EXERCISES

for

"TRANSDUCERS AND ARRAYS FOR UNDERWATER SOUND" (Springer 2007)

by

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This document is designed to supplement the book "Transducers and Arrays for Underwater Sound" by providing specific exercises that illustrate and amplify the material in the text. The degree of difficulty and effort required by these exercises varies from easy and quick to difficult and time consuming, and is indicated only roughly by: *lowest, **moderate, ***highest. Some of the exercises require a simple calculation with a specific numerical answer, but the solution often requires the reader to become familiar with use of the data in the Appendix. Other exercises are more complex and may require use of simplifying assumptions and approximations that are usually discussed in the text. Some exercises also call for more extensive computations to be attempted depending on the reader's interests and the availability of specialized computer programs for transducer analysis or finite element modeling programs (see references in Chapter 7). A set of answers is given in a separate section after the exercises. In some cases, when the exercises require considerable effort to complete all the numerical and graphical work, it is not practical to present the answers in complete detail. In these cases the answers consist of some discussion of the main issues, some directions for approaching the problem and enough numerical results to allow the reader to evaluate his understanding. In many cases the answers also contain additional discussion related to the practical relevance of the exercise.

EXERCISES

Chapter 1

1.1 ** The free simple harmonic oscillator is basic to most transducers. Its equation of motion is given by Eq. (1.1) with the external mechanical and electrical forces removed by setting F = 0 and V = 0. Show that the solution for the displacement of the mass represents oscillations at the angular frequency $[(K_m/M)-(R/2M)^2]^{1/2}$, and that these oscillations diminish in amplitude exponentially with time with a decay factor R/2M.

1.2 ** Calculate the directivity factor of a transducer that has a far field intensity directivity function of $(A + B\cos\theta)^2$. What is the DI if A = B =1? What value of the ratio B/A gives the maximum DI? Sketch the directivity pattern for some specific values of A and B. See Section 4.56 for other examples of similar directivity patterns.

1.3 * Calculate the time average of the product of the two harmonically time varying quantities $x_1 = X_1 e^{j(\omega t+\phi_1)}$ and $x_2 = X_2 e^{j(\omega t+\phi_2)}$ (see Appendix A.3), and show that it is equal to ½ *Re* ($x_1 x_2^*$).

1.4 ** A ship approaching a harbor in poor visibility, before the invention of radio direction finders, Loran or GPS, knows its position only within a circle of two mile diameter. At the harbor entrance are a bell buoy and a nearby foghorn and underwater sound source located close together that blast simultaneously at one minute intervals. The navigator has his eyes, ears and an omnidirectional underwater listening device. Derive a formula for the distance of the ship from the buoy, d , in terms of the speeds of sound in water, c_w , and air, c_a , and the time interval between hearing the two sounds.

1.5 *** For magnetic field transducers the equations

$$F_b = Z_{mr}^H u + N_{em} I$$

$$V = N_{me} u + Z_0 I$$

correspond to Eqs. (1.8) and (1.9) for electric field transducers, where H is the magnetic field (proportional to I), Z_0 is the clamped electrical impedance and Z_{mr}^H is the open circuit mechanical impedance. Go through the steps following Eqs. (1.8) and (1.9) to derive the short circuit mechanical impedance, $Z_{mr}^B = Z_{mr}^H + N^2/Z_0$, and the free electrical impedance, $Z_f = Z_0 + N^2/Z_{mr}^H$. Then follow the steps in Section 1.41 to derive the expressions for the coupling coefficient squared, k^2 , for magnetic field transducers in Eq. (1.18).

1.6 ** Use Table A.2 of the Appendix and list the ratio of the ρc values to that of water for tungsten, steel, PZT-4, aluminum and magnesium. Consider a sandwich transducer with PZT-4 in the center as described in Section 3.41. Of the listed materials which would you put on the water side and which on the opposite side for the best match to water and, separately, for a less expensive “cost effective” match to water?

1.7* Show that Eq. (1.25) may be written as the expression given in Appendix A.13, Eq. (A13.28). Calculate the source level for a transducer with an input power of 1,000 watts and efficiency of 50% operating as an omnidirectional radiator with DI = 0 dB and also as a directional radiator with DI = 6 dB. What is the reduced value of power needed in the directional case to achieve the source level of the omnidirectional case?

1.8* Show that we may write the far field intensity as $I_0 = D_f W/4\pi r^2$ and also as $I_0 = D_f u_r^2 R_r/4\pi r^2$ where u_r is the rms velocity.

1.9**Use the pressure expression for a piston in a rigid baffle, Appendix A.13, Eq. (A13.31), to obtain the on axis (θ = 0) intensity expression. For the case of a piezoelectric ceramic transducer, eliminate the velocity and write the expression in terms of the voltage, mechanical impedance and the electromechanical turns ratio.

1.10** Of the two intensity expressions, given in Exercises 1.8 and 1.9, one is directly dependent on the directivity factor while the other is not, but depends on the velocity or voltage. Does this mean that if the intensity is calculated using the velocity or voltage that the D_f should not be additionally used? Show that both expressions are equivalent by use of Appendix A.13, Eq. (A13.17) for a piston in a rigid baffle.

