin, and describes the applications of this method to periodic magnetic Schrödinger operators on a Riemannian manifold, which is the universal covering of a compact manifold.

K-theory, C^*-algebras, and groups: In the paper *"The Group of Unital C^*-extensions"* by **V. Manuilov** and **K. Thomsen** it is shown that there is a natural six-terms exact sequence which relates the group which arises by considering all semi-split C^*-extensions of an algebra A by B to the group which arises from unital semi-split extensions of A by B. The paper *"The Thom Isomorphism in Gauge-equivariant K-theory"* by **V. Nistor** and **E. Troitsky** is devoted to the study of gauge-equivariant K-theory. In particular, they introduce and study products, which help to establish the Thom isomorphism in gauge-equivariant K-theory. They construct push-forward maps and define the topological index of a gauge-invariant family. The paper *"Bundles of C^*-algebras and the $KK(X;-,-)$-bifunctor"* by **E. Vasselli** is an overview of C^*-algebra bundles with a $\mathbb{Z}$-grading, with particular emphasis on classification questions. In particular, author discusses the role of the representable $KK(X;-,-)$-bifunctor introduced by Kasparov. As an application, Cuntz-Pimsner algebras associated with vector bundles are considered, and a classification in terms of K-theoretical invariants is given in the case of the base space being an n-sphere. **J. Brodzki** and **G.A. Niblo** in the paper *"Approximation Properties for Discrete Groups"* give a short survey of approximation properties of operator algebras associated with discrete groups. Then they demonstrate directly that groups that satisfy the rapid decay property with respect to a conditionally negative length function have the metric approximation property. The paper *"On the Hopf-type Cyclic Cohomology with Coefficients"* by **I.M. Nikonov** and **G.I. Sharygin** is devoted to the Hopf-type cyclic cohomology with coefficients. They calculate it in a couple of examples and propose a general construction of a coupling between algebraic and coalgebraic versions of such cohomology with values in the usual cyclic cohomology of an algebra.

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