19

Conclusion. Critical Analysis of Some Basic Concepts of Micromechanics

T is strange,- but true, for Truth is always strange-
Stranger than fiction ...

-Lord Byron. *Don Juan.*

If you can look into the seeds of time,
And say which grain will grow and which will not,
Speak then to me ...

-William Shakespeare. *Macbeth.*

Let us discuss the main scheme as well as the short sketch of limitations and ideas as well as possible generalizations of some basic concepts of the linear version of the MEFM and some related methods (see Chapters 7-14). This sketch does not pretend to be rigorous and may contain controversial statements, too personal or one-sided arguments which are deliberately presented with the aim to give free reins to our imagination.

Let us consider the basic equations of linear elasticity of composites in micropoint (see Chapter 8 for details):

$$\nabla\boldsymbol{\sigma} = \mathbf{0}, \quad \boldsymbol{\sigma}(\mathbf{x}) = \mathbf{L}(\mathbf{x})\boldsymbol{\varepsilon}(\mathbf{x}), \quad \boldsymbol{\varepsilon}(\mathbf{x}) = \mathrm{Def}\,\mathbf{u}(\mathbf{x}). \tag{19.1}$$

Substituting (19.1_2) and (19.1_3) into the equilibrium equation (19.1_1) we obtain a differential equation:

$$\nabla\mathbf{L}^c\nabla\mathbf{u}(\mathbf{x}) = -\nabla\mathbf{L}_1(\mathbf{x})\nabla\mathbf{u}(\mathbf{x}) \tag{19.2}$$

with the constant coefficients in the left-hand side that allows the general integral equation (7.19)

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \langle\boldsymbol{\varepsilon}\rangle(\mathbf{x}) + \int \mathbf{U}^c(\mathbf{x}-\mathbf{y})[\boldsymbol{\tau}(\mathbf{y}) - \langle\boldsymbol{\tau}\rangle]d\mathbf{y}. \tag{19.3}$$

where the tensor $\mathbf{U}^c = \nabla\nabla\mathbf{G}^c$ is defined by the Green function $\mathbf{G}^c$ (3.3) for the infinite homogeneous medium with an elastic modulus $\mathbf{L}^c$.

Equation (19.3) can be rearranged in the form of an infinite system of integral equations (8.8)–(8.11) that can be solved by the different approximate methods considered in Chapters 8 and 9. The main hypothesis of many micromechanical methods called effective field hypothesis **H1)** (8.14) and (8.15) allows the infinite system for integral equations with respect the smooth effective fields for any shape and structure of inclusions [see Eqs. (8.18) and (8.19)]. However, a generality of the last statements is an illusion because arbitrariness of the inclusion shape does not guarantee a protection from the necessity of using of Eshelby tensor in the MEFM. Indeed, the notion Eshelby tensor has a fundamental conceptual sense rather than only an analytical solution of some particular problem. Exploiting the Eshelby tensor concept in the MEFM is based on the ellipsoidal shape of the correlation hole v_{ij}^0 rather than on the inclusion shape v_i. An abandonment of the assumption of the v_{ij}^0's ellipsoidal shape leads of necessity to the inhomogeneity of the effective field $\overline{\boldsymbol{\sigma}}_i$ acting on the inclusion v_i that is prohibited for the classical version of the MEFM (see Sections 9.4).

For statistically homogeneous medium subjected to homogeneous boundary conditions, two-particle approximation of a closing hypothesis **H2** (8.39):

$$\langle \widetilde{\boldsymbol{\sigma}}(\mathbf{x})_{1,2}\rangle_i = \langle \overline{\boldsymbol{\sigma}}(\mathbf{x})\rangle_i = \text{const.} \quad (i = 1, 2). \tag{19.4}$$

reduces this system to the system of algebraic Eqs. (8.40). Further simplification is accomplished by "quasi-crystalline" approximation by Lax (8.65):

$$\langle \overline{\boldsymbol{\sigma}}_i(\mathbf{x})|v_i, \mathbf{x}_i; v_j, \mathbf{x}_j\rangle = \langle \overline{\boldsymbol{\sigma}}_i\rangle, \quad \mathbf{x} \in v_i \tag{19.5}$$

which is weaker than Mori-Tanaka assumption (8.90):

$$\overline{\boldsymbol{\sigma}}_i \equiv \langle \boldsymbol{\sigma}\rangle^{(0)}. \tag{19.6}$$