Chapter 2

2.1 * Perform a few calculations to become familiar with some of the important parameters of transducers. Consider an ideal transducer operating in the 33 mode of piezoelectric ceramic Type I material (PZT-4, see Appendix A.5) of bar length 1.27 cm and cross sectional area 1.6 cm^2 with rigid backing on the rear end and an ideal piston of area 8 cm^2 on the front end. Assume the piston face radiates into the water and the remaining part is isolated from the water by a water-tight housing. Calculate the free, $C_f = \varepsilon_{33}{}^T A_0/L$, and clamped, C_0, capacitance.

2.2 * Ignore the mass of the piezoelectric ceramic of Ex. 2.1 and calculate the short circuit mechanical compliance $C^E = s^E{}_{33}L/A_0$, the open circuit compliance C^D, the short circuit resonance frequency, f_r, and open circuit anti-resonance frequency, f_a, for a total piston mass and radiation mass of M = 0.1 kg. Also, calculate the effective coupling coefficient from f_r and f_a. Why does $k_{eff} = k_{33}$ in this case?

2.3 ** Calculate the electromechanical turns ratio, N, and the corresponding mechanical force, F, for 1 volt and 5 kV drive for the transducer of Ex. 2.1. Then determine the piston velocity, u, at mechanical resonance for a total mechanical loss resistance R = 1.2x10^3 and radiation resistance R_r = 1.2x10^3 Ns/m. What is the mechanoacoustic efficiency?

2.4 ** Calculate the power output, W, and average surface intensity, $I_s = W/A_0$, at resonance for 5 kV for the transducer of Ex. 2.1. Assume a directivity factor of $D_f = 2$ and calculate the source level in dB re 1 μPa and the far field intensity, I_0, at 1 meter and compare with the surface intensity. Explain the difference in the values of the intensities I_s and I_0.

2.5 * To gain a little more familiarity with transduction consider Eq. (2.9) with external force $F_b = 0$ and at low frequency where the mass and resistance terms are negligible compared with the stiffness term in the impedance. Show that under the usual linear case where $x_3 << L$ the strain $S_3 = d_{33}E_3$ and the displacement $x_3 = d_{33}V$. Calculate this linear strain for Type I piezoelectric ceramic with typical maximum electric field of E_3 = 4 kV/cm (10.2 kV/inch).

2.6 * Using the results of Exercise 2.5 calculate the corresponding displacement and voltage for a length, L, of 1.27 cm.

2.7 * Consider the piezoelectric and magnetostrictive cases of Table 2.1 and show that the third column can be obtained from the first and second columns.

2.8 * Calculate the values of the electromechanical turns ratio N for piezoelectric and magnetostrictive cases of Table 2.1 with $A_0 = 1.6\ cm^2$ and $L = 1.27$ cm for Type I piezoelectric material (see Appendix A5) and Terfenol – D magnetostrictive material (see Appendix A.7) with n = 100 coil turns.

2.9 * Calculate the piezoelectric force for voltage V = 10 volts and the magnetostrictive force for current I = 0.10 amps for the N values of Exercise 2.8.

2.10** Derive Eq. (2.82), $Q_m = \omega_r M/R$, from the definition $Q_m = 2\pi$(Total Energy)/(Energy dissipated per cycle at resonance). See Eq. (8.5) and Section 8.2 for further discussion of Q_m.

2.11* Show that, on using resonance relations, the expression $Q_m = \omega_r M/R$ may also be written as $Q_m = 1/\omega_r R C_m^E$ or $Q_m = (M/C_m^E)^{1/2}/R = (MK_m^E)^{1/2}/R$ where C_m^E is the short circuit mechanical compliance and K_m^E is the short circuit mechanical stiffness .

2.12 * Although the mechanical Q may be calculated for a 100% efficient transducer, the mechanoacoustical efficiency, η_{ma} , usually must be measured or estimated because the internal mechanical resistance is not known. Accordingly, show that the mechanical Q may be written as $Q_m = Q_m'\eta_{ma}$ where $Q_m' = \omega_r(M + M_r)/R_r$ is the mechanical Q for $\eta_{ma} = 100\%$. If the calculated transducer Q_m' is 5 what is Q_m for $\eta_{ma} = 80\%$?

2.13 ** Show that the mechanical Q may be written as $Q_m = [\omega_r M/R_r + \omega_r M_r/R_r]\,\eta_{ma}$ where the first term in the brackets is the Q_m of the transducer without radiation mass loading and the second term is called the radiation Q and is due to the radiation mass and radiation resistance alone. Also show that for the case of a spherical radiator that the second term may be written as $1/k_r a$ where the wave number $k_r = \omega_r/c$ and a is the radius of the sphere. Determine the Q_m for a spherical transducer where $\omega_r M/R_r = 4$ and $\eta_{ma} = 80\%$ for the cases where $k_r a = 0.1$, 1 and 10.

2.14* Calculate the electroacoustic efficiency of an electric field transducer at resonance for $\eta_{ma} = 80\%$, $k_{eff} = 0.5$, for $Q_m = 1$ and 10 for $\tan\delta = 0.01$ and, under high electric drive conditions, for $\tan\delta = 0.10$. Make the same calculation for a magnetostrictive transducer with coil $Q_0 = 100$ and 10 (instead of $\tan\delta = 0.01$ and 0.10 respectively) but with negligible eddy current losses.

2.15* Construct a table or graph of the power factor, P_f, as a function of B/G from 0 to 10. Show that in particular for B/G = 0, 1 and 10 the power factor $P_f = 1$, 0.707 and $\approx$ 0.1 respectively. Show that in the limit $P_f \approx 1$ for $B/G \ll 1$ and $P_f \approx G/B$ for $G/B \ll 1$.

