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Introduction: Extending the Rasch Model

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1.1 Introduction

The present volume is a collection of chapters on research and development work on extensions of the Rasch model (RM; Rasch, 1960) that have focused on relaxing some fundamental constraints of the original RM, while preserving many of the unique features of the model. More specifically, the volume presents extensions of the RM in which certain homogeneity assumptions on the item level and the population level have been relaxed. With these two types of assumption intact, the original RM decomposes the probability of item responses in two independent components: an item-specific difficulty parameter that is constant across all examinees in the population, and one ability parameter for each examinee that is the same across all items in a given assessment.

These homogeneity assumptions, however, are the ones not met in many practical applications of the RM, since either some or all of the items may function differently in different subpopulations, or the responses of subjects to these items may depend on more than one latent trait. This turns out to be an issue, for example, if item types are mixed, if the content of items varies somewhat, and/or if items are assessed in complex populations of examinees that come from different backgrounds such as different educational systems.

The volume addresses these issues in two ways, first by presenting chapters on recent extensions to the RM and second by providing chapters on applications of these extensions in educational or psychological contexts. The model extensions presented here have been actively developed and studied by various researchers, who have contributed to pioneering theoretical developments on extending the RM to multiple populations and multidimensional abilities. These researchers are often long-term advocates of applying these models to substantial research questions in the social sciences. Many researchers with backgrounds in other well-established statistical fields likewise took the RM as a basis for extending "their" models, frequently with a specific substantive question in mind. Several chapters in this volume are contributed by the original developers of such model extensions, who took a mathematical model and made it more flexible to suit applied research questions.

This direction of development—from a theory-driven substantive research question or a hypothesis to a model extension that reflects this theory—is guiding the structure of most contributions in this volume. The different chapters describe this process by referring to exemplary theories or research questions under investigation, then outline the required features of the model extension used to investigate these questions, and finally describe the path taken to extend or choose a model and to plan and carry out the analysis. To reflect this interplay between substantive theory and model development, the first part of this volume includes papers presenting work on extending MRMs and multiple group RMs—relaxing the person homogeneity assumptions—as well as multivariate RMs that relax the item homogeneity assumption to fit typical questions arising in applied research. The second part of this volume consists of chapters that present the models developed in the first part in a variety of applications in empirical educational research and a number of areas of psychological research.

1.1.1 The Rasch Model

This section introduces a basic set of assumptions and a general framework for latent variable models for item response data. The conventions introduced here can be found in most subsequent chapters, except where the extensions developed in subsequent chapters are more easily derived using a different notation.

Assume there are n examinees, $E_1, \ldots, E_n$, drawn randomly from a population, who respond to a set of I test items. Let $x_{vi} \in \{0, 1, \ldots, m_i\}$ denote the integer-coded response of examinee v to item i, that is, the actual behavioral response is mapped to an element of a set of successive integers starting from 0.

If the responses to item i take on only the two values 0 and 1, we speak of dichotomous data and refer to the dichotomous RM; if the responses can take on more than two integer values, say $x_{\cdot i} \in \{0, 1, 2, 3, 4\}$, the RM has to be specified for polytomous ordinal data to model responses of this type appropriately. In this volume, both the dichotomous RM and the RM for polytomous data will be used frequently, and it will often not be explicitly specified whether item responses are assumed to be dichotomous or polytomous. We ensure that this will not lead to ambiguities by using a specific method of introducing the RM in a mathematical form that can be used for both dichotomous and polytomous data while meeting certain common foundational assumptions of the RM.

Given the above definitions, denote the observed item responses of an examinee v by $\boldsymbol{x}_v = (x_{v1}, \ldots, x_{vI})$, that is, a vector with integer components in the finite space $\Omega_{\boldsymbol{x}} = \prod_{i=1}^{I} \{0, \ldots, m_i\}$ of possible response patterns for these I test items. The RM is derived by assuming certain unobserved quantities in addition to the observed quantities $x_{v1}, \dots, x_{vI}$ for each examinee v and each item i, and by specifying certain assumptions about the relation of these, yet to be specified, unobserved quantities to the probability of observing a response pattern $\boldsymbol{x} \in \Omega_{\boldsymbol{x}}$.

