ANSWERS

for

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by

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Chapter 1

1.1 Linear differential equations of this type can be solved by assuming a solution in the form $x = e^{\gamma t}$ and determining γ in terms of the coefficients:

$$\gamma = -R/2M \pm [R^2 - 4MK_m]^{1/2}/2M = -R/2M \pm j[K_m/M - (R/2M)^2]^{1/2}$$

$$x = e^{-\alpha t}e^{\pm j\omega_0 t}, \quad \alpha = R/2M, \quad \omega_0 = [K_m/M - (R/2M)^2]^{1/2}$$

where $j = \sqrt{-1}$.

1.2 $D_f = (A + B)^2/(A^2 + B^2/3)$. For A = B, DI = 4.8 dB, this pattern is called a cardioid. Maximum DI = 6 dB for B/A = 3.

1.3 Note that $\mathrm{Real}(x_1x_2^*) = X_1X_2\cos(\varphi_1 - \varphi_2)$.

1.4 $d = \Delta t c_a c_w/(c_w - c_a)$

1.5 $k^2 = 1 - K_m^H/K_m^B = 1 - L_0/L_f = N^2/K_m^B L_0 = N^2/K_m^H L_f$

1.6 Tungsten 55.7, steel 26.9, PZT 14.7, aluminum 9.3 and magnesium 5.9. The best match is with a tungsten tail and a magnesium head (water end). This provides greater output and more bandwidth because of the better matched head and the higher impedance tail. A steel tail and aluminum head is often sufficient, less expensive and commonly used in many applications.

1.7 The power output $W = \eta_{ea}W_i$. SL = 30 dB – 3 dB + 0 + 170.8 = 197.8 dB// 1 μPa @ 1m and SL = 30 dB – 3 dB + 6 + 170.8 = 203.8 dB// 1 μPa @ 1m . Because of the 6 dB increase in DI, 6 dB less or ¼ the power would be needed, - truly significant.

1.8 From Eq. (1.20) $I_0 = D_f W/4\pi r^2$. The output power $W = u_r^2 R_r$ where u_r is the rms output velocity and R_r is the radiation resistance.

1.9 From Appendix A.13, Eq. (A13.31) $p(r, 0) = j\, \omega\, \rho\, \pi a^2\, u\, e^{-jkr}/2\pi r$ and the intensity, $I_0 = |p|^2/\rho c$, may then be written as $I_0 = (ka)^2 \rho c \pi a^2 u_r^2/4\pi r^2 = (ka)^2 \rho c \pi a^2 N_{em}^2 V^2/|Z_{mr}^E|^2 4\pi r^2$.

1.10 Yes, the D_f should not be used if the intensity is calculated through the velocity, voltage or, for that matter, the current. It is only to be used if the intensity is based on the power. Appendix A.13, Eq. (A13.17) may be written as $R_r D_f = (ka)^2 \pi a^2 \rho c$ and, when substituted in the solution for Exercise 1.9, yields $I_0 = D_f u_r^2 R_r/4\pi r^2 = D_f W/4\pi r^2$, as it should.

Chapter 2

2.1 The free permittivity $\varepsilon_{33}^T = K^T\, \varepsilon_0$ with K^T the relative dielectric constant found in Appendix A 5. (The dielectric constant for free space $\varepsilon_0 = 10^{-9}/36\pi$.). Capacitances: C_f = 0.145 and C_0 = 0.074 nF.

2.2. Mechanical compliance: $C^E = 1230 \times 10^{-12}$ and $C^D = 627 \times 10^{-12}$ m/N. Resonance frequencies: f_r = 14.4 kHz, f_a = 20.1 kHz and k_{eff} = 0.70, the same as k_{33} because no degrading effects were included in this idealized calculation.

2.3 Electromechanical turns ratio N = 0.235 (newton/V). Velocity u = 0.0979×10^{-3} and 0.4895 m/s for 1V and 5 kV. Mechanoacoustic efficiency η_{ma} = 50%.

2.4 Power out W = 143 watts. Intensities: I_s = 180 kW/m^2 =18 W/cm^2 and I_0 = 22.8 W/m^2 = .00228 W/cm^2. The difference in intensity is a result of the two areas used for I_s and I_0 and the directivity factor D_f.

2.5 The strain $S_3 = 115.6 \times 10^{-6}$.

2.6 The voltage = 5080 volts and the displacement = 1.47×10^{-6} m.

2.7 $K_m^E C_f = (A_0/s_{33}^E L)(A_0 \varepsilon_{33}^T/L) = (N/k)^2$ and $K_m^I L_f = (A_0/s_{33}^H L)(n^2 \mu_{33}^T A_0/L) = (N/k)^2$.

2.8 Piezoelectric ceramic turns ratio = 0.235 and the magnetostrictive turns ratio = 491.

2.9 Piezoelectric ceramic force = 2.35 newton and the magnetostrictive force = 49.1 newton.

2.10 At resonance the total energy is equal to the peak kinetic energy given by $Mu^2/2$ where u is the peak velocity. The energy dissipated per cycle is $(Ru^2/2)T$ where the period $T = 1/f_r = 2\pi/\omega_r$. The result is $Q_m = 2\pi(Mu^2/2)/[(Ru^2/2)2\pi/\omega_r] = \omega_r M/R$.

2.11 At resonance $\omega_r M = 1/\omega_r C_m^E$ and $\omega_r = 1/(MC_m^E)^{1/2}$. Substitution for $\omega_r M$ leads to the first result and substitution for ω_r to the second result with $K_m^E = 1/C_m^E$.

2.12 Use Eqs. (2.83) and (2.105) to eliminate the total resistance, $R_m + R_r$. Numerical answer is $Q_m = 4$.

2.13 Use Eqs. (2.83), (2.105) and (2.106). Numerical answers are $Q_m = 11.2$, 4.0 and 3.28 respectively.

2.14 Using Eqs.(2.112) and (2.115) the electroacoustic efficiency $\eta_{ea} = 76.9\%$ and 79.7% for tan δ = 0.01 and $\eta_{ea} = 57.1\%$ and 76.9% for tan δ = 0.10 for Q_m = 1 and 10 respectively. Note that for $k_{eff}^2 Q_m >> \tan\delta$ or $k_{eff}^2 Q_m Q_0 >> 1$, the electroacoustic efficiency $\eta_{ea} \approx \eta_{ma}$ at resonance. Numerical results are the same for the magnetostrictive case since $Q_0 = 1/\tan\delta$. This exercise shows the benefits of using materials with low electrical losses.

2.15 Numerical and analytical evaluation of Eq. (2.91). Limiting expressions of Eq. (2.91) can be obtained from the binomial expansion of $P_f(B/G)$ and $P_f(G/B)$.