2.16 ** The effective coupling coefficient may be obtained from measurable quantities by use of the equations $k^2 = 1 - (f_r/f_a)^2$ or $k^2 = 1/(1 + Q_mQ_e)$. Prove the last very important equation.

2.17 * Calculate the input power at resonance for a transducer with k = 0.5, C_f = 10 nF, Q_m = 5 and f_r = 10 kHz for an RMS input voltage of 1 and 100 volts assuming that electrical losses are negligible. What is the efficiency and output power if the internal mechanical resistance is one half the radiation resistance?

Chapter 3

3.1. * Calculate the in-air ring mode short circuit resonance frequencies of a thin-walled, 0.508 cm (0.2 inches) Type 1 (PZT-4) piezoelectric ceramic ring transducer of mean diameter 10.16 cm (4 inches) and height 5.08 cm (2 inches) operating in the 31 mode. Compare the result with the resonance frequency of a thin walled spherical transducer operating in the planar mode with the same diameter and wall thickness. Use the frequency constants in Appendix A.6.

3.2. ** Calculate the in–air short circuit resonance frequencies for the ring transducer of Exercise 3.1 operating in the 33 mode. Calculate the change in resonance frequency and coupling coefficient if the 33 mode is obtained by electrode striping the ring, replacing 10 % of the active circumferential length of short circuit elastic modulus, s^E_{33}, by Type I material, but with open circuit modulus s^D_{33}. Use equations in Section 3.22.

3.3. * Calculate the approximate in–water resonance short-circuit frequencies and mechanical Q_m's for the ring and spherical transducers of Exercise 3.1. This resonance is the frequency of maximum output for a constant voltage transmitting response. Assume the ring has end caps that are mechanically isolated from the ring with air on the inside and approximate spherical loading to obtain the water mass loading for the ring.

3.4. *** Evaluate the ring parameters of Exercise 3.1 for the equivalent circuit of Fig. 3.4 and compute the TVR. Compare the results with a finite element model if available.

3.5. * Calculate the volume and mass figures of merit (FOM_v and FOM_m) for the transducer of Exercise 3.1.

3.6. ** Determine the Helmholtz resonance frequency (often called cavity resonance) of the ring of Exercise 3.1. Assume the ring is small compared to the wavelength at this frequency and use the piston in a rigid baffle low frequency radiation mass loading $M_r = 8\rho a^3/3$ or use the water field added tube length $\Delta L = 8a/3\pi$ (see Exercise 3.7). Assume symmetry about a plane midway through the ring; thus the effective cavity depth is L/2 not L.

3.7. * Show that the water load added length, $\Delta L = 8a/3\pi$, is equivalent to the low frequency radiation mass loading $M_r = 8\rho a^3/3$. Justify the Exercise 3.6 statement "We assume symmetry about a plane midway through the ring; thus the effective cavity depth is L/2 not L."

3.8. *** Consider the Tonpilz projector illustrated in Fig. 3.17 with piston area 0.0162 m^2, head mass M_h = 10 kg, tail mass M_t = 40 kg and a drive stack consisting of four Type 1 (PZT-4) piezoelectric ceramic rings of mean diameter 10.16 cm (4 inches), height 0.508 cm (0.2 inches) and wall thickness 1.016 cm (0.4inches) operating in the 33 mode and wired as illustrated in Fig. 3.17. Also assume a tie rod with one-tenth the stiffness of the stack and insulators plus glue joints with a total of 10 times the stiffness of the stack. Calculate the in-air short circuit resonance frequency, the turns ratio N, the free capacitance C_f and the effective coupling coefficient.

A) Ignore the effects of the tie rod, insulator, glue joints and mass of the stack.
B) Ignore the effects of the tie rod, insulator, glue joints, but include stack mass.
C) Include effects of the tie rod, insulator, glue joints and stack mass.

3.9. *** Use the equivalent circuit of Fig. 3.20 to calculate the response of the projector of Exercise 3.8, case A. Then use the equivalent circuit of Fig. 3.19 to calculate the TVR and maximum SPL response of the projector of Exercise 3.8. Assume equivalent sphere radiation loading and piston in a rigid baffle radiation loading. Compare results. Assume 100 % and then 80% mechanoacoustic efficiency. Determine k_{eff} and Q_m. Compare results from circuit model with transmission line, matrix or FEA models. Calculate the maximum power output and the FOM_v for maximum electric field voltage drive.

3.10* Determine the in-air frequency diameter constants, fD, for thin walled Terfenol-D and Galfenol magnetostrictive ring transducers under open, f^H, and short, f^B, circuit conditions (see Appendix A.7).

3.11* Calculate the in-air ring mode open circuit and short circuit resonance frequencies and free inductances of thin walled, 0.508 (0.2 inches) ideally biased magnetostrictive Terfenol-D and Galfenol ring transducers of mean diameter 10.16 cm (4 inches) and height 5.08 cm (2 inches) operating in the 33 mode with a coil of 100 turns. Compare the resonance frequency results with the PZT - 4 ring of Exercise 3.1.

3.12*** In Section 3.32 an equation for estimating the effective coupling coefficient, $k_e^2 = 2k^2/(1 + k^2)$ was given for the hybrid transducer for the case where the piezoelectric ceramic and magnetostrictive drivers have the same coupling coefficient. This approach evaluated the coupling coefficient from the total mechanical energy stored and the shared electrical energy at resonance for the two sections wired in parallel. Using the same approach develop an expression for k_e^2 where the drivers have two different coupling coefficients, k_1 and k_2, and show that it reduces to the original expression for $k_1 = k_2$.

