

2

Heat Transfer

2.1. Introduction

2.1.1. *General Aspects*

The main object of this chapter is to give a brief account of the mathematical methods of determining the temperature distribution with time and position in packaged foods while being heated and cooled. This is a prerequisite to establishing a *process* which will ensure the microbiological safety of the product and is also organoleptically acceptable. This requires an examination of the modes of heat transfer in different parts of the processing operation.¹

2.1.2. *Mechanisms of Heat Transfer*

There are three modes of heat transfer, which contribute to the overall heat transfer process in differing proportions: conduction, convection and radiation. Conduction is the transfer of heat by molecular motion in solid bodies. Convection is the transfer of heat by fluid flow, created by density differences and buoyancy effects, in fluid products. Radiation is the transfer of electromagnetic energy between two bodies at different temperatures. In Figure 2.1 the main modes for heat transfer in the processing of packaged foods are illustrated.

The first mode is heat transfer to the container or packaging from the heating and cooling medium; the main modes of heat transfer to be considered for the various heating media are given in Table 2.1. Heating with pure steam, or microwaves, is very effective and does not present any appreciable resistance to heat transfer; consequently, it does not need to be taken into account in the overall heat transfer. In the case of all other media it is necessary to take the convective or radiative heat-transfer coefficient into account. Convective-heat transfer rates depend largely on the velocity of flow of the media over the container, and this is an important factor to be controlled in all processing operations. This subject is dealt with in more detail in Chapter 8.

¹ The reader is encouraged to consult the very useful text on heat transfer and food products by Hallström *et al.* (1988). This is an excellent guide to the basic principles of heat transfer and its application to food processing.

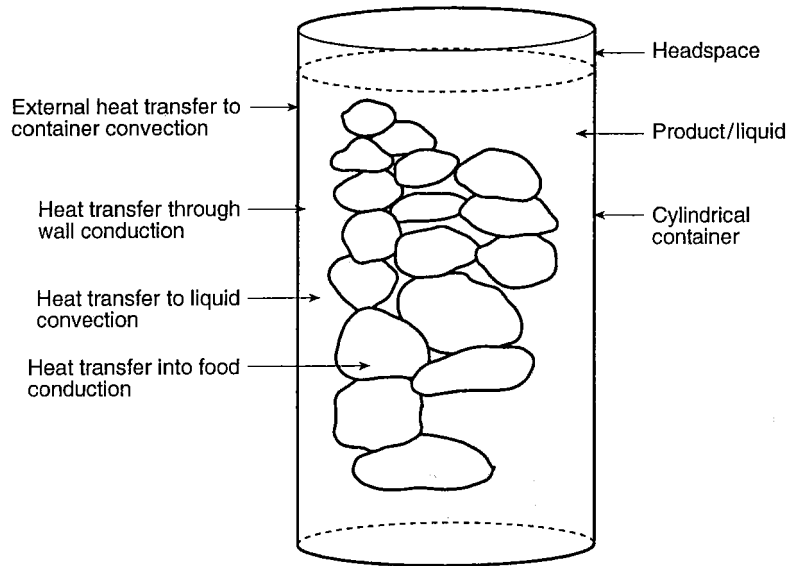


FIGURE 2.1. Heat transfer to food product in a cylindrical container.

The second mode of heat transfer is through the container wall; for metallic containers of normal thickness, the thickness and the thermal conductivity of the material are such that there is no appreciable resistance to heat transfer. However, for glass bottles and plastic containers there is a significant resistance, and this should be considered in determining the overall heat transfer resistance.

The third mode of heat transfer is into the product from the container wall; this depends on the consistency of the food material and is discussed in detail elsewhere (see Chapter 5). Fluid products or solid particulates covered with a fair amount of fluid heat or cool rapidly by convection, while other products of a more solid consistency heat mainly by conduction. In between there are products that heat/cool by a combination of conduction and convection, and some that start with convection heating and finish in conduction mode because of physico-chemical

TABLE 2.1. Heat transfer modes for containers being heated or cooled.

Media	Main mode	Resistance
Steam (air-free)	Condensation	Effectively none
Steam-air mixtures	Convection	Increases with increasing air content
Air	Convection	High
Water, boiling	Convection	Low
Water, hot	Convection	Decreases with increasing water velocity
Water, cold	Convection	Medium
Flame/infrared	Radiation	Low
Fluidized bed	Convection	Medium, depends on degree of agitation
Microwave	Radiation	None

changes. Thus the internal mechanisms of heat transfer are complicated. From a theoretical point of view it is only possible at the present time to deal with simple heat transfer mechanisms; however, empirical methods (see Chapter 5) allow the processor to calculate temperature distributions without being too concerned about the mechanism.

When dealing with heat transfer theory, it should be noted that a distinction is made between (a) steady-state heat transfer, which involves constant temperatures of heat transfer media, and the product, e.g. heating and cooling in continuous-flow heat exchangers; and (b) unsteady-state heat transfer, which implies that the temperatures are continuously changing. It is type (b) with which we are concerned in this book, i.e. the determination of time–temperature profiles at specified points in the container. From a practical point of view, a satisfactory process is determined at the slowest point of heating in the packaged food, and this makes calculation easier, since with conduction heating products, the center point of the food mass is taken as the slowest point of heating, or critical point. It is not sufficient in processing packaged foods just to achieve a given temperature at the slowest point of heating, but to achieve it for a given time, specified either by calculation or experimental investigation.