2.16. $Q_m Q_e = (1/\omega_r C^E R)(\omega_r C_0/G) = C_0/C^E N^2 = C_0/k^2 C_f = (1 - k^2)/k^2$

2.17. For 100 volts rms the input power is 7.85 watts. The mechanoacoustic efficiency is 66.7 % and the output power is 5.24 watts.

Chapter 3

3.1. Ring: 41/4 =10.25 kHz. Sphere: 68/4 =17.0 kHz.

3.2. 33 mode segmented ring: 36.5/4 = 9.125 kHz. Assuming no fringing from striping the effective coupling coefficient can be found from Eq. (3.18) which gives k_e = 0.70/1.028 = 0.68 and the resonance frequency is reduced to 1.026x9.125 kHz = 9.362 kHz. The actual electric field in a striped ring is very complicated, and this model gives only a rough approximation of how striping reduces the coupling coefficient from the value for a segmented ring where the electric field is nearly uniform. Measurements indicate that the effective coupling could be about 10 % lower than the value calculated here.

3.3. For the ring: Water resonance ≈ 0.72x10.25 = 7.4 kHz and $Q_m \approx 3.9\eta_{ma}$ or ≈ 3 for $\eta_{ma} = 0.8$ using Eqs. (3.14) and (3.16). For the sphere use Eqs. (3.24) and (3.25) and find water resonance $\approx 15\text{kHz}$ and $Q_m \approx 3.05\eta_{ma}$.

3.4. Use the equations in Section 3.21 to obtain:

$C^E = 3.8\text{x}10^{-10}$ m/newton
N = 3.19 newton/volt
M = 0.62 kg
C_0 = 36 nF
$G_0 = 1.6\ \omega\ \text{x}10^{-10}$ mho (using $k_{31} = 0.33$ and tan $\delta = 0.004$)

$R_m = 0$ for $\eta_{ma} = 1$

The equivalent sphere model for radiation impedance is given and discussed in Eqs. (3.13), (10.44) and (A13.24). This radiation impedance model was used to calculate the in-water resonance frequency in Exercise 3.3. The TVR is obtained from Eqs. (3.17) and (3.7) as $p/V = \omega\rho_0 AN/[4\pi|Z_m||1+jk_r a_s|]$ which approaches $\omega^2 C^E \rho_0 AN/4\pi$ at low frequency.

3.5. Use equations from Section 3.1. Eq. (2.112) gives η_{em} at resonance; using k_{31} , $\tan\delta$ and Q_m from Exercises 3.3 and 3.4 shows that $\eta_{em} \approx 1$. Assuming $\eta_{ma} = 0.8$ makes $\eta_{ea} \approx 0.8$. The mass and volume of the ring are 0.62 kg and $4\text{x}10^{-4}$ m^3 ; assume that end caps and waterproofing together increases the total mass and total volume to 1kg and $6\text{x}10^{-4}$ m^3. Then, assuming a maximum electric field in the ceramic of 4 kV/cm, the equations in Section 3.1 give:

$$(FOM)_V = 75 \text{ watts/Hz m}^3$$
$$(FOM)_m = 45 \text{ watts/kHz kg}$$

3.6. One approach is to assume the radiation mass increases the effective tube length to .0254 +0.85x0.0483 = 0.0664m. For quarter wavelength resonance λ = 4x0.0664 = 0.266m, and the resonance frequency = c/λ = 1500/0.266 = 5.6 kHz.

Another approach is to use $\omega_h^2 = 1/C_2 M_2$ from Section 3.25 where C_2 is the spring of length L and M_2 is the approximate radiation mass. This gives the resonance frequency as 7.1 kHz. (see Exercise and Answer 3.7). If the length of the ring is short compared to the acoustic wavelength the latter approach is probably more accurate. For a longer ring the former approach may be more accurate. Since this is a radiation problem that has not been solved analytically, accurate results would require finite element numerical calculations.

3.7. Think of the radiation mass as a rigid mass of $M = 8\rho a^3/3$ attached to the end of the spring of length L/2 formed by the fluid in the cavity. Alternatively, think of it as an extension of the fluid in the cavity of radius a, length ΔL and mass $\rho\pi a^2\Delta L$. Equate the two masses and solve for ΔL. These two ways of approximating the radiation mass loading are not equivalent as seen in Exercise 3.6. Because of symmetry, a rigid plane could be inserted through the ring at half the height of the ring, without affecting the radiation loading.

3.8. A) With M_r = M_s = 0 and neglecting the tie rod we have $\omega_r = [(1+M_h/M_t)/M_h C^E]^{1/2}$ where C^E = $0.97\text{x}10^{-10}$ m/newton giving ω_r = $(1.29\text{x}10^9)^{1/2}$ and f_r = 5.71 kHz; N = 12.2 newton/volt, C_f = 29.3 nF, and k_e = 0.70 = k_{33} because no degrading effects have been included.

B) Use Eqs. (3.38) and (3.39); M_s = 0.048 kg which has a negligible effect on M and therefore on f_r and k_e . This is expected since the wavelength in the ceramic at f_r is about 50 cm while the length of the ceramic stack is only about 2 cm.

C) Modify Eq. (3.38) to include insulators and glue as well as the tie rod. This is done in Section 8.42 where it is shown that the effective compliance is

$$C_e = C_{tr}(C^E + C_i)/(C^E + C_{tr} + C_i),$$

where C_i is the insulator/glue compliance. Using $C_{tr} = 10C^E$ and $C_i = 0.1\ C^E$ gives $C_e = 0.991C^E$ which causes a very small increase in the resonance frequency, by the factor $(0.991)^{-1/2}$. The effect of these compliance changes on the effective coupling coefficient, k_e, is discussed in Section 8.42 and given by Eq. (8.31). In this case the result is that k_e is reduced from 0.7 to 0.65.

3.9.Calculate the velocity in the mechanical branch for a given input voltage. Then use Eq. (3.17) for the far field pressure.

3.10 Fundamental extensional ring resonance occurs for the circumference $\pi D = \lambda = c/f$ where c is the bar speed of sound. Thus $fD = c/\pi$. From Appendix A.7, for Terfenol-D, $f^H D = 0.54$ kHz m = 21.3 kHz in, $f^B D = 0.764$ kHz m = 30.1 kHz in. while for Galfenol $f^H D = 0.86$ kHz m = 33.9 kHz in, $f^B D = 1.08$ kHz m = 42.5 kHz in.