Chapter 4

4.1. ** Calculate the in-air ring-mode open-circuit resonance frequency and the low frequency receiving response and free capacitance of a thin-walled, 0.508 cm (0.2 inches) Type 1 (PZT-4) piezoelectric ceramic ring hydrophone of mean diameter 10.16 cm (4 inches) and height 5.08 cm (2 inches) operating in the 31mode. Compare results with a thin walled spherical hydrophone operating in the planar mode, with the same wall thickness and diameter. What is the length mode resonance frequency of the ring? Assume air backing with isolated end caps on the ends of the ring.

4.2. *** Determine the receiving response through the anti-resonance of the ring hydrophone of Exercise 4.1. Assume equivalent sphere radiation loading and spherical diffraction constant. Calculate the electrical impedance and use reciprocity to determine the TVR. Compare with the TVR obtained directly in Exercise 3.4.

4.3. *** Consider two small ring Type I hydrophones one-tenth the size of the one in Exercise 4.1 and separated by 5.08 cm (2 inches). First, calculate anti-resonance and the low frequency RVS value for each alone. Are these results scalable from the results of Exercise 4.1? Next, determine the low frequency summed sensitivity and the low frequency differenced sensitivity for a plane wave incident in the axial direction. Calculate the frequency at which you would expect this vector sensor to differ from an expected cosine beam pattern. Calculate the frequencies at which the summed and differenced modes yield a null in the axial direction.

4.4. *** Determine the low frequency electrical noise voltage, equivalent noise pressure and signal to noise ratio for a 1 μPa plane wave signal on the MRA for the ring hydrophone of Exercise 4.1. Repeat for the wide band noise response through anti-resonance. Assume an electrical dissipation factor of 0.004, equivalent sphere radiation loading and diffraction constant.

4.5. ** Determine the low frequency electrical noise voltage and equivalent noise pressure for the summed and differenced dual hydrophones of Exercise 4.3. Assume an electrical dissipation factor of 0.004.

4.6. ** The Tonpilz of Exercise 3.8 (case A), as shown in Fig. 3.13, is to be also used as a hydrophone. Calculate the low frequency signal sensitivity and equivalent noise pressure for a dissipation factor of 0.004. Assume array baffle conditions and that $D_a = 2$. Sketch and explain the operation of the TR switch for achieving transmit and receive operation from a single transducer.

4.7. ** Calculate the first four short circuit extensional ring mode resonance frequencies of the hydrophone of Exercise 4.1 for Type I material operating in the 31 mode. How many electrodes are needed to detect (or excite) each of these four modes? What is the highest mode that can be excited below the length mode resonance?

4.8. *** The internal thermal noise for a 100% efficient hydrophone can be represented by the mechanical radiation resistance of the hydrophone, although it is caused by a thermal noise pressure in the medium acting on the surface of the hydrophone, as described by Mellen [48]. Radiation resistance is usually thought of as a measure of how a transducer radiates, but in this case, it is also a measure of how the transducer receives. What is the basic physical reason for this dual role of radiation resistance?

4.9. * As an example of the usefulness of Eq. (4.56) use the known radiation resistance in Eq. (10.44) and directivity factor for a monopole sphere to find the diffraction constant for a spherical hydrophone.

Chapter 5

5.1. * Consider two small pistons of radius, a, with $ka \ll 1$, separated by a center-to-center distance, d, in a large rigid baffle. Show that the total radiation resistance of each approaches $2R_{ii}$ as kd becomes small. Why is this case similar to the case of a single piston approaching a rigid wall?

5.2. * Show that Eq. (5.20c) follows from Eqs. (5.20a) and (5.20b).

5.3. * Consider a large array of square pistons of side length L and with spacing L/10 between the pistons. What is the packing factor? What is the average radiation resistance of the individual pistons? What is the total radiation resistance of an array of N pistons?

5.4. * Show that Eq. (5.29) reduces to $Z = F/U$ for a uniform velocity distribution.

5.5. * Explain why the product theorem is useful in array beam pattern analysis? Would it be useful in determining the DI or the radiation impedance?

5.6. *** A close packed array of ideal square pistons shows no grating lobes when it is not steered, because the individual piston patterns null out the array grating lobes. But when the array is steered grating lobes appear at angles such that they are only partially nulled out. Show this by calculating the beam pattern for a line array of ten close packed, square pistons with side length of one-wavelength and center-to-center spacing of one wavelength steered to 45^0 (see Eqs. (5.10a) and (5.10b)).

5.7. ** Show that Sinc $(kd/2) = 2\sin(kd/2)/kd$ approaches 1 for $kd/2 \ll 1$. Likewise show that $J_1(ka)$ goes to $ka/2$ for $ka \ll 1$; and thus, $2J_1(ka)/ka$ also approaches 1 for $ka \ll 1$.

5.8. ** Determine the value of α/k_d required for a parametric array full beam width of 4^0 and calculate the corresponding beam pattern.

5.9. ** To see extreme effects of acoustic coupling consider a small transducer located at the center of a circle of radius d of N identical transducers being driven to make all have the same velocity. Use Eqs. (5.16) and (5.23b) and show that the total radiation resistance of the center transducer could become negative for N > 5 when $\lambda < 4d/3$ and also in other frequency regions for greater values of N. What does negative radiation resistance mean physically? What is the total radiation reactance of the center transducer when $\lambda = 2d$.