The dichotomous RM assumes that there is a real-valued parameter θ_v for each examinee, referred to as person parameter, and real-valued β_i for each item, subsequently referred to as item difficulty. For the probability of a response x_{vi}, the RM assumes

$$P_{vi}(X = x_{vi}) = P(x_{vi}|\theta_v, \beta_i) = \frac{\exp(x_{vi}(\theta_v - \beta_i))}{1 + \exp(\theta_v - \beta_i)} \tag{1.1}$$

for all examinees $v = 1, \dots, N$ and all items $i = 1, \dots, I$.

This equation can easily be extended to polytomous responses by writing the model as

$$P(x_{vi}|\theta_v, \beta_{i\cdot}) = \frac{\exp\left(x_{vi}\theta_v - \beta_{ix_{vi}}\right)}{1 + \sum_{x=1}^{m_i} \exp\left(x\theta_v - \beta_{ix}\right)} \tag{1.2}$$

with real-valued β_{ix} for $i = 1, \dots, I$ and $x = 1, \dots, m_i$ and θ_v real-valued as above. The model as defined in Equation 1.2 is suitable for observed variables $x_{vi} \in \{0, \dots, m_i\}$ with an integer $m_i > 0$.

The definition of the RM ensures that the probability of responding with category x rather than with $x - 1$ is strictly increasing with increasing person parameter θ. For the item parameters, strictly decreasing monotonicity holds, with increasing difficulty threshold γ_{ix}, a response in the upper (x) of two adjacent categories $(x, x - 1)$ decreases in probability. These *monotonicity properties (MO)* are among the defining characteristics of the RM.

For the second defining characteristic, it is convenient to write

$$\alpha(\theta_v, \beta_{i\cdot}) = -\ln\left(1 + \sum_{x=1}^{m_i} \exp(x\theta_v - \beta_{ix})\right)$$

and to write the RM as

$$P(X = x_{vi}|\theta_v, \beta_{i\cdot}) = \exp\left(x_{vi}\theta_v - \beta_{ix_{iv}} + \alpha(\theta_v, \beta_{i\cdot})\right). \tag{1.3}$$

In addition to the monotonicity in item and person parameters, the RM assumes *local independence (LI)*, i.e., it is assumed that, for an examinee with person parameter θ, the responses $\boldsymbol{x} = (x_1, \dots, x_I)$ are independently distributed given θ. That is,

$$P(\boldsymbol{x}|\theta) = \prod_{i=1}^{I} P(X = x_i|\theta, \beta_i)$$

for all θ. This, with the above definitions, yields after some elementary transformations

$$P(\boldsymbol{x}|\theta) = \exp(t_v\theta)\exp(\boldsymbol{\alpha}(\theta,\boldsymbol{\beta}))\exp\left(\sum_{i=1}^{I}\beta_{ix_{iv}}\right) \tag{1.4}$$

with $t_v = \sum_{i=1}^{I} x_{vi}$ and $\boldsymbol{\alpha}(\theta_v,\boldsymbol{\beta}) = \sum_{i=1}^{I}\alpha(\theta_v,\beta_{i\cdot})$.

Note that in Equation 1.4, the probability of a response pattern $\boldsymbol{x}$ in the RM has been written as a product of three terms. Note that one of the terms, $\exp(\alpha(\theta,\beta))$, does not depend on the observed data, and another one is the same for all response patterns that share the same total score t. This property will be used in the next section, which talks about conditional inferences in the RM.

To estimate parameters, maximum likelihood methods can be applied. Initial approaches to the estimation problem have been based on maximizing a likelihood function for the observed data matrix $(x_{vi})_{i=1\ldots I,v=1\ldots N}$ jointly for the θ_v and the β_{ix} parameters. To avoid undesirable properties of the joint estimation (Neyman & Scott, 1948), later approaches applied modified likelihood equations that eliminated the person parameter θ and thus allow one to maximize for the item parameters only. By eliminating the "nuisance" parameters θ_v, which are increasing in number with sample size N, the consistency of item parameter estimates can be ensured. This is done either by assuming a distribution for the person parameter θ and integrating over this distribution (marginal maximum likelihood—MML) or by conditioning on some available observed quantity, a sufficient statistic (Bickel & Doksum, 1977) that allows one to eliminate the nuisance parameters.

