

Preface

Problems linking the shape of a domain or the coefficients of an elliptic operator to the sequence of its eigenvalues are among the most fascinating of mathematical analysis. One of the reasons which make them so attractive is that they involve different fields of mathematics: spectral theory, partial differential equations, geometry, calculus of variations Moreover, they are very simple to state and generally hard to solve! In particular, one can find in the next pages more than 30 open problems!

In this book, we focus on extremal problems. For instance, we look for a domain which minimizes or maximizes a given eigenvalue of the Laplace operator with various boundary conditions and various geometric constraints. We also consider the case of functions of eigenvalues. We investigate similar questions for other elliptic operators, like Schrödinger, non-homogeneous membranes or composites.

The targeted audience is mainly pure and applied mathematicians, more particularly interested in partial differential equations, calculus of variations, differential geometry, spectral theory. More generally, people interested in properties of eigenvalues in other fields such as acoustics, theoretical physics, quantum mechanics, solid mechanics, could find here some answers to natural questions. For that purpose, I choose to recall basic facts and tools in the two first chapters (with only a few proofs). In chapters 3, 4 and 5, we present known results and open questions for the minimization problem of a given eigenvalue $\lambda_k(\Omega)$ of the Laplace operator with Dirichlet boundary conditions, where the unknown is here the domain Ω itself. In chapter 6, we investigate various functions of the Dirichlet eigenvalues, while chapter 7 is devoted to eigenvalues of the Laplace operator with other boundary conditions. In chapter 8, we consider the eigenvalues of Schrödinger operators: therefore, the unknown is no longer the shape of the domain but the potential V . Chapter 9 is devoted to non-homogeneous membranes and chapter 10 to more general elliptic operators in divergence form. At last, in chapter 11, we are interested in the bi-Laplace operator.

Of course no book can completely cover such a huge field of research. In making personal choices for inclusion of material, I tried to give useful complementary references, in the process certainly neglecting some relevant works. I would be grateful to hear from readers about important missing citations.

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