3.11. Use the results from Ex. 3.10 with the ring diameter 4 inches. For Terfenol-D: f^H = 5.3 kHz and f^B = 7.5 kHz while for Galfenol: f^H = 8.6 kHz and f^B = 10.6 kHz. From Ex 3.1 the short circuit PZT-4 resonance, f^E = 10.25 kHz, is nearly the same as the Galfenol short circuit resonance but higher than the Terfenol-D short circuit resonance. The free inductance $L_f = \mu_{33}^T n^2 A_c/\pi D$. The cross sectional area $A_c = 0.258\text{x}10^{-3}\ \text{m}^2$, $\pi D = 0.319$ m, n = 100 and $\mu_{33}^T = \mu_0 \mu_r^T$. For Terfenol-D the free inductance L_f = .095 mH , while for Galfenol the considerably higher value L_f = 2.64 mH is obtained.

3.12 See Section 3.32. Start with $k_e^2 = (E_{m1} + E_{m2})/\ (E_{m1} + E_{m2} + E_e)$, define $k_1^2 = E_{m1}/(E_{m1} + E_e)$ and $k_2^2 = E_{m2}/(E_{m2} + E_e)$, write $E_{m1}/E_e = k_1^2/(1 - k_1^2)$ and $E_{m2}/E_e = k_2^2/(1 - k_2^2)$ to get the desired expression $k_e^2 = (k_1^2 + k_2^2 - 2\ k_1^2 k_2^2)/(1 - k_1^2 k_2^2)$. For $k_1 = k_2 = k$ we get $k_e^2 = (2k^2 - 2k^4)/(1 - k^4) = 2k^2(1 - k^2)/(1 - k^2)(1 + k^2) = 2k^2/(1 + k^2)$ as expected.

Chapter 4

4.1. This ring has the same dimensions as the ring in Exercise 3.1; therefore, f_a = 10.25/0.94 kHz = 10.9 kHz; RVS = -185 dB/1V//1μPa; C_f = 36.7 nF; f_r(length mode) = 65/2 = 32.5 kHz, f_a (length) = 32.5/0.94 = 34.6 kHz.
Sphere: f_a = 17/0.81 = 20.9 kHz, RVS = -185 dB/1V//1μPa, C_f = 73.4 nF.

4.2. Eq. (4.10) gives the general expression for the sensitivity of the cylinder as a function of frequency where D_a for a sphere is given in Eq. (4.53) and ω_a is changed from the value in Exercise 4.1 by the radiation mass. Fig. 4.11 is an example of the wideband sensitivity.

For the low frequency case: RVS = -185 dB/1V//1μPa as found in Exercise 4.1 or $M = g_{31}\ a$, from which the TVR can be found using reciprocity and Eq. (12.33) for low frequency where $Z_0 = 1/\ j\omega C_f$. TVR = TCR/Z_0 = $(\rho_0 f / 2Z_0)M = 2\pi^2 a^2 g_{31} \rho_0 f^2 L \varepsilon_{33}^T / t$.

From Exercise 3.4 the low frequency TVR was found directly to be $\omega^2\rho_0 ANC^E/4\pi$. These two results for TVR are equal as can be seen by using the expressions for N and C^E and $d_{31} = g_{31}\varepsilon_{33}^T$.

Similarly, using Eq. (12.36) and Exercise 4.1 we have

$$\begin{aligned} TVR &= -185 + 20\log(36.7\text{x}10^{-9}) + 40\log f + 310 \\ &= -23 + 40\log f \ \ \text{dB}//\mu\text{Pa} @ 1\text{m}/\text{V} \end{aligned}$$

The RVS and TVR for all frequencies up to anti-resonance requires extensive calculation. Results at low frequency and at f_a are the most important.

4.3. f_a = 109 kHz, RVS = -205 dB, Yes, the scaling factor is 10 in this case. If summed in parallel RVS = -205 dB, while if summed in series -199 dB. If series differenced, RVS = - 199 + 20 log ($\pi sf/c_0$) = - 278.5 + 20 log f dB. Deviation expected in the vicinity of quarter wavelength separation at frequency of 7.38 kHz. Axial null summed modes at one-half wavelength at frequency of 14.76 kHz. Axial null differenced modes at one wavelength at frequency of 29.53 kHz.

4.4. Use low frequency Eq. (4.67b) yielding equivalent noise pressure of 29 – 10 log f dB re 1 μPa for a 1 Hz band and Eq. (4.58) for a noise voltage of -156 – 10 log f dB re 1 volt for a 1 Hz band. Equivalent circuit or analytical calculations are needed for wide band noise response.

4.5. From Eq. (4.58), $10\log < V_n^2 > = -198 + 10\log 2R_h$ for both summed and differenced cases since the incoherent noise voltages add. Using $R_h = \tan\delta/\omega C_f$ for low frequency gives 10 log $2R_h$ = 45 – 10 log f and $< V_n^2 >$ = -153 – 10 log f .

From Eqs. (4.80a,b), using RVS = -199 dB for the series summed case and -278.5 +20 log f for the series differenced case from Exercise 4.3 gives

$$\begin{aligned} 10 \log <p_{on}^2> &= 46 - 10 \log f, \\ 10 \log <p_{dn}^2> &= 125 - 30 \log f. \end{aligned}$$

These results illustrate the much higher levels of equivalent noise pressure for the differenced case at low frequency.

4.6., RVS = -167.4 dB. Equivalent noise pressure = 12.8 – 10log f dB in one cycle band using C_f = 29.3 nF and Eq. (4.67b). Operation of the T/R switch is discussed in Section 12.61 and shown in Fig. 12.21.

4.7. Resonance f_r: 10.25, 14.49, 22.92, 32.41 kHz. From Exercise 4.1 f_r(length mode) = 32.5 kHz which is only slightly above the n = 3 mode and may seriously distort the directivity pattern of that mode.

4.8. The reciprocal nature of transducers gives the radiation resistance this dual role. See Appendix A.17 for further discussion.

4.9. $D_a = [1+(ka)^2]^{-1/2}$. The direct derivation of this result, which is somewhat lengthy, is given in Section 11.31.

Chapter 5

5.1. Use $R_1 = R_{11} + R_{12}$ and $R_{12} \approx R_{11}(\sin kd)/kd$ from Eq. (5.23b). A rigid wall creates an image of the piston approaching it, making it equivalent to an array of two elements.

5.2. Use $V_i/I_i = (Z_e)_i$ and $Z_i = F_i/u_i$.

5.3. Packing factor $pf = L^2/(L + L/10)^2 = 0.826$. See Section 5.31. The average radiation resistance of one piston in the array is $\rho c L^2 pf$ and the total radiation resistance of an array of N pistons is $\rho c N L^2 pf = \rho c N(L + L/10)^2 (pf)^2 = \rho c$ (total array area)$(pf)^2$.

