Preface

Boundary Element Methods (BEM) play an important role in modern numerical computations in the applied and engineering sciences. Such algorithms are often more convenient than the traditional Finite Element Method (FEM), since the corresponding equations are formulated on the boundary, and, therefore, a significant reduction of dimensionality takes place. Especially when the physical description of the problem leads to an unbounded domain, traditional methods like FEM become unalluring.

A numerical procedure, called Boundary Element Methods (BEM), has been developed in the physics and engineering community since the 1950s. This method turns out to be a powerful tool for numerical studies of various physical phenomena. The most prominent examples of such phenomena are the potential equation (Laplace equation) in electromagnetism, gravitation theory, and in perfect fluids. A further example leading to the Laplace equation is the steady state heat flow. One of the most popular applications of the BEM is, however, the system of linear elastostatics which can be considered in both bounded and unbounded domains. A simple model for a fluid flow, the Stokes system, can also be solved by the use of the BEM. The most important examples for the Helmholtz equation are the acoustic scattering and the sound radiation.

It has been known for a long time that boundary value problems for elliptic partial differential equations can be reformulated in terms of boundary integral equations. The trace of the solution on the boundary and its conormal derivative (Cauchy data) can be found by solving these equations numerically. The solution of the problem as well as its gradients or even high order derivatives are then given by the application of Green's third formula (representation formula); this method based on Green's formula is called the direct BEM approach. Another possibility is to use the property that single or double layer potentials solve the partial differential equation exactly for any given density function. Thus, this function can be used in order to fulfill the boundary conditions. The density function obtained this way has, in general, no physical meaning. Therefore, these boundary element methods are called indirect.

When boundary integral equations are approximated and solved numerically, the study of stability and convergence is the most important issue. The most popular numerical methods are the Galerkin methods which perfectly fit to the variational formulation of the boundary integral equations. The theoretical study of the Galerkin methods is now completed and provides a powerful theoretical background for BEM. Traditionally, however, the collocation methods were widely used, especially in the engineering community. These methods provide an easier practical implementation compared with the Galerkin methods. However, the stability and convergence theory for collocation methods is available only for two-dimensional problems. Furthermore, the error analysis of the collocation methods for three-dimensional problems, when assuming their stability, shows that the rate of convergence of the Galerkin methods is better, when assuming that the solution is smooth enough.

In any case, a numerical procedure applied to the boundary integral equation leads to a linear system of algebraic equations. The matrix of this system is in general dense, i.e. almost all its entries are different from zero, and, therefore, have to be stored in computer memory. It is clear that this is the main disadvantage of the BEM compared with FEM which leads to sparse matrices. This quadratic amount of computer memory sets very strong, unattractive bounds for the discretisation parameters and, often, force the user to switch to the out–of–core programming. However, so called fast BEM have been developed in the last two decades. The original methods are the Fast Multipole Method and the Panel Clustering; another example is the use of wavelets. Furthermore, the Adaptive Cross Approximation (ACA) was introduced and successfully applied to many practical problems in the last years.

The purpose of this book is twofold. The first goal is to give an exact mathematical description of various mathematical formulations and numerical methods for boundary integral equations in the three-dimensional case in an uniform and possibly compact form. The second goal is a systematic numerical treatment of a variety of boundary value problems for the Laplace equation, for the linear elastostatics system, and for the Helmholtz equation. This study will illustrate both the convergence of the Galerkin methods corresponding to the theory and the fast realisation of BEM based on the ACA method. We restrict our numerical tests to some more or less artificial surface examples. The simplest one is the surface of the unit sphere. Furthermore, two TEAM examples (Testing Electromagnetic Analysis Methods) will be considered besides some other non-trivial surfaces.

This book is subdivided into four parts. Chapter 1 provides an overview of the direct and indirect reformulations of second order boundary value problems by using boundary integral equations, and it discusses the mapping properties of all boundary integral operators involved. From this, the unique solvability of the resulting boundary integral equations and the continuous dependence of the solution on the given boundary data can be deduced. Chapter 2 is concerned with boundary element methods, especially with the Galerkin method. The discrete version of the boundary integral equations from Chapter 1 and their variational formulations lead to systems of linear equations with different matrices. The entries of these matrices are explicitly derived for all integral operators involved. Chapter 3 describes the Adaptive Cross Approximation of dense matrices and provides, in addition to the theory, some first numerical examples. The largest part of the book, Chapter 4, contains some results of numerical experiments. First, the Laplace equation is considered, where we study Dirichlet, Neumann, and mixed boundary value problems as well as an inhomogeneous interface problem. Then, two mixed boundary value problems of linear elastostatics will be presented, and, finally, many examples for the Helmholtz equation are described. We consider again Dirichlet and Neumann, interior and exterior boundary value problems as well as multifrequency analysis. Many auxiliary results are collected in three appendices.

The chapters are relatively independent of one another. Necessary notations and formulas are not only cross-referred to other chapters but usually repeated at the appropriate places.

In 2003, Prof. Allan Jeffrey approached us with the idea to write a book about fast solutions of boundary integral equations. It has been delightful to write this book and we are also very thankful for his providing the opportunity to get this book published.

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