Preface

Time series and random fields are main topics in modern statistical techniques. They are essential for applications where randomness plays an important role. Indeed, physical constraints mean that serious modelling cannot be done using only independent sequences. This is a real problem because asymptotic properties are not always known in this case.

The present work is devoted to providing a framework for the commonly used time series. In order to validate the main statistics, one needs rigorous limit theorems. In the field of probability theory, asymptotic behavior of sums may or may not be analogous to those of independent sequences. We are involved with this first case in this book.

Very sharp results have been proved for mixing processes, a notion introduced by Murray Rosenblatt [166]. Extensive discussions of this topic may be found in his *Dependence in Probability and Statistics* (a monograph published by Birkhaüser in 1986) and in Doukhan (1994) [61], and the sharpest results may be found in Rio (2000) [161]. However, a counterexample of a really simple non-mixing process was exhibited by Andrews (1984) [2]. The notion of weak dependence discussed here takes real account of the available models, which are discussed extensively. Our idea is that robustness of the limit theorems with respect to the model should be taken into account. In real applications, nobody may assert, for example, the existence of a density for the inputs in a certain model, while such assumptions are always needed when dealing with mixing concepts. Our main task here is not only to provide the reader with the sharpest possible results, but, as statisticians, we need the largest possible framework. Doukhan and Louhichi (1999) [67] introduced a wide dependence framework that turns out to apply to the models used most often. Their simple way of weakening the independence property is mainly adapted to work with stationary sequences.

We thus discuss examples of weakly dependent models, limit theory for such sequences, and applications. The notions are mainly divided into the two following classes:

- The first class is that of "Causal" dependence. In this case, the conditions may also be expressed in terms of conditional expectations, and thus the powerful martingale theory tools apply, such as Gordin's [97] device that allowed Dedecker and Doukhan (2003) [43] to derive a sharp Donsker principle.

- The second class is that of noncausal processes such as two-sided linear processes for which specific techniques need to be developed. Moment inequalities are a main tool in this context.

In order to make this book useful to practitioners, we also develop some applications in the fields of Statistics, Stochastic Algorithms, Resampling, and Econometry. We also think that it is good to present here the notation for the concepts of weak dependence. Our aim in this book was to make it simple to read, and thus the mathematical level needed has been set as low as possible. The book may be used in different ways:

- First, this is a mathematical textbook aimed at fixing the notions in the area discussed. We do not intend to cover all the topics, but the book may be considered an introduction to weak dependence.

- Second, our main objective in this monograph is to propose models and tools for practitioners; hence the sections devoted to examples are really extensive.

- Finally, some of the applications already developed are also quoted for completeness.

A preliminary version of this joint book on weak dependence concepts was used in a course given by Paul Doukhan to the Latino Americana Escuela de Matemática in Merida (Venezuela). It was especially useful for the preparation of our manuscript that a graduate course in Merida (Venezuela) in September 2004 on this subject was based on these notes. The different contributors and authors of the present monograph participated in developing it jointly. We also want to thank the various coauthors of (published or not yet published) papers on the subject, namely Patrick Ango Nzé (Lille 3), Jean-Marc Bardet (Université Paris 1), Odile Brandière (Orsay), Alain Latour (Grenoble), Hélène Madre (Grenoble), Michael Neumann (Iena), Nicolas Ragache (INSEE), Mathieu Rosenbaum (Marne la Vallée), Gilles Teyssière (Göteborg), Lionel Truquet (Université Paris 1), Pablo Winant (ENS Lyon), Olivier Wintenberger (Université Paris 1), and Bernard Ycart (Grenoble). Even if all their work did not appear in those notes, they were really helpful for their conception. We also want to thank the various referees who provided us with helpful comments either for this monograph or for papers submitted for publication and related to weak dependence.

We now give some clarification concerning the origin of this notion of weak dependence. The seminal paper [67] was in fact submitted in 1996 and was part of the PhD dissertation of Sana Louhichi in 1998. The main tool developed in this work was combinatorial moment inequalities; analogous moment inequalities are also given in Bakhtin and Bulinski (1997) [8]. Another close definition of weak dependence was provided in a preprint by Bickel and Bühlmann (1995) [17] *anterior to* [67], also published in 1999 [18]. However, those authors aimed to work with the bootstrap; see Chapter 13 and Section 2.2 in [6]. The approach of Wu (2005) [188] detailed in Remark 3.1, based on $\mathbb{L}^2$-conditions for causal Bernoulli shifts, also yields interesting and sharp results.

This monograph is essentially built in four parts:

Definitions and models

In the first chapter, we make precise some issues and tools for investigating dependence: this is a *motivational* chapter. The second chapter introduces formally the notion of *weak dependence*. *Models* are then presented in a long third chapter. Indeed, in our mind, the richness of examples is at the core of the weak dependence properties.

Tools

Tools are given in two chapters (Chapters 4 and 5) concerned respectively with *noncausal* and *causal* properties. Tools are first used in the text for proving the forthcoming limit theorems, but they are essential for any type of further application. Two main tools may be found: *moment* bounds and *coupling* arguments. We also present specific *tightness criteria* adapted to work out empirical limit theorems.

Limit theorems

Laws of large numbers (and some applications), *central limit theorems*, *invariance principles*, *laws of the iterated logarithm*, and *empirical central limit theorems* are useful limit theorems in probability. They are precisely stated and worked out within Chapters 6–10 .

Applications

The end of the monograph is dedicated to applications. We first present in Chapter 11 the properties of the standard *nonparametric techniques*. After this, we consider some issues of *spectral estimation* in Chapter 12. Finally, Chapter 13 is devoted to some miscellaneous applications, namely *applications to econometrics*, the *bootstrap*, and *subsampling* techniques.

After the table of contents, a useful short *list of notation* allows rapid access to the main weak dependence coefficients and some useful notation.

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