5.4. The uniform velocity distribution u_i^* factors out of the integral and is canceled by the denominator U_i^* leaving $Z_i = \int p(r_i) ds_i/U_i = F_i/U_i$.

5.5. It allows simplification and further physical interpretation by factoring a common function leaving the product of this function and that for an array of point sources. This interpretation may be helpful for DI calculations, if the integral of the product of the squares of the two functions is easier to evaluate. It may also be helpful for radiation resistance calculations since radiation resistance is directly related to the far field, and possibly for radiation reactance calculations (see Section 11.14).

5.6 Eq.(5.10a) gives $p(\theta) = \text{Sinc}[\pi N(\sin\theta - \sin\theta_0)]$ which can be evaluated for a steering angle of $\theta_0 = 45$ degrees. Eq. (5.10b) shows that a grating lobe will occur at $\sin\theta = -0.293$ or at -17 degrees, and the individual transducer beam pattern factor in Eq. (5.8) shows that it will be reduced by about 2.6 dB.

5.7. The function Sinc $x \approx (x - x^3/6 + \bullet\bullet\bullet)/x = 1 - x^2/6 + \bullet\bullet\bullet \rightarrow 1$ as $x \rightarrow 0$. Also the function $J_1(x) \approx x/2 - x^3/16 \bullet\bullet\bullet$ and thus, $2J_1(x)/x \rightarrow 1$ as $x \rightarrow 0$.

5.8. The quantity $\alpha/k_d = \sin^2(BW/4) = 3.05 \times 10^{-4}$. Then plot the normalized beam pattern function $p(\theta) = [1 + 1.1\text{x}10^7 \sin^4(\theta/2)]^{-1/2}$.

5.9 The total radiation resistance of the center transducer for $\lambda = 4d/3$ is

$$R_1 = R_{11}[1 + N\sin kd/kd] = R_{11}[1 - 2N/3\pi]$$

which is negative for N > 5, meaning that the center transducer is absorbing power radiated by the other transducers. The total radiation reactance of the center transducer when $d = \lambda/2$ is $X_1 = X_{11} - NR_{11}/\pi$, which could also become negative for large enough N, reducing the total mass on the center transducer and making its resonance frequency higher than that of the others.

5.10 Using Eqs. (5.16) and (5.23b) gives:

$$Z_1 = Z_{11} + Z_{12}e^{j\mu} = Z_{11} - R_{11}je^{-j(kd-\mu)}/kd$$

$$Z_2 = Z_{22} + Z_{21}e^{-j\mu} = Z_{11} - R_{11}je^{-j(kd+\mu)}/kd$$

$$R_1 = R_{11}[1+\sin(kd-\mu)/kd], \qquad X_1 = X_{11} + R_{11}\cos(kd-\mu)/kd$$
$$R_2 = R_{11}[1+\sin(kd+\mu)/kd], \qquad X_2 = X_{11} + R_{11}\cos(kd+\mu)/kd$$

For steering to end-fire, $\mu = kd$, which gives

$$R_1 = R_{11}, \qquad X_1 = X_{11} + R_{11}/kd$$
$$R_2 = R_{11}[1+\sin 2kd/kd], \qquad X_2 = X_{11} + R_{11}\cos 2kd/kd$$

Note that driving transducers in an array to achieve the same velocities, as in Exercises 5.9 and 5.10, is not accomplished by driving with the same voltages, since acoustic coupling affects the radiation impedances, and thus the total mechanical impedance, in a way that depends on the velocities. The calculations described in Section 5.21 are needed to determine the required voltages.

Chapter 6

6.1. Substitute $m_1(x_0) = m_0 e^{jkx_0 \sin\gamma}$, where γ is the steering angle and m_0 is a uniform sensitivity , into the one dimensional version of Eq. (6.5a) where θ is the beam pattern angle. Integrate from –L/2 to L/2, where L is the line length, and normalize to obtain the beam pattern function Sinc [(kL/2)sin θ – (kL/2)sin γ)], which shows that the wave number k sin θ is displaced by k sin γ which means the beam is steered from $\theta = 0$ to $\theta = \gamma$.

6.2. Waves traveling in a medium at a speed lower than the sound speed in that medium are called non-acoustic waves. Hydrophone arrays designed to receive acoustic waves in water often must be installed in locations where noise in the form of non-acoustic waves is present. Therefore the response of the array to non-acoustic waves is critical in determining the signal to noise ratio. (See Section 6.3).

6.3. The intensity increase indicated by the DI of a projector array is referenced to the average (omni-directional) radiation. Therefore, in a receiving array only array gains that are determined by isotropic (omnidirectional) noise could be consistent with this definition of DI. Because array gain depends on noise, while DI does not, array gain and

DI depend on frequency and array geometry in different ways; therefore, they generally do not have the same value (see Exercises 6.4 and 6.5 and the first paragraph of Section 6.2).

6.4. For $kD = \pi$: $D_f = N = 6$, DI = 7.8 dB for all steering angles
For $kD = \pi/2$: $D_f = 3.19$, DI = 5.04 dB for no steering
$D_f = 3.23$, DI = 5.07 dB for 30 degree steering
$D_f = 6$, DI = 7.8 dB for 90 degree steering
For $kD << 1$: $D_f = 1$, DI = 0

6.5. Use $\sin 2x = 2\sin x \cos x$ to show that $D_f(N, kD, 0°) = D_f(N, 2kD, 90°)$.

6.6. For all the arrays of Exercise 6.4 the array gain in isotropic, incoherent noise is the same, 10log 6 = 7.8 dB. This can be seen from the examples at the end of Section 6.2 and from Eq. (6.12) for the steered cases with the signal arriving from the steered direction, i.e., $kd_{ij}\cos\theta = -\phi_{ij}$ which makes $\rho^{s}_{ij} = 1$ for all hydrophone pairs.

6.7. Using the properties of carbon steel from Appendix A.2, the coincidence frequency is approximately 36 kHz. The evanescent pressure wave amplitude decays by a factor of 0.47 at 1 cm from the plate, or 6.6 dB.

6.8. A noise model must be assumed as the basis for calculating a spatial correlation function. The simplest model for isotropic noise assumes that the noise consists of uncorrelated plane waves arriving with equal intensity from all directions. Thus, starting with the cross correlation function for one plane wave arriving from a specific direction as given in Eq. (6.12) with $\phi_{ij} = 0$, and averaging over all directions gives the result in Eq. (6.16). Two other noise models that give the same spatial correlation are: 1) The noise originates from a thin spherical shell of point noise sources, or 2) from a uniform spherical volume distribution of point noise sources. In both cases, when the radius of the sphere is very large and the spatial correlation is calculated is at the center of the sphere, the result is also Eq. (6.16).