The Mori-Tanaka hypothesis (19.6) is probably the most popular hypothesis of micromechanics which at the same time exerted most deleterious impact on the development of this subject. Harmful effect is explained not only by the loss of the effective modulus symmetry (see Subsections 8.3.3 and Section 18.3) in some general cases but, that is crucial, by self-closing nature of the hypothesis (19.6). The seeming simplicity and illusory assuredness hide the ways of generalization of this hypothesis while an extension of "quasi-crystalline" approximation (19.5) to the closing hypothesis (19.4) is obvious.

Summarizing what we said above, we will present now the contractions of some concepts and assumptions erroneously recognized in micromechanics as basic ones: a) constitutive equation, b) homogeneous comparison medium, c) Green function, d) Eshelby tensor, e) effective field hypothesis **H1**.

The fallacy in these recognitions will be justified in an inverse order.

Indeed, we demonstrated in the book two methods of analyses of the inhomogeneous effective fields $\overline{\boldsymbol{\sigma}}_i$. In the first case of statistically homogeneous medium subjected to the homogeneous boundary conditions, the inhomogeneity of $\overline{\boldsymbol{\sigma}}_i$ takes place if the correlation hole v_{ij}^0 is nonellipsoidal that leads to the problem of the estimation of stress distribution in the domain v_{ij}^0 undergoing to the constant eigenstrains $\boldsymbol{\beta}_1(\mathbf{x}) \equiv \text{const}$, $\mathbf{L}_1(\mathbf{x}) \equiv \mathbf{0}$ $(\mathbf{x} \in v_{ij}^0)$ (see Subsections 4.2.3 and Section 9.4). A more general second method applicable for the analysis of both statistically homogeneous and statistically inhomogeneous fields in both the infinite and bounded media, and for any shape of v_{ij}^0 is reduced to the estimation of perturbations produced by the inclusion v_j in both inside and outside in some vicinity of the inclusion v_j (see Eq. (14.63)). In so doing these perturbations evaluated by any either numerical or analytical method available for a researcher are reduced to the classical Eshelby internal and external tensors only in some particular simplest cases.

Thus the popular effective field hypothesis **H1** is not fundamental and can be easily avoided if it is known the numerical solution just for one inclusion obtained by any numerical method available for researcher. Known numerical methods such as FEA, VIE, BIE, and the complex potentials, which can be used for micromechanical analysis, have a series of advantages and disadvantages. It is crucial for the analyst to be aware of their range of applications. This solution only in simplest cases is expressed through the Green function and Eshelby tensor which are not, in general, necessary tools in micromechanics. In the case of taking of multiparticle interactions into account, a solution for a single inclusion should be complemented by a solution for n interacting inclusions in the comparison medium that also can be performed without the notions Green function and Eshelby tensor (see Chapters 10 and 14) by the use of operator representations of these solutions. In so doing, the existence of the mentioned operators was justified through the Green function's technique.