MML estimation is prevalent in more general IRT models since these often do not have simple sufficient statistics. However, the specific form of the RM as given in Equations 1.1 and 1.2 ensures that the total score t_v is a sufficient statistic for the person parameter θ_v, and similarly for the item-category totals. This property of the RM, the *sufficiency of total (ST)* scores for the item and person parameters, is the third defining characteristic of RMs. The impact of this sufficiency is elaborated on in the following subsection on the conditional (on total score) form used in the conditional maximum likelihood estimation (CML) of the RM.

1.1.2 Conditional Inferences in the Rasch Model

The sufficiency of the total score *(ST)* ensures that the RM can be written in a conditional form, based on the observed distribution of the sufficient statistic. The conditional form of the RM no longer contains the person parameter and can be used to draw conditional inferences about model data fit and to estimate item parameters without assumptions about the distribution of person parameters in the population by plugging in the observed counts of the total score.

The derivation of the RM in conditional form is based on Equation 1.4. For a given θ, the probability of observing a total score t is

$$P(t|\theta) = \sum_{\boldsymbol{x}|t} P(\boldsymbol{x}|\theta),$$

which is the sum over all conditional probabilities of response patterns $\boldsymbol{x}$ with the same total score t. As it is easily seen in Equation 1.4, all probabilities in the above sum share the terms $\exp(t\theta)$ and $\exp(\boldsymbol{\alpha}(\theta, \boldsymbol{\beta}))$, since these do not depend on the specific response pattern $\boldsymbol{x}$, but only on θ and t (and β, which is of lesser concern at this point).

Conditional inference in the RM uses the specific form of $P(\boldsymbol{x}|\theta)$ from Equation 1.4, which separates terms that depend on the observed data $\boldsymbol{x}$ from terms that depend only on the total score t or do not at all depend on the observed data. Then, after some algebra, we may write

$$P(\boldsymbol{x}|t,\theta) = \frac{P(\boldsymbol{x}|\theta)}{P(t|\theta)} = \frac{\exp(-\sum_{i=1}^{I} \beta_{ix_i})}{\sum_{\boldsymbol{x}'|t} \exp(-\sum_{i=1}^{I} \beta_{ix'_i})}.$$

The right-hand side of the above expression is independent of θ and contains only the response vector $\boldsymbol{x}$ and the item parameters $\boldsymbol{\beta}$. Integrating over the person parameter distribution using $P(x|t) = \int_\theta P(x|t,\theta)p(\theta)d\theta$ yields

$$P(\boldsymbol{x}|t) = \frac{\exp(-\sum_{i=1}^{I} \beta_{ix_i})}{\sum_{\boldsymbol{x}'|t} \exp(-\sum_{i=1}^{I} \beta_{ix'_i})}, \tag{1.5}$$

which is the probability of a response vector $\boldsymbol{x}$ in the conditional form of the RM. This is not to be confused with the integration over the ability distribution commonly used for more general IRT models in conjunction with MML estimation methods (Bock & Aitkin, 1981). In contrast to MML estimation, the integration mentioned above to arrive at the expression in Equation 1.5 does not actually take place during estimation; it is utilized as an algebraic equivalence to get rid of the θ on the left side of the expression.

In this conditional form of the RM, we have an expression for the probability of a response pattern $\boldsymbol{x}$, given total score t that is independent of θ. This eliminates the need either to estimate the ability θ for each examinee or to assume a specific form of ability distribution when estimating item parameters.

The conditional form of the RM is quite useful when item parameters have to be estimated from observed data. The independence of specific assumptions about the ability distribution is ensured in the conditional estimation of parameters. This sets the RM apart from other models for item response data, since most other models such as the two- and three-parameter item response theory (IRT) models need additional assumptions about the distribution of person parameters for estimating item parameters.

Conditional inferences play an important role in the RM (Fischer & Molenaar, 1995) and in many of the extensions of the RM presented in this volume. These extensions preserve the defining characteristics of the RM in a way that enables one to use the RM (or its extensions) in conditional form.