6.9. Imagine a coordinate system with two unit vectors, V_1 and V_2, starting from the origin and pointing in different directions given by θ_1, ϕ_1 and θ_2, ϕ_2. The vector dot product of unit vectors is equal to the cosine of the angle between them, γ, and can be evaluated by calculating the sum of the products of the vector components. Thus, using $V_{1x} = \sin\theta_1 \cos\phi_1$, etc., gives

$$\cos\gamma = V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1 - \phi_2)$$

This general relation also contains the direction cosines of any specified vector. For example, the direction cosine of V_1 with respect to the x-axis is given for $\theta_2 = 90°$ and $\phi_2 = 0°$ as $\cos\gamma_x = \sin\theta_1\cos\phi_1$.

6.10 Using Eq. (6.16) for the spatial correlation of ambient noise gives:

1) $AG = 10\log\{4N^2/[\sum\sum\rho_{ij}^{na} + 3N]\}$
2) AG = 10logN
3) $AG = 10\log\{36/[3 + 4\sin kD/kD + 2\sin 2kD/2kD + 9]\}$
4) AG = 10log 2.47 = 3.93 dB

6.11 . Use Eq. (5.23b) to determine the total radiation resistance of the array,

$$R_r = 2\{\rho c\pi a^2[(ka)^2/2][1 + \sin kD/kD]\}$$

Use Eq. (6.8b) to determine D_f for a two element line array in free space and multiply by 2 for a line array in a plane baffle, $D_f = 2/(1+\sin kD/kD)$. Determine D_a from Eq. (4.51) for a plane wave arriving on the MRA of the array, $D_a = 2$. Show that these results satisfy Eq. (4.56).

Chapter 7

7.1. For $M_1 = 4$ kg, $f_r = 36$ kHz and $Q_m = 4.7$, while for $M_1 = 8$ kg, $f_r = 34$ kHz and $Q_m = 4.0$. It's probably not worth going from 4 kg to 8 kg as there is only a 6 % reduction in frequency and a 15 % reduction in Q_m. Better to increase the length of the piezoelectric material.

7.2. The anti-resonance frequency $\omega_a^2 = 1/MC^D = \omega_r^2/(1 - k^2)$ since $C^D = C^E(1 - k^2)$.

7.3. The sine and binomial expansions yield $1/[\sin kL] \approx 1/[kL - (kL)^3/6] \approx [1 + (kL)^2/6]/kL = 1/kL + kL/6$ which when multiplied by $-j\rho cA_0$ yields $- j\omega M/6$ for the second term.

7.4. At very low frequencies $1/\omega C$ becomes very large and open circuiting this branch leaving just M/2 + M/2 = M.

7.5. Eliminate u_2 in Eq. (7.36a) and F_2 in Eq. (7.36b) and then identify coefficients.

7.6. For kL << 1, $j\rho cA_0 \tan kL/2 \approx j\omega M/2$ and $-j\rho cA_0/\sin kL \approx 1/j\omega C^E$.

7.7. Substitute $Z_m = j(\omega M - 1/\omega C) + R$ and $Y_0 = j\omega C_0 + \omega C_f/\tan\delta$ into Eq. (7.70).

7.8. The far field pressure at 1 m is $p = NAfp_p/c = 4300$ μPa.

Chapter 8

8.1. $f_r = 32$ kHz and $Q_m = 3.3$ when the bar mass is ignored; $f_r = 28$ kHz and $Q_m = 3.8$ with an added bar mass of 0.3 kg.

8.2. Divide by N^2C^E/C_f. Subtract both sides of Eq. (4.23) from 1.

8.3. For k = 0.5, k_e = 0.482, for - 3.6% change; change in f_r is + 5 %; original $f_a = f_r/0.87 = 1.15f_r$, new $f_a = 1.05f_r/0.876 = 1.19f_r$ where f_r is the original resonance frequency

8.4. For k = 0.5, k_e = 0.477, for – 4.5% change, change in f_r is -5 %, original $f_a = f_r/0.87 = 1.15f_r$, new $f_a = 0.95f_r/0.879 = 1.08f_r$

8.5. For k = 0.5, k_e = 0.458 for – 8.5% change, change in f_r is 0.5 % , original $f_a = 1.15\ f_r$ new $f_a = 1.005f_r / 0.89 = 1.13f_r$

8.6. Segmented bar with k = 0.5: k_e = 0.46, for – 8% change. End electroded bar with k = 0.5, k_e = 0.45, for – 10 % change.

8.7. For k = 0.5 and n = 1, k_e = 0.45; n = 2, k_e = 0.225, n = 3, k_e = 0.15.

8.8. The modes are given by $(kL)_n = n\pi/2$ with n = odd integers, and the dynamic stiffness is $K_{dn} = K_b n^2\pi^2/8$. For the length expander bar, $k_{edn}^2 = 8k_{33}^2/n^2\pi^2$ and for the segmented bar, $k_{edn}^2 = k_{33}^2/[k_{33}^2 + (n^2\pi^2/8)(1-k_{33}^2)]$.

8.9. Determine an effective stiffness for each mode from the modal frequency, i.e. $\omega_n^2 = \omega_0^2(1+n^2) = K_n/M = K^E(1+n^2)/M$, $K_n = K^E(1+n^2)$. Then, assuming C_0 and N are the same for each mode, and following Section 8.43 where it is noted that the dynamic increase in stiffness, $(K_n - K^E)$, has the same effect as a stress rod, use of Eq. (8.29) gives

$$k_{edn}^2 = k_{31}^2/[1+(1-k_{31}^2)n^2].$$

The values for n = 0, 1, 2, 3 are 0.33, 0.24, 0.16, 0.11.

Chapter 9

9.1. The equation becomes $x_3/L = d_{33}V/(L + x_3/L)$ leading to $(1 + x_3/L)x_3/L = d_{33}V/L$ yielding the quadratic result $S_3^2 + S_3 - d_{33}V/L = 0$ with exact solution $S_3 = [-1 + (1 + 4d_{33}V/L)^{1/2}]/2$. The four term binomial expansion for $(1 + y)^n \approx 1 + n\,y + n\,(n - 1)\,y^2/2 + n(n-1)(n-2)\,y^3/6$ leads to $S_3 = d_{33}V/L - (d_{33}V/L)^2 + 2(d_{33}V/L)^3 + \bullet\bullet\bullet$.