5.10. *** As a simple example of the effects of beam steering on the radiation impedance of transducers in an array consider a line array of two small transducers driven to have the same velocity amplitude but steered by a velocity phase difference of μ, i.e., $u_2 = u_1 e^{j\mu}$. How does the total radiation impedance of the two transducers differ, in general, and specifically for end fire steering?

Chapter 6

6.1. ** Show, starting from the Fourier Transform formulation of the array output in Eq. (6.5a) (also see Appendix A.11), that a linear phase shift in the array sensitivity distribution results in a wave vector displacement that corresponds to steering the beam.

6.2. * What is a non-acoustic plane wave? Why do we need to consider the wave vector response of an array of hydrophones?

6.3. ** What are the physical situations in which the array gain and the directivity index could have the same value? What are the basic reasons why they usually are not the same?

6.4 *** Use Eq. (6.8b) to calculate the DI of an unshaded line array of small hydrophones for N = 6 and kD = $\pi, \pi/2$ and kD << 1, and for steering angles of 0, 30 and 90 degrees.

6.5 ** Show from Eq. (6.8b), for unshaded line arrays for any values of kD and N, that the D_f is the same when the array is unsteered and when the array is steered to 90 degrees at 2kD, i.e., at twice the frequency or twice the spacing.

6.6. ** Calculate the array gains for the arrays in Exercise 6.4 in isotropic, incoherent noise.

6.7. ** Calculate the coincidence frequency of a steel plate with thickness 0.635 cm (0.25 inches). At frequencies below the coincidence frequency flexural waves in a submerged plate produce evanescent pressure waves in the water traveling parallel to the plate that decay with distance from the plate. At 20 kHz how much does the evanescent wave amplitude decay at 1 cm from the surface of the plate?

6.8. *** Develop the spatial correlation function for isotropic noise given by Eq. (6.16).

6.9. ** The discussion of the triaxial vector sensor (see Fig. 6.24) makes use of certain relationships between angles. Show that in general $\cos \gamma_n = \cos \theta \cos \theta_n + \sin \theta \sin \theta_n \cos (\varphi - \varphi_n)$ and that specifically for $\varphi = \varphi_n$, $\cos \gamma_n = \cos (\theta - \theta_n)$.

6.10. *** Consider Eq. (6.29) for an array of N hydrophones under conditions where all four types of noise have the same intensity at each hydrophone, i.e., $f_a = f_s = f_f = f_h = ¼$. Also consider the structural, flow and hydrophone internal noise to be incoherent, while ambient noise is isotropic and partially coherent with spatial correlation given by Eq. (6.16), and the signal is a plane wave from the broadside direction. What is the array gain: 1) for any configuration of N hydrophones? 2) for a line array of N with half wavelength spacing? 3) for a line array of N = 3 with spacing D? 4) for a line array of N = 3 with quarter wavelength spacing?

6.11 *** Note that the development of the generalized diffraction constant in Section 11.31 suggests that Eq. (4.56) holds for arrays as well as individual transducers, although it does not explicitly show it. Investigate the validity of this suggestion by considering a simple array of two small circular pistons in a plane, rigid baffle and calculating the total radiation resistance of the array, the diffraction constant of the array and the directivity factor of the array and showing that they satisfy Eq. (4.56).

Chapter 7

7.1. ** Consider the dual mass mechanical resonator of Fig. 7.4. Assume loss resistance R_1 is negligible and $R_2 << \omega_r M_1$. Calculate the approximate resonance frequency for stiffness $K = 4 \times 10^{10}$ N/m and head mass $M_2 = 1$ kg for tail masses $M_1 = 1$, 2, 4 and 8 kg. Also calculate the mechanical Q_m with $R_2 = 6 \times 10^4$ Ns/m for the above values of M_1. Does there appear to be much benefit gained in doubling the tail mass from 4 kg to 8 kg, considering the weight increase in the transducer?

7.2. * The equivalent circuits of Fig. 7.12a and 7.13 show a single resonance under short circuit conditions (or constant voltage drive) at the mechanical resonance frequency where $\omega_r M = 1/\omega_r C^E$. Show that anti resonance, $\omega_a = \omega_r/(1 - k^2)^{1/2}$ is obtained under open circuit conditions where, in this case, C_0/N^2 is now in series with C^E reducing the compliance to C^D raising the stiffness and the resonance frequency to the value ω_a.

7.3. ** Expand the function $-j\rho c A_0/\sin(kL)$ and obtain $-j\rho c A_0(1/kL + kL/6)$ for small kL using trigonometric and binomial expansions; and thus, verify the $-M/6$ in Fig. 7.25.

7.4. * Demonstrate that the equivalent circuit of Fig. 7.25 may be used to represent an ideal mass, M, or any mass at low enough frequencies. Show that at very low frequencies the reduced equivalent circuit for a force F at terminals 2 and an air loaded free condition, i.e. a short circuit, at terminals 1 is simply a mass M, as it should be.

7.5. ** Show that Eqs. (7.36a,b) may be written in the transfer matrix form of Eqs.(7.37a,b).

7.6. * Show that the transmission line equivalent circuit of Fig. 7.28 may be reduced to the simple lumped mode representation of Fig. 7.14a (without G_0) for $kL = \omega L/c << 1$ if the – M/6 term is ignored, with a free condition at one end ($F_0 = 0$) and a radiation load of $F_n/u_n = R_r + j\omega M_r$ on the other end.