1.1.3 Some Notation for Extended Rasch Models

This section introduces notation that allows one to specify the RM in the presence of multiple populations and for multiple scales simultaneously. Using this approach, many extensions presented in this volume can be viewed as models that assume that the RM holds, with the qualifying condition that it holds with a different set of parameters in different populations or with a different ability (person) parameter for each of a set of distinguishable subsets (scales) of test items.

Assume that there is a many-to-one classification g that maps the person index v to $v \to g(v) = c \in \{1, \ldots, C\}$, so that each examinee v is member of exactly one of C populations (classes, groups). In the ordinary RM, $C = 1$, and therefore, the population index c is not needed. Also, assume that there is a real-valued θ_{vk} for all v and multiple scales $k = 1, \ldots, K$, and let $\boldsymbol{\theta}_v = (\theta_{v1}, \ldots, \theta_{vK})$ be the k-dimensional person parameter.

Let $\boldsymbol{x}_v = (x_{v1}, \ldots, x_{vI})$ be the vector of observed responses for examinee $v \in \{1, \ldots, N\}$. As above, the categorical responses x_{vi} may be dichotomous or polytomous ordinal responses, i.e., assume $x_{vi} \in \{0, \ldots, m_i\}$. Note that we keep most of the notation intact; v denotes the examinee index, and N is the total number of observations. Since there is more than one set of items, the index k denotes the scale, and the items $i = 1, \ldots, I$ are mapped onto the k scales.

One additional constructive element has to be included. Each item may belong to exactly one component of ability, say the kth component of $\boldsymbol{\theta}$, or it may be considered an item that taps into one or more of the K-person parameter components. In the case that the items belong to more than one ability component k, we speak about *within-item multidimensionality*. Otherwise, if each item belongs to exactly one ability component, we talk about *between-item multidimensionality* (compare also Chapter 4 in this volume).

Within-item multidimensionality refers to the assumption that responses to each item may require multiple ability components (more than one skill or ability component is required for each item) while between-item multidimensionality refers to the assumption that each item can be solved using only one skill, but different subsets of items may require different skills.

For the case of within-item multidimensionality, each item i is characterized by a vector $\boldsymbol{q}_i = (q_{i1}, \ldots, q_{iK})$ that represents the load of each scale on the ith item. The collection of these vectors into a matrix Q represents the design of the assessment instrument. The matrix Q determines which items load on which scales. In the RMs presented here, this design matrix consists of zeros and ones, predetermined by the researcher. More specifically, the Q-matrix entries are a hypothesized structure of relationships between required skills and items, and the matrix entries (loadings) are fixed, not estimated.

Therefore, we may write for the case of within-item multidimensionality

$$P_i(x|\boldsymbol{\theta}_v, c = g(v)) = \frac{\exp\left(x(\boldsymbol{q}_i^T \boldsymbol{\theta}_v) - \beta_{ixc}\right)}{1 + \sum_{y=1}^{m_i} \exp\left(y(\boldsymbol{q}_i^T \boldsymbol{\theta}_v) - \beta_{iyc}\right)}$$

with $\boldsymbol{q}_i^T\boldsymbol{\theta} = \sum_k q_{ik}\theta_k$.

For the case of between-item multidimensionality (each item "loads" on one scale only), we can define the probability of a response x to item i in scale k by an examinee v with $c = g(v)$ as

$$P_i(x|\boldsymbol{\theta}_v, c = g(v)) = \frac{\exp(x\theta_{vk} - \beta_{ixc})}{1 + \sum_{y=1}^{m_i} \exp(y\theta_{vk} - \beta_{iyc})}$$

with real-valued β_{ixc} for $x = 1, \ldots, m_i$, and $\beta_{i0c} = 0$. The two definitions above are compatible, since the between-item multidimensionality is a special case of the within-item multidimensionality. If each item loads on only one scale, the cross product $\boldsymbol{q}_i^T\boldsymbol{\theta}$ reduces to the one term $\theta_{\cdot k}$ for which $q_{ik} = 1$, since all other $q_{ik'}$ are equal to 0.

Obviously, if c and k were not present, the above equation would resemble the ordinary RM from the previous section. Many of the extensions treated in this volume can be expressed in ways that add a population index (like c), or a scale index (like k) to the ordinary RM.