The challenge of modern micromechanics is a development of the general methodology incorporating the solution for multiple interacting inhomogeneities obtained by highly accurate numerical methods into the most general scheme of analytical micromechanics (see Chapters 10-12, and 14). A fundamental difference between the proposed methodology and the ones published earlier is a systematic analysis of statistical distributions $\langle \boldsymbol{\sigma}(\otimes\boldsymbol{\sigma})^n\rangle^{(k)}(\mathbf{x})$ ($n = 0, 1, \ldots;\ k = 0, 1, \ldots, N;\ \mathbf{x} \in v^{(k)}$) of local microfields rather than only effective properties based on the average fields inside the phases $\langle\boldsymbol{\sigma}\rangle^{(k)}$. Moreover, this approach allows us to estimate the conditional moments of stresses inside the fixed inclusions v_i: $\langle \boldsymbol{\sigma}(\otimes\boldsymbol{\sigma})^n\rangle_i(\mathbf{x}^l)$ ($\mathbf{x}^l \in v_i \subset v^{(k)}$), which are more informative than $\langle \boldsymbol{\sigma}(\otimes\boldsymbol{\sigma})^n\rangle^{(k)}(\mathbf{x})$ ($\mathbf{x} \in v^{(k)}$) (see Chapter 14). Obviously, the analysis of nonlinear phenomena such as plasticity, creep, and damage should use precisely $\langle \boldsymbol{\sigma}(\otimes\boldsymbol{\sigma})^n\rangle_i(\mathbf{x}^l)$ ($\mathbf{x}^l \in v_i \subset v^{(k)}$), rather than $\langle \boldsymbol{\sigma}(\otimes\boldsymbol{\sigma})^n\rangle^{(k)}(\mathbf{x})$ ($\mathbf{x} \in v^{(k)}$) (see Chapters 15 and 16), which is a too rough descriptor of the stress distribution especially in the case of statistically inhomogeneous media in bounded domains (see Chapters 10, 13, and 14).

Furthermore, popular assumptions about homogeneity of the comparison medium and their unboundedness (19.2) were in reality accepted exclusively for introduction of a much more powerful tool of micromechanical research such as Green's function; see Eq. (19.3). However, the eventual abandonment of the Green function concept (see Chapters 10, 11, and 14) removed any restrictions on the comparison medium in the case of linear elastic problems. A comparison medium can either coincide with the whole space or be bounded by any surface (either simple connected or multiply connected). Elastic properties and eigenstrains of the comparison medium can be either homogeneous or inhomogeneous. A single restriction on the inhomogeneity of a comparison medium is a deterministic nature of this inhomogeneity. So, a comparison medium can have either continuously varying elastic moduli or piecewise constant elastic properties such as e.g. periodic structure (see Chapter 11) or two joint elastic half spaces. The bounded or unbounded medium with one or a few fixed macroinclusions (e.g. macrocracks) can also be considered as a comparison medium. However, at any rate, we need only to know the numerical solutions for the mentioned comparison medium containing one and a few interacting micro inclusions with any their possible location in the comparison medium.

At last, must we really know a local constitutive law (19.1_2) if we are going to estimate the effective elastic moduli of composite materials? At first glance this question is beyond the scope of common sense. However, despite the apparent paradoxicality, the correct answer to this question is no, in the following sense. Indeed, it was demonstrated in Section 18.2 that we don‘t need to know anything about either local or nonlocal nature of constitutive laws of inclusions. We only need to know that these laws are linear and also need to know the concentrator factors $\mathbf{A}_\sigma(\mathbf{x})$ and $\mathbf{B}_\sigma(\mathbf{x})$ (18.10) (which can be found directly, e.g., from MD simulations) at least for one inclusion in a comparison medium mentioned above.

Shortening what we said above, we can conclude that for linear elastic problems for microinhomogeneous medium of random structure:

a) we do not need to know a constitutive law for the inclusions,
b) we do not need to assume the homogeneity and infiniteness of a comparison medium,
c) we do not need to know the Green tensor and Eshelby tensor, and
d) we do not need to assume homogeneity of the effective fields $\overline{\boldsymbol{\sigma}}(\mathbf{x})$.

What we really need is the numerical solutions for one and a few interacting inclusions with any their possible location in the comparison medium described above.

Thus, in the framework of a unique scheme of the proposed MEFM, we have undertaken in this book an attempt to analyze the wide class of statical and dynamical, local and nonlocal, linear and nonlinear multiscale problems of composite materials with deterministic (periodic and nonperiodic), random (statistically homogeneous and inhomogeneous, so-called graded) and mixed (periodic structures with random imperfections) structures in bounded and unbounded domains, containing coated or uncoated inclusions of any shape and orientation and subjected to coupled or uncoupled, homogeneous or inhomogeneous external fields of different physical natures.

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