7.7. * Evaluate the A, B, C, D parameters of Eq. (7.70) using the electrical and mechanical parameters of Fig. 7.12a.

7.8. * The finite element evaluation of the broad side far field pressure from a large planar array (with most of the elements under array loaded conditions) could require very large run times at each frequency. Consider the alternative approach of first evaluating the pressure of a single element by approximating its environment by a rigid fluid filled wave guide with a ρc absorber at the end (see Section 7.46). Then calculate the far field pressure, reduced to 1 meter, for an array of 100 elements each of area 6.45 cm^2 (one inch square) for a single element wave guide pressure of 0.01 Pa at 10 kHz.

Chapter 8

8.1. * Consider the single mass resonator of Fig. 8.2. Calculate the resonance frequency for bar stiffness $K = 4 \times 10^{10}$ N/m and head mass M = 1 kg by first ignoring the bar mass and then considering that the bar contributes an additional 0.3 kg. Also, calculate the mechanical Q_m with $R_2 = 6 \times 10^4$ Ns/m for the two cases.

8.2. * Show that the square of the coupling coefficient $k^2 = N^2C^E/C_f$ where the free capacity $C_f = N^2C^E + C_0$, C_0 is the clamped capacity and the mechanical short circuit compliance $C^E = 1/K^E$ is consistent with Eq. (8.22) and $C_0 = C_f(1 - k^2)$ is consistent with Eq. (8.23).

8.3. ** Determine the percentage change in the coupling coefficient if a stress rod with a stiffness that is one-tenth the stiffness of the piezoelectric ceramic short circuit stiffness is used with original coupling coefficient values of 0.9, 0.7, 0.5 and 0.3. What is the percentage change in the resonance and anti-resonance frequencies? Assume a simple lumped mass system.

8.4. ** Determine the percentage change in the coupling coefficient if an electrical insulator with a stiffness that is 10 times greater than the stiffness of the piezoelectric ceramic is used between the head mass and the piezoelectric ceramic. Evaluate with original coupling coefficient values of 0.9, 0.7, 0.5 and 0.3. What is the percentage change in the resonance and anti-resonance frequencies? Assume a simple lumped mass system. Tabulate results and compare with Exercise 8.3.

8.5. ** Determine the percentage change in the coupling coefficient and the resonance and anti-resonance frequencies when both the stress rod and the electrical insulator of Exercises 8.3 and 8.4 are used. Tabulate results and compare with individual effects.

8.6. ** Calculate the effective dynamic coupling coefficient for the fundamental mode of a 33 mode segmented bar for k_{33} values of 0.9, 0.7, 0.5 and 0. 3. Tabulate the percentage reduction. Compare results for the case of the length expander bar with electrodes only on the ends.

8.7. * Calculate the effective dynamic coupling coefficients for the second and third modes of the end electroded bar of Exercise 8.6.

8.8. ** Derive an expression for the dynamic stiffness of the n^{th} mode of the fixed-free bar from Eq. (8.12). Then derive expressions for the effective coupling coefficient of the n^{th} mode for both the length expander bar and the segmented bar following the procedure in Section 8.43.

8.9 *** Derive an expression for the dynamic stiffness of the n^{th} mode of the 31 mode ring based on the expression for the short circuit resonance frequencies, $f_n = f_0(1+n^2)^{1/2}$. Then derive an expression for the effective coupling coefficient following the procedure in Section 8.43. Calculate k_{edn} for the n = 0,1,2,3 modes and for $k_{31} = 0.33$.

Chapter 9

9.1. *** Consider Eq. (2.8) with external force $F_b = 0$ at low frequencies where the acceleration and velocity terms are negligible compared with the displacement term. Show that this leads to the low frequency nonlinear equation for the strain, $S_3^2 + S_3 = d_{33}(V/L)$. Determine the exact solution. . Assume $4d_{33}V/L < 1$ and use the binomial series to obtain the three term expansion solution showing terms that could generate second and third harmonics under high sinusoidal drive.

9.2. * Calculate the strain for a Type I piezoelectric material in Exercise 9.1 with a typical maximum electric field of V/L = 4 kV/cm (10.2 kV per inch) and compare this result with the linear result given by the first term of the expansion for S_3. Calculate the percentage difference.

9.3. ** What are the physical nonlinear conditions for even and for odd harmonic generation?

9.4. *** One general objection to operating below the fundamental resonance of high power Tonpilz transducers is the higher harmonic distortion generated as compared to operating above resonance. What is the physical reason for this? As the most extreme example of this effect explain how operating at one half or one third the fundamental resonance frequency can greatly increase the second or third harmonic distortion.

9.5. ** Why does the presence of significant harmonic distortion indicate the possibility of transducer failure under high drive?

9.6*** The most common nonlinear force in transducers is the square law electric or magnetic drive force. Results that are applicable to many cases can be derived from a simplified form of Eq. (9.9) containing only the linear strain term and the quadratic electric field term, $T = cS - eE^2$. Use this expression in the equation of motion [as in Eq. (9.13)] with a drive voltage of $V = V_0 + V_1 \cos\omega t$, and find the static, fundamental and second harmonic drive terms and displacements. Show, when $V_0 = 0$, that there is no displacement at the drive frequency, but there is a static displacement and a displacement at twice the drive frequency. Thus a bias voltage, V_0 , is necessary to have any linear output with this force law.

9.7 ** In Exercise 9.6 calculate the ratio of the second harmonic displacement to the fundamental displacement at the resonance frequency, at one half the resonance frequency and well below resonance.