9.2. Exact quadratic solution for S_3 gives 1.1558664×10^{-4}. Neglecting the nonlinearity gives the approximate solution, $S_3 = d_{33}\,V/L = 1.156 \times 10^{-4}$ with an error of only 0.01% , which is much smaller than the uncertainty in d_{33} . The specific nonlinear mechanism included here, at very low frequency, is the variation of the electric field as the length of the material varies with the voltage held constant. At higher frequencies other nonlinear mechanisms would usually have more significant effects.

9.3. The strain versus field curve must be symmetric for even harmonic generation and anti-symmetric for odd harmonic generation. This can be seen from the nonlinear solution in Exercise 9.1, although that exercise was stated for nearly static conditions. If the applied voltage is $V = V_0 \cos\omega t$ the second term of the solution, which is symmetric, gives a strain proportional to $\cos^2\omega t = (1+\cos 2\omega t)/2$, i.e., static and second harmonic strain components. Similarly, the third anti-symmetric term of the solution, proportional to $\cos^3\omega t = (3\cos\omega t + \cos 3\omega t)/4$, gives a third harmonic and a change in the fundamental.

9.4. Below resonance the voltage response increases (typically at 12 dB/octave) and flattens out above resonance. If a transducer is operated below resonance, the harmonics will have additional output relative to the fundamental, since they occur at higher frequencies where the voltage response is greater. Operating at $f_r/2$ and $f_r/3$ augments the second and third harmonics, since then they occur at the transducer resonance frequency where the voltage response is usually maximum (see the pressure harmonics in Figs. 9.4 and 9.5 and Exercises 9.6 and 9.7).

9.5. Materials generally produce harmonics when the strain approaches and exceeds the elastic limit or when the electric or magnetic field approaches breakdown. In this region the transducer efficiency at the drive frequency decreases since part of the input power is transferred to power in the harmonics. Thus an increase of input power is required to achieve a goal based on linearity, which brings the strain or field closer to the limits of the material and may lead to mechanical or electrical failure of the transducer.

9.6. The equation of motion is: $M\ddot{x} + R\dot{x} + (Ac/L)x = (Ae/L^2)V^2 = (Ae/L^2)(V_0^2 + 2V_0V_1\cos\omega t + V_1^2\cos^2\omega t)$
$= (Ae/L)^2(V_0^2 + \frac{1}{2}V_1^2 + 2V_0V_1\cos\omega t + \frac{1}{2}V_1^2\cos 2\omega t)$ which shows the static, fundamental and second harmonic drive terms. The solution for the displacement for each drive term can be found separately since this is a linear equation. The results are:

$$|x_0| = (Ae^2/cL)(V_0^2 + \frac{1}{2}V_1^2)$$
$$|x_1| = (Ae/L)^2(2V_0V_1)/\omega|Z_m(\omega)|$$
$$|x_2| = (Ae/L)^2V_1^2/4\omega|Z_m(2\omega)|$$

9.7. The ratios can be found from the solutions in Exercise 9.6 or from Eq. (9.36) and Table 9.2:

$|x_2/x_1| = V_1/4V_0$, well below resonance

$|x_2/x_1| = (V_1/16V_0)(4+9Q_m^2)^{1/2}$, at one half the resonance frequency

$$\left|x_2 / x_1\right| = (V_1 / 4V_0)/(4+9Q_m^2)^{1/2} \quad , \quad \text{at the resonance frequency}$$

Note that for $Q_m = 10$ the ratio at one half the resonance frequency is 7.5 times greater, and at the resonance frequency 30 times smaller, than it is well below resonance, showing that harmonic distortion and its effects can vary strongly with frequency.

9.8. Let U_{m1} and U_{m2} be the converted mechanical energy in the fundamental and in the second harmonic and U_e be the stored electrical energy. Define the square of the electromechanical coupling coefficient, generalized to nonlinear conditions, as the ratio of the mechanical energy in the fundamental to the total input energy:

$$k_{nl}^2 = U_{m1}/(U_{m1} + U_{m2} + U_e) = k^2/(1 + U_{m2}k^2/U_{m1})$$

where $k^2 = U_{m1}/(U_{m1} + U_e)$ is the squared coupling coefficient when nonlinear effects are negligible. Using the displacement ratio from Exercise 9.6, and noting that the mechanical energy is proportional to displacement squared, gives at low frequency,

$$U_{m2}/U_{m1} = \left|x_2/x_1\right|^2 = (V_1/4V_0)^2 \; .$$

When operating a biased transducer the usual practice is to keep the alternating drive voltage less than the bias voltage ($V_1 < V_0$) to avoid significant nonlinear effects. The results from Exercise 9.6 show that, even for $V_1 = V_0$, the effect on coupling is very small at low frequency (for k = 0.7, k_{nl} = 0.696) and even smaller at resonance. But the effect is much greater at one half the resonance frequency (for k = 0.7 and Q_m = 10, k_{nl} = 0.42). This is a specific example of the effects pointed out in Exercise 9.4.

9.9 For the component at

$$\begin{aligned} \omega&: \quad x_1 \propto V^2, \qquad u_1 \propto \omega V^2 \\ 2\omega&: \quad x_2 \propto V^2/2, \quad u_2 \propto \omega V^2 \\ 3\omega&: \quad x_3 \propto V^2, \qquad u_3 \propto 3\omega V^2 \\ 4\omega&: \quad x_4 \propto V^2/2, \quad u_4 \propto 2\omega V^2 \end{aligned}$$

9.10 Since the radiated pressures are proportional to the accelerations the four pressure components are: $p_1 \propto \omega^2 V^2$, $p_2 \propto 2\omega^2 V^2$, $p_3 \propto 9\omega^2 V^2$, $p_4 \propto 8\omega^2 V^2$.

Chapter 10

10.1. Substitution yields $\partial^2 p/\partial x^2 = -k^2 p$. Particle velocity = u = (jk/jωρ)p = p/ρc showing that characteristic impedance is ρc. Characteristic impedance ratio of PZT to water is 22.2/1.5 = 14.8.

10.2. For $L = \lambda$, $kL = 2\pi$. $S = \lambda/2$. The line with $L = \lambda$ can be thought of as two lines, each $\lambda/2$ long, with centers $\lambda/2$ apart, and thus canceling in the axial direction as the two point sources do.

10.3. Line: BW = 51^0 and DI = 3 dB. For piston: BW = 58^0 and DI = 9.9 dB. The higher DI of the circular piston occurs because it radiates mainly in one direction, while the line radiates omnidirectionally in the plane perpendicular to its axis. BW is caused by partial cancellation as the observation point is moved away from the MRA. The fact that the piston has area, while the line does not, causes the piston's greater beam width when the diameter is the same as the line length. This can be seen by considering the piston to be a collection of parallel strips. Note that cancellation for those strips near the diameter is about the same as it is for the line. However, for those shorter strips near the edge of the piston, cancellation is less than for the line, thus giving a broader beam for the piston.