In the equations, the probability of the outcome depends on v only through $\boldsymbol{\theta}_v$ and through $c = g(v)$, so that we may write

$$P_i(x|\boldsymbol{\theta}, c) = \frac{\exp\left(x(\boldsymbol{q}_i^T\boldsymbol{\theta}) - \beta_{ixc}\right)}{1 + \sum_{y=1}^{m_i} \exp\left(y(\boldsymbol{q}_i^T\boldsymbol{\theta}) - \beta_{iyc}\right)} \tag{1.6}$$

by omitting the v in the equation. This holds, since all examinees v, v' with identical $\boldsymbol{\theta}_v = \boldsymbol{\theta}_{v'}$ and $c = g(v) = g(v')$ have the same response probabilities in the model above.

For a response vector $\boldsymbol{x} = (x_1, \ldots, x_I)$, the probability of this variable is defined by Equation 1.6 above and the usual assumption of local independence, that is,

$$P(\boldsymbol{x}|\boldsymbol{\theta}, c) = \prod_{i=1}^{I} P_i(x_i|\boldsymbol{\theta}, c)$$

with the same definitions as before, i.e, $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_K)$ and $c \in \{1, \ldots, C\}$, and $P_i(x_i|\boldsymbol{\theta}, c)$ as defined above.

For between-item multidimensionality, the conditional form of the RM is easily derived in this framework as well, but it will be obviously dependent on the scale k and the population c. In that case, the conditional RM becomes

$$P(\boldsymbol{x}_k|t_k, c) = \frac{\exp(-\sum_{i|k(i)=k} \beta_{ix_ic})}{\sum_{\boldsymbol{x}'_k|t_k} \exp(-\sum_{i|k(i)=k} \beta_{kix'_{ki}c})} \tag{1.7}$$

with $\boldsymbol{x}_k$ denoting the projection of the response vector that contains only items of scale k. The total scale score t_k is the corresponding sum over only those items belonging to the kth scale. The conditional RM for scale k in population c allows one to estimate item parameters for this scale in this population, using conditional maximum likelihood estimation methods (Fischer & Molenaar, 1995; von Davier & Rost, 1995).

1.1.4 Are These Extensions Still Rasch Models?

Critics of extensions such as the ones presented in this volume may argue that these models are no longer RMs, since some basic assumptions of the original model are modified. Even within the group of researchers who use the original RM, there are arguments as to what is the right way to do so. In this volume, the majority of extensions of the RM are based on the assumption that the original RM holds in exhaustive and mutually exclusive subsets of the item universe and the examinee population. This means that each examinee belongs to one subpopulation where the RM holds, possibly with a unique set of item parameters. The same is true for most extensions presented here for each item; that is, it is assumed that each of the items belongs to one subset (subscale) for which the original RM holds, but there may be more than one subscale. A Rasch purist could still analyze these subscales separately, or analyze subpopulations separately in this case. Such an approach would retain all the assumptions of the RM by using a more constrained definition of the target population and/or the item universe. However, if a joint analysis is desired, an extended model that accommodates differences between items and subpopulations is required.

The first rule of statistical modeling is that no model ever “really”’ fits the data. This is true and can be shown empirically by rigorously testing models in sufficiently large samples. Still, there is hope in the sense that some models provide useful summaries of data, so that these summaries are predictive for some outside variable that was assessed concurrently or even some future outcome. Model extensions are aimed at improving these capabilities; they are aimed at improving predictions by including a more complex description of the observed variables (that is, the item responses), the examinees involved, or both. This more complex description relates to an increased number of model parameters that often make either items’ response functions or population distributions more flexible.

Which of these extensions are legitimate? And for whom? This may often depend on which *group (or subpopulation)* the researcher who judges these extensions belongs in (von Davier, 2006). There are, of course, common statistical issues that pose problems for any model extension, such as a lack of identifiability, which all professional groups would agree disqualifies a model from further consideration. Apart from these, the selection of which extensions are permissible, and which catapult the specific model outside of the group of “extended” RMs stays somewhat subjective.