9.8*** When significant harmonic distortion is present a modified definition of the electromechanical coupling coefficient should be considered, because part of the input energy is converted to harmonics which do not contribute to the desired fundamental output. Such a definition might be: k_{nl}^2 equals the ratio of converted mechanical energy at the fundamental frequency to the total input energy. Express this definition analytically and evaluate it numerically using the second harmonic results from Exercise 9.7.

9.9 ** Square law transducers are seldom used without bias, because the frequency content of the output differs so much from that of the input. Using the same equation of motion as Exercise 9.6 show that the output of a square law transducer driven with voltage $V = V_1 \cos\omega t + V_2 \cos 2\omega t$ contains four different frequency components. What are those frequencies and what are the relative displacement amplitudes and velocity amplitudes at low frequency if $V_2 = V_1$?

9.10.* Convert the relative displacement amplitudes calculated in Exercise 9.9 to radiated sound pressure to show the enhancement of the higher frequency pressure components relative to the displacement components (see the discussion in Section 9.21).

Chapter 10

10.1. * Show that the plane wave representation for the pressure $p = p_0 e^{-j(kx-\omega t)}$ satisfies the one dimensional Helmholtz wave equation $\partial^2 p/\partial x^2 + k^2 p = 0$. Determine the particle velocity $u = -(1/j\omega\rho)\partial p/\partial x$ and show that, in this case, the characteristic impedance $p/u = \rho c$. What is the characteristic impedance for Type I (PZT-4) piezoelectric ceramic and how does it compare with the value for water?

10.2. ** Show the condition under which the first axial null occurs for a continuous line of length L. What is the condition for the first axial null for a line array of two point sources separated by distance s? Why the difference in lengths?

10.3. ** Use the approximate formulas to calculate the DI and beam width for a line of length $L = \lambda$. Compare this result with the approximate result for a circular piston in an infinite rigid baffle with diameter $D = \lambda$. Why is the beam width larger and DI higher for a circular piston?

10.4. * Calculate the beam widths and DI for the line and piston in Exercise 10.3 using the exact expressions and compare with the approximate results.

10.5. *** Calculate the beam width for a vibrating ring on an infinite rigid cylinder of length $L = \lambda$ and diameter $D = \lambda$ and compare these results with that of a thin line of length $L = \lambda$. Note in Eq. (10.34) that L is half the length of the vibrating ring, while in Eq. (10.22) L is the whole length of the thin line. Also note that in Eq. (10.34) θ is measured from the axis of the cylinder, while in Eq. (10.22) $\alpha = \pi/2 - \theta$ is measured from the perpendicular to the line.

10.6. * Calculate the values of ka for which there is a pressure null at the center of a circular piston set in an infinite rigid baffle.

10.7. ** Consider two spheres vibrating at the same frequency one as a dipole source with radius a_d and the other as an omni source with radius a_o. Determine the approximate value of ka_o of the omni sphere in order for it to have the same value of radiation resistance as the dipole sphere under the conditions of both ka_o and $ka_d \ll 1$ and also for both ka_o and $ka_d \gg 1$. Why are the radii so different for $ka \ll 1$? Use appropriate approximations.

10.8. * Calculate the diffraction constant for a circular piston in an infinite rigid plane by using Eq. (4.56) with D_f and R_r from Eqs. (10.31) and (10.52).

10.9. ** Calculate the diffraction constant for a sphere vibrating in the dipole mode by using Eq. (4.56). Use R_r from Eq. (10.47) and D_f from the answer to Exercise 1.2 with $A = 0$ and $B = 1$. Find the maximum value of D_a in this case and the value of ka at which the maximum occurs.

10.10. *** An acoustic radiation problem is defined by stating the boundary condition on the normal velocity. In some cases the solution can be completed in terms of known solutions of the wave equation. In other cases a numerical finite element solution is the only practical approach. A clear statement of the boundary condition is the essential starting point for both approaches. Write out the normal velocity boundary condition for a free-flooding ring transducer and consider how it could be solved. Make a sketch showing the various vibrating surfaces in a cylindrical coordinate system.

Chapter 11

11.1. ** Of the 11 coordinate systems in which the wave equation is separable (see Reference 3) only 6 or 7 have been used in modeling practical transducers and arrays.

Consider the geometrical shapes that occur in practice and think of specific shapes that can be accommodated by certain coordinate systems as a way of identifying the most useful systems.

11.2. ** Show that Eq. (11.8) follows from Eq. (11.9) for n = 1. Also see Exercise 6.9 regarding the development of Eq. (11.8).

11.3. **Derive Eq. (11.4) from Eqs. (11.2), (11.3a) and (11.3b) using the orthogonality relation for Legendre polynomials:

$$\int_0^\pi P_n(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = 2/(2m+1) \text{ for } n = m, \text{ and } = 0 \text{ for } n \neq m.$$

11.4. *** The boundary condition in Exercise 10.10 for a free flooding ring simplifies considerably for an end-capped cylinder where the inside velocity, u_i, is not involved and u_t applies to the entire top. But the problem is still not solvable because of the finite length of the cylindrical surface. In such cases, under low frequency conditions, a useful model can be based on spherical wave functions. In the lowest order approximation the cylinder is replaced by a monopole sphere with the same radiating area as the cylinder as discussed in Section 3.21. A better approximation is possible by using a sphere with uniform normal velocity in the polar regions, representing the velocity of the end caps, and a different uniform normal velocity in the equatorial region, representing the velocity of the sides of the cylinder. When the end caps are considered motionless this boundary condition is described by:

$$u = 0\ , \quad r = a\ , \quad 0 < \theta < \theta_1 \quad \text{and} \quad 180° - \theta_1 < \theta < 180°$$

$$u = u\ , \quad r = a\ , \quad \theta_1 < \theta < 180° - \theta_1$$

where a is the radius of the sphere and also the radius of the ring being modeled. Express the solution of this problem as a series of spherical wave functions and determine the coefficients. Consider the solution for ka small enough that only two terms are sufficient. What is the effect of the second term on the beam pattern?