10.4. For the line the exact BW = 52.9^0 and the approximate BW = 51^0; error of 3.6% for one wavelength long. A better approximation for DI, given by Eq. (10.23b), is 2.5 dB rather than the approximate 3dB. For the circular piston the exact BW = 62^0 and the approximate BW = 58^0 ; error of 6.5% for one wavelengh diameter. The exact Eq. (10.31) yields DI = 9.7 dB rather than the approximate 9.9 dB.

10.5. Normalize Eq. (10.34) by dividing by the value at $\theta = 90^0$ giving

$$p(\theta)/p(90^0) = \mathrm{Sinc}(kL\cos\theta)H_0'(ka)/\sin\theta H_0'(ka\sin\theta).$$

This ratio can be simplified by use of the approximation , $H_0'(x) = (2/\pi x)^{1/2}$, valid for x > 1/2 (see Morse and Ingard, Reference 17, p.360), and it becomes

$$p(\theta)/p(90^0) = \mathrm{Sinc}(kL\cos\theta)(\sin\theta)^{1/2}.$$

Note that the first factor is the line function (for length 2L). Now with $kL = ka = \pi$, find θ for a pressure amplitude ratio of 0.707 (i.e. -3 dB) . The quantity 2θ gives the beam width. This must be done by trial and error, and it is important to start with a good guess. In this case, since the ring is the same length as the line in Exercise 10.4, the beam width will be similar; so 50^0 is a good initial estimate. Thus $\theta = (\pi - BW)/2 = 65^0$, $\mathrm{Sinc}(\pi\cos\theta) = 0.73$, $(\sin\theta)^{1/2} = 0.95$ and the pressure ratio = 0.69, close to the desired value. A trial beam width of 46^0 gives a pressure ratio of 0.74. The correct beam width is about 48^0 .

10.6. Eq. (10.36) with z = 0 gives nulls when sin(ka/2) = 0 and the radius $a = n\lambda$, n = 1, 2, 3, • • • .

10.7. For ka << 1, $ka_o = (ka_d)^{3/2}/(12)^{1/4}$. For ka >> 1, $ka_o = ka_d/\sqrt{3}$. When the size is small compared to the wavelength the dipole is a much poorer radiator than the monopole because of strong cancellation from its two out-of-phase parts. Thus to radiate the same power the dipole must have much more radiating area or much greater velocity.

10.8. $D_a = 2$ for a plane wave arriving perpendicular to the plane.

10.9 $D_a = (ka)^2/[4+(ka)^4]$ for a plane wave arriving parallel to the axis of the dipole. This D_a has a maximum value of 0.25 when $ka = \sqrt{2}$.

10.10. Consider a free-flooding ring of length L, inner radius a and outer radius b. Use a cylindrical coordinate system with the center of the ring at the origin. This makes the z = 0 plane a plane of symmetry, and the boundary conditions only need to be stated for half the ring. The boundary values of the normal velocity are:

$$
\begin{array}{lll}
u = 0, & \text{for } z = 0, & 0 < r < a \quad \text{and} \quad b < r < \infty \\
u = u_t, & \text{for } z = L/2, & a < r < b \\
u = u_i, & \text{for } 0 < z < L/2, & r = a \\
u = u_o, & \text{for } 0 < z < L/2, & r = b
\end{array}
$$

where u_t, u_i and u_o are uniform outward normal velocities on the top, inside and outside surfaces of the ring. These velocities are related by the mechanics of the ring; e.g., if the ring has very thin walls, $u_i \approx -u_o$. Consider trying to make the cylindrical wave functions in Eq. (10.13) satisfy these conditions. The ring has cylindrical symmetry and only the functions for m = 0 are needed, but all values of k_z are allowed. A solution in terms of these wave functions does not exist (see Chapter 11).

Chapter 11

11.1. Spherical coordinates are the most useful because the constant coordinate surfaces are finite and therefore capable of fitting real transducers. Thus exact solutions are available for spherical transducers with any normal velocity distribution and exact solutions for quantities such as radiation impedance have been derived from them. Oblate spheroidal, prolate spheroidal and ellipsoidal coordinates also have the important advantage of finite constant coordinate surfaces, although the wave functions are not as well developed. Rectangular and cylindrical coordinates are commonly used, but their constant coordinate surfaces are not finite, and radiation from finite flat surfaces such as pistons can only be solved by assuming they are part of an infinite rigid plane. Similarly, cylindrical radiators must be assumed to have infinite rigid extensions. Radiation from common shapes such as a rectangular box or a cylinder of finite length cannot be solved by expansion in wave functions except as an approximation.

11.2. Use $P_1(x) = x$ and let $x = \cos\theta_i$ etc.

11.3. Satisfying the boundary condition in Eq. (11.2) requires the integral:

$$\int_0^{\pi} u(\theta) P_m(\cos\theta)\sin\theta d\theta = u_0 \int_0^{\theta_{0i}} P_m(\cos\theta)\sin\theta d\theta$$

$$= \frac{u_0}{(2m+1)}[P_{m+1}(\cos\theta_{0i}) - P_{m-1}(\cos\theta_{0i})].$$

11.4. Use Eq. (11.1) with m = 0 because of the azimuthal symmetry of the boundary condition, leading to Eq. (11.3a). The boundary condition is satisfied, in the same way that led to Eq. (11.4), using the orthogonality relation given in Exercise 11.3. Using the relation $P_n(-x) = P_n(x)$ for n an even integer and $P_n(-x) = -P_n(x)$ for n an odd integer gives the coefficients:

$$A_n = [\rho c u / j h_n'(ka)][P_{n+1}(\cos\theta_1) - P_{n-1}(\cos\theta_1)], \qquad \text{for n even}$$
$$A_n = 0, \qquad \text{for n odd} .$$

Vanishing of the odd coefficients is a necessary feature of the solution because the boundary condition in this Exercise is symmetric about the equatorial plane, while the boundary condition in Exercise 11.3 does not have this symmetry. Using $P_0(\cos\theta) = 1$ and $P_2(\cos\theta) = (3\cos 2\theta - 1)/4$ the first two terms of the solution for the pressure are:

$$p(r,\theta) = A_0 h_0^{(2)}(kr) + A_2(3\cos 2\theta - 1)h_2^{(2)}(kr)/4 .$$

The second term shows the departure from omnidirectionality that results from this model of an end capped ring transducer.