As mentioned above, most extensions in this volume maintain basic features of the RM such as the conditional sufficiency of raw scores (either in subpopulations, or as subscores based on subsets of items), the conditional independence assumption, and the monotonicity assumption. Conditional independence is given up in only one of the chapters, mainly to account for differences in point-biserial correlations among items, which would otherwise be modeled by allowing a discrimination parameter. Monotone increasing characteristic functions, in both item easiness (negative difficulty) and in the person parameter, are the basis for all the models presented in this volume.

Maybe more interesting than the question whether the extensions presented here may still be called (extended) RMs is whether these models add value to the statistical analysis of item response data. In many cases, adding parameters to a model and increasing model-data fit is easy to do, but the added value of doing so has to be well established in order to justify the added complexity for the given purpose of the analysis. Molenaar (1997) has expressed this in very understandable terms that may be paraphrased as “IRT models are great, even if they never fit the data. But does it matter?” The RM (and its extensions) set the stage for answering Molenaar’s question. However, the question whether it matters has to be qualified as, “Does it matter for the specific purpose one has in mind?”

Applications aimed at variance decomposition using background variables ask a different question, and therefore may require consideration of a different type of model extension, than applications aimed at deriving a rank order of students applying to a higher-education facility. The former purpose is explanatory and tests hypotheses about relationships between variables, whereas the latter classifies students as admitted versus not admitted. One application is concerned with the best possible representation of variance components, whereas the other is concerned with the best possible point estimate for each student in order to provide the most accurate classification, given data and model. The chapters in this volume derive extensions of the RM with specific purposes in mind. The reader is kindly asked to view the chapters with that in mind, in order to see the scope of applicability of the specific extensions and to explore the different fields in which the simple and elegant form of the RM has proven useful as the foundational basis for a more complex statistical model.

1.2 Overview and Structure of This Volume

Most if not all extensions presented in this volume were created after encountering the need to model data that are more complex than the RM in its “pure” form can handle. Some extensions address specific questions and were driven by some specific research context, whereas other extensions address more general considerations as to which model assumptions may limit the applicability of the RM to more complex assessment data.

The chapters within this volume introduce specific extensions or applications and cross reference to other appropriate chapters. References to work published outside this volume are also provided to encourage further reading and to provide a broader view of this area of research as consisting of interconnected fields. In this view, it is less important whether a statistical tool such as hierarchical linear models uses the RM for categorical dependent variables or whether the RM adopts a more complex population structure that reminds one of a hierarchical linear model. We hope that it becomes evident that no matter what prompted a particular development, the merger of the RM with other statistical methods creates interesting, useful, and rigorously testable models with applications in a variety of fields. This approach should provide some guidance for readers and help them to build a cognitive map of the different extensions of the RM.

This format is applied to the more general chapters as well as to the more applied chapters, which either contain an overview of relevant applications or illustrate certain extensions using exemplary studies from various areas of research.

1.2.1 General Rasch Model Extensions

The first part of this volume covers the ideas guiding these model extensions and tries to create a framework that helps the reader understand the specific tools these model extensions provide for researchers. These more conceptual chapters are an attempt to showcase more generally some ways to think about deriving model extensions from demands that cannot be fulfilled by a model that assumes a very strict structure. This part also contains a chapter that provides some insight into how the expected payoff of extending the RM can be tested.

The first chapter in this part (Chapter 2) is the most conceptual in the sense that it lays out what kind of inferences require models that include strong homogeneity assumptions. Chapter 3 outlines how evidence for the need for more complex models can be collected and evaluated statistically. This chapter introduces procedures for testing whether the added complexity of extended RMs actually helps to describe and understand the data better. This, in our understanding, is a fundamental requirement of analysis with complex statistical models, since the added complexity requires more resources for reporting as well as additional effort for researchers who want to make sense of the results or who want to use the outcomes in subsequent analysis. Chapter 4 presents an overview of flexible families of multivariate RMs. These multivariate RMs are based on the assumption that there is a hypothesis about the dimensional structure of each observed variable, i.e., each item is related to one or more of the multiple abilities through a design matrix defined a priori. This design matrix is often referred to as a Q-matrix in models for student profiles (Tatsuoka, 1983) and resembles the structural basis for a confirmatory analysis of a multivariate model. Chapter 5 introduces a very useful way to specify, estimate, and study extensions of RMs. This chapter shows how RMs and their extensions can be framed in terms of loglinear models and how these models can be estimated using software for loglinear models. The final chapter (Chapter 6) in the first part of this volume describes the family of discrete mixture distribution RMs (mixed Rasch models, [MRMs]; Rost, 1990; von Davier & Rost, 1995) and HYBRID RMs. This chapter provides an outline of the basis for these models as derived from IRT and the RM and as integrated with latent class analysis (LCA). This unique way of modeling offers tools to, among other things, handle differential item functioning (DIF) as well as to test for multidimensionality in the context of discrete mixture distribution models.