11.5. *** The Hilbert transform was used in Sec. 11.14 to obtain the radiation reactance from the radiation resistance of a sphere of radius, a, operating in a uniform omni-directional mode. Obtain the radiation reactance from the radiation resistance of the same sphere vibrating in a dipole mode given by Eq. (10.47).

11.6. *** Show that for an unbaffled plane array, the pressure in the plane of the array is one-half the pressure for the same array in a large rigid baffle. Use the fact that, for the same array in a large pressure release baffle, the pressure in the plane of the array is zero.

11.7. ** Equation (11.52) gives the diffraction constant, D_a, for a sphere of radius, a. Why does D_a decrease from unity as the frequency is increased rather than increase as in the case of a piston. Why does D_a decrease as 1/ka for ka >> 1?

11.8** Beam pattern results for a piston set in a pressure release sphere show a significant reduction in the back radiation compared to a rigid sphere as shown in Fig. 11.15. Explain why this happens.

11.9. * Table I shows the radiation resistance for a linear array of three equally space pistons in a rigid baffle. Why do the two outside pistons have the same radiation resistance and the same velocity?

11.10. *** Consider the basis of the important Eq. (4.56) by following its derivation in Section 11.31 and, specifically, by showing that Eq. (11.53) follows from the acoustic reciprocity relation in Eq. (11.34) and the definition of clamped force in Eq. (11.50).

11.11.** The diffraction constant, D_a , in Eq. (4.56) is for an incoming wave arriving on the MRA of the hydrophone, and the D_f is referred to the same MRA. The directional diffraction constant, $D_a(\theta,\phi)$, in Eq. (11.57) is D_a referred to the MRA multiplied by the square root of the normalized intensity directivity function. Use of $D_a(\theta,\phi)$ in any of the expressions for hydrophone output [such as Eqs. (4.10) or (4.17)] gives the output as a function of direction. The frequency dependence of D_a and of the hydrophone mechanism together determine the final frequency dependence of the output. In an isotropic noise field the mean squared noise output is proportional to the average of $D_a^2(\theta,\phi)$ which is D_a^2/D_f . What is $D_a(\theta,\phi)$ for a tonpilz hydrophone with a circular head mounted in a rigid baffle, and what is its value at $\theta = 0$?

Chapter 12

12.1. * Show that if Z = R + jX, then the conductance $G = R/(R^2 + X^2)$ and the susceptance $B = -X/(R^2 + X^2)$. Show that if the electrical Q_e is defined as B/G, then Q_e is also equal to -X/R.

12.2. * Show that the Van Dyke equivalent circuit of Fig. 12.1 reduces to the circuit of Fig. 12.1a where the free capacity $C_f = C_0 + C_e$.

12.3. ** Convince yourself that the measurements of C_f, $\tan\delta$, f_r, f_a and $|Y_{max}|$ are sufficient to determine the parameters of the Van Dyke circuit of Fig. 12.1. What measurement or measurements are necessary to evaluate the electromechanical transformer turns ratio N of the equivalent circuit of Fig. 7.12a?

12.4. ** Explain why the formula $k_{eff} = (1 + Q_mQ_e)^{-1/2}$ is more suitable for determining k_{eff} under water loaded conditions (with measurements of B vs. G or response of B and G) while the formula $k_{eff} = [1 - (f_r/f_a)^2]$ is more suitable for in-air measurements of |Y| or |Z|.

12.5. ** Explain the change in the response curve slopes from the TVR curve of Fig. 12.11a to the TCR curve of 12.12a to the RVS curve of 12.13a.

12.6. ** Why does the TCR curve of a magnetostrictive transducer look like the TVR curve of a piezoelectric ceramic transducer?

12.7. * Shunt tuning a transducer does not change the TVR and series tuning does not change the TCR and RVS. Why is this so?

12.8. ** The Transmit/Receive, "TR", switch of Fig. 12.21 has at times been criticized for the diode noise introduced in the RVS. With no restrictions on you, how would you avoid this?

12.9. *** What should the smallest dimension of a water-filled tank be for pulsed measurements of transducers which go as low as 2 kHz and have a Q_m of no more than 5, and also for a Q_m of no more than 1. Assume two steady state cycles are needed for measurement.

12.10. * Determine the far field Rayleigh distance for a 19.35 cm (7.6 inch) diameter piston transducer operating at 10 kHz. Determine the approximate far field distance for a close packed seven element circular array of the same transducers.

12.11. * The in-air frequencies of maximum and minimum admittance magnitudes (f_r and f_a respectively) of a transducer are measured to be 10 kHz and 11.55 kHz. The in-water mechanical Q_m is measured to be 3.0. What is the in-water electrical Q_e ?

12.12. * The measured TVR level for a transducer is 140 dB and the measured impedance is 400 – j300 ohms, all at 10 kHz. What is the RVS?

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