11.5. Follow the procedure in Section 11.14 but use the radiation resistance of a dipole given by the real part of Eq. (10.47).

11.6. Use reasoning similar to that in Section 10.33 for small pistons. Imagine an array constructed such that it vibrates exactly the same on both sides (case 1, the rigid baffle). Imagine the same array surrounded by pressure release material, or lying on the surface of water, or constructed to vibrate exactly the opposite on both sides (case 2, the pressure release baffle). Now consider the same array mounted such that it vibrates only on one side (case 3, no baffle). As in Section 10.33, superposition of the velocity boundary conditions for cases 1 and 2 shows that $u_3 = (u_1 + u_2)/2$, and it follows that the entire pressure fields of the three cases are related in the same way, i.e. , $p_3 = (p_1 + p_2)/2$. But in the plane of the array $p_2 = 0$, and so $p_3 = p_1/2$ in the plane of the array. The approximation in Eq. (11.44b) is also consistent with this result.

11.7. Note that the D_a for the piston holds for a wave at normal incidence. As the wavelength decreases the piston acts as a plane rigid baffle with the reflected and incident waves in phase, which raises the pressure on the surface and increases the D_a . While for the sphere the phase distribution of both the incident and scattered waves becomes more nonuniform as the wavelength decreases, which reduces the average pressure on the surface and decreases the D_a . It is reasonable for D_a to decrease as 1/ka as ka increases, because 1/ka is proportional to the number of wavelengths contained in the diameter of the sphere; the more wavelengths, the more completely the pressure cancels.

11.8. The pressure along the pressure release surface is forced to be zero, but the velocity of that surface is not zero. The motion of the pressure release surface is similar to an array of dipoles that results in radiation in the back direction out of phase with the direct radiation from the piston.

11.9. The radiation resistance and velocity must be equal because of symmetry. The symmetry of the results is a valuable means of checking the validity of the calculation, which can also be used for large arrays. Programs designed for handling large arrays should also be validated by applying them to small arrays for which the results can be estimated by other means.

11.10 In Eq. (11.34) p_2 is the pressure exerted on transducer 1 by transducer 2, including the incident wave from 2 and the scattered wave from 1, i.e., $p_2 = [p_i(r) + p_s(r)]$, while u_1 is the normal velocity as a function of position on transducer 1, $u_1(r)$. Thus, using Eq. (11.50), the left side of Eq. (11.34) can be written

$$\iint_{S_1} p_2 u_1 dS_1 = u_1 \iint_{S_1} [p_i(r) + p_s(r)](u_1(r)/u_1)dS_1 = u_1 A_1 p_b\,.$$

In Eq. (11.53) $p_1(r,\theta,\phi)$, the pressure from transducer 1, is constant over the surface of the very small transducer 2. Thus the right side of Eq. (11.34) can be written

$$\iint_{S_2} p_1 u_2 dS_2 = p_1(r,\theta,\phi)\iint_{S_2} u_2 dS_2 = p_1(r,\theta,\phi)u_2 A_2\,,$$

consistent with Eq. (11.53).

11.11. $D_a(\theta,\phi) = 4J_1(ka\sin\theta)/ka\sin\theta \to 2$ for $\theta = 0$.

Chapter 12

12.1. The admittance $Y = 1/Z = 1/(R + jX) = (R - jX)/(R^2 + X^2) = G + jB$

12.2. At low frequencies, well below resonance, $1/j\omega C_e >> j\omega L_e + R$. Also, $C_f = C_0 + N^2C^E$.

12.3. A measurement of both an electrical and mechanical quantity is needed to determine N; such as, voltage and force or voltage and acceleration with a known mass.

12.4. In air the damping is usually small allowing an accurate determination of f_r and f_a from measurements that clearly show the maxima and minima of $|Z|$ or $|Y|$; however, an accurate measurement of Q_m and Q_e is difficult in air where the resonant peak is very narrow. With much higher in-water damping Q_m and Q_e are much smaller and are easily measured, while the maxima and minima of |Z| and $\left|Y\right|$ are broadened making it difficult to determine f_r and f_a accurately.

12.5. TCR is given by S = p/I = Zp/V where p/V gives the TVR. The electrical impedance is, except at resonance, approximately given by a function like $1/j\omega C$. Thus the TCR response is modified by a function with a slope of - 6 dB/ octave. The RVS is obtained from the TVR through reciprocity that includes an additional slope change of – 6 dB/octave.

12.6. The magneotstricitive transducer is the dual of the piezoelectric transducer. Thus, one acts like the other with current and voltage interchanged.

12.7. A parallel inductor does not change the input voltage. A series inductor does not change the input current. RVS is for open circuit conditions which is equivalent to a very high impedance constant current condition.

12.8. Use a separate hydrophone or construct a separately wired hydrophone as part of the projector.

12.9. At 2 kHz the wavelength $\lambda = 0.75$ m. If we need 2 cycles to make a measurement, the differential distance, Δ, between the transmitted direct path and the reflected path, would be $\Delta = 0.75(5 + 2) = 5.25$ meters. For a mid-tank projector-hydrophone separation, r, (see Fig. 12.26) the distance $\Delta = 2H - r$ where 2H is the total reflected path and H is the hypotenuse of the right triangles. Thus for a separation of 1 meter $H = (5.25 + 1)/2 = 3.125$ m. The distance to the surface, w/2, is then $[(3.125)^2 - (0.5)^2]^{1/2} = 3.08$ m, and the water tank depth, w, should be 6.2 m (20 ft) for $Q_m \leq 5$ and frequency ≥ 2 kHz. For $Q_m \leq 1$, $\Delta = 2.25$ m, $H = (2.25 + 1)/2 = 1.625$ m, and the tank depth needs to be only 3.09 m (10.1 ft).

12.10. At 10 kHz the wavelength $\lambda = 0.15$ m. The Rayleigh distance $z_f \geq (0.1935)^2/0.3 = 0.125$m (4.92 inches). For a close packed 7 element array $z_f \geq (3 \times 0.1935)^2/0.3 = 1.123$ m (44.2 inches), which is nine times the distance needed for a single piston. Note that the Rayleigh distance is less than the diameter for one piston but almost twice the width of the array.

12.11 The effective coupling coefficient is $k = [1 - (f_r/f_a)^2]^{1/2} = 0.5$. Since $Q_m Q_e = (1 - k^2)/k^2$, $Q_e = 1$.

12.12 The RVS can be obtained from the reciprocity relation in Eq. (12.35) where $|Z| = (400^2 + 300^2)^{1/2} = 500$ ohms. The result is that the RVS = 140 + 54 – 80 -294 = -180 dB//V/μPa .

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