1.2.2 Model Extensions for Specific Purposes

The second part of the volume covers models that were created in response to a specific problem or research question. Overlap with the first part is intentional, since some of the extensions treated here, even if originally developed for a specific research question, grew into a broader class of models with applications in a variety of fields.

The first chapter in this part (Chapter 7) describes a model that allows one to study developmental processes using repeated measures. This chapter introduces the saltus model, an extension of the RM that allows one to study changes in difficulty of tasks over different developmental stages. Chapter 8 in this part introduces stochastically ordered MRMs for identifying diagnostic cutscores. Chapter 9 is dedicated to an extension of the HYBRID model that allows one to study speededness phenomena in detail. This chapter modifies mixture distribution RMs introduced in the first part of the volume by imposing complex equality constraints on them to model the switch between systematic and random response at a certain point in the response process. Chapter 10 is a specialization of the multidimensional approach also already introduced in the first part of the volume. This chapter covers different types of potential applications of these multidimensional RMs. The fifth chapter in Part II, Chapter 11, relates the RM and the MRM to discrete latent trait models, namely to located latent-class models, and compares parameter estimates from these different latent-variable models.

The following chapter (Chapter 12) introduces MRMs for longitudinal data. Interestingly, several contributions in this volume use loglinear models, initially described in Chapter 5, as the common language to describe developments based on multivariate or mixture-distribution Rasch models. These loglinear models with unobserved grouping variables are a useful tool that lends itself nicely to treating this kind of missing-data problem. Chapter 13 extends the RM to allow for differences in discriminations across the range of items by introducing an interaction rather than a slope parameter. In contrast to the two-parameter logistic model, the interaction model used in Chapter 13 retains some of the conditional inference features of the RM. The final chapter in Part II (Chapter 14) is an extension of the RM to complex samples from hierarchically organized populations that do not lend themselves easily to drawing simple random samples. This situation is often encountered in large-scale educational assessments and other survey assessments. Here we might also assume the development from the other side of the statistical toolbox, namely that the model basis was a hierarchical linear model that was extended by a Rasch-type measurement model.

1.2.3 Applications of Extended Rasch Models

The third part of this volume is dedicated to chapters that provide insight into exemplary applications of extended RMs in various fields of research. There is a strong link between these chapters and the previous parts, since the applied work shows how statistical tools that are based on the RM can help to pose and answer specific questions on data from complex assessments and or populations.

The first chapter in this part (Chapter 15) presents a variety of applications of extended RMs such as mixture distribution RMs in the area of cognitive psychology. Chapter 16 applies mixture RMs to the task of detecting faking and response distortions with the aim of identifying candidates who try to present themselves in a specific way. Chapter 17 talks about applications of multidimensional RMs in an international educational survey assessment.

Chapter 18 talks about applications of RMs and extensions of RMs to studying developmental issues. This chapter presents an overview of areas of application and the limitations of these approaches. Chapter 19 compares an item response model that uses a parsimonious way to account for guessing by estimating a constrained three-parameter logistic model with the application of mixture-distribution RMs to identify and correct for guessing behavior.

Chapter 20 covers extended RMs developed for modeling strategy shifts. This chapter extends previous work on strategy differences and helps one to understand how such complex models can be conveniently specified in the framework of loglinear models. Chapter 21 integrates principles of graphical models and mixture distribution RMs and presents an application to health science data. The last chapter in this volume (Chapter 22) presents some applications of RMs and extensions of RMs to data from sports science and applied psychology in the motor domains.

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