2.2. Heat Transfer by Conduction

2.2.1. Introduction

Energy transfer by conduction takes place when different parts of a solid body are at different temperatures. Energy flow in the form of heat takes place from the hotter, more energetic state, to the colder, less energetic state. The quantity of heat transferred under steady-state conditions is given by

$$Q = k \frac{T_1 - T_2}{x} At, \quad (2.1)$$

where

Q = quantity of heat (J or N m);

T = temperature (K or °C), with subscripts 1, 2 referring to the two parts of the body;

t = time (s);

x = the distance (m) of separation of the two points;

A = the cross-sectional area (m²) for heat flow;

k = the thermal conductivity (Wm⁻¹ K⁻¹).

Differentiating with respect to time gives the rate of heat flow:

$$\frac{dQ}{dt} = k \frac{T_1 - T_2}{x} A, \quad (2.2)$$

This equation can be written more simply in a differential form

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}. \quad (2.3)$$

This relates the rate of heat flow dQ/dt to the temperature gradient in the material dT/dx , and is known as the one-dimensional heat conduction equation expressed in Cartesian coordinates. The quantity $(dQ/dt)/A$ is known as the heat flux, and is measured in joules per square meter per second.

2.2.2. Formulation of Problems Involving Conduction Heat Transfer

The main object of this section is to indicate the mathematical basis of the problems encountered in the determination of the temperature distribution in heating canned foods by conduction. The treatment is necessarily brief, and further information can be found in the standard texts, e.g. Ingersoll, Zobel and Ingersoll (1953), Carslaw and Jaeger (1959), Arpaci (1966), Luikov (1968), and Ozisik (1980).

The basis of all unsteady-state conduction heat transfer equations is Fourier's equation, established by the French physicist Jean Baptiste Joseph Fourier (1768–1830) (Fourier 1822) and written as

$$\rho c \frac{\partial T}{\partial t} = \nabla k \nabla T \quad (2.4)$$

where ρ is the density (kg m^{-3}), c the specific heat or heat capacity ($\text{J kg}^{-1} \text{K}^{-1}$) and ∇ the differential operator (del, also known as nabla), where

$$\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z.$$

Equation (2.4) implies that the thermal conductivity is a function of temperature, an assumption which is not usually made in heat transfer calculations in order to simplify the calculations. Consequently, a simpler equation is generally used,

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

or

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \quad (2.5)$$

where α is the thermal diffusivity, $k/\rho c$ ($\text{m}^2 \text{s}^{-1}$) and ∇^2 is the Laplace operator, given by

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2.$$

The physical significance of this property is associated with the speed of heat propagation into the solid. Materials with high values, such as metals, heat rapidly, whereas food materials and water have comparatively low values and heat much more slowly. Table 2.2 gives some data for food products. More

TABLE 2.2. Some values of thermal diffusivities of various products.

Product	Temperature (°C)	Thermal diffusivity ($\times 10^7 \text{ m}^2 \text{ s}^{-1}$)	Reference
(i) Food products			
Apple sauce	105	1.61	Uno and Hayakawa (1980)
Apple pulp: Golden Delicious	29	1.50–1.62	Bhowmik and Hayakawa (1979)
Cherry tomato pulp	26	1.46–1.50	Bhowmik and Hayakawa (1979)
Tomato ketchup	—	1.20 ± 0.02	Gouvaris and Scholefield (1988)
Tomato: Ace var.	42.9	1.22–1.88	Hayakawa and Succar (1983)
Carrots	138	1.82–1.88	Chang and Toledo (1990)
Pea purée	—	1.59	Bhowmik and Tandon (1987)
Pea purée	—	1.54	Lenz and Lund (1977)
Pea & potato purée	—	1.48	Masilov and Medvedev (1967)
Potato purée	—	1.30 ± 0.04	Gouvaris and Scholefield (1988)
Potato (78% water)	60–100	1.39–1.46	Tung <i>et al.</i> (1989)
Potato	42.9	1.42–1.96	Hayakawa and Succar (1983)
French bean & chicken purée	—	1.62	Patkai <i>et al.</i> (1990)
Mixed vegetables & beef purée	—	1.63	Patkai <i>et al.</i> (1990)
Tuna fish/oil	115	1.64	Banga <i>et al.</i> (1993)
Mushrooms in brine	—	1.18	Akterian (1995)
Ham, processed	—	0.94	Smith <i>et al.</i> (1967)
Ham salami	—	1.52	Choi and Okos (1986)
Beef purée	—	1.75	Lenz and Lund (1977)
Meat hash	—	1.52	Choi and Okos (1986)
Meat sauce	69–112	1.46 ± 0.05	Olivares <i>et al.</i> (1986)
Meat croquette	59–115	1.98 ± 0.22	Olivares <i>et al.</i> (1986)
Meat, ground	20	1.26–1.82	Tung <i>et al.</i> (1989)
Pork purée	—	1.94	Lenz and Lund (1977)
Meat/tomatoes/potatoes	65–106	1.57 ± 0.20	Olivares <i>et al.</i> (1986)
Meat/potatoes/carrots	58–113	1.77 ± 0.15	Olivares <i>et al.</i> (1986)
Cooked chickpeas/pork sausages	71–114	1.90 ± 0.03	Olivares <i>et al.</i> (1986)
Chicken & rice	65–113	1.93 ± 0.21	Olivares <i>et al.</i> (1986)
Chicken/potatoes/carrots	72–109	1.70 ± 0.03	Olivares <i>et al.</i> (1986)
Lasagne (73.6% water)	60–100	1.32–1.70	Tung (1989)
Water	0–100	1.338–1.713	Evans (1958)
(ii) Simulants			
Acrylic plastic ellipsoids	—	1.19	Smith <i>et al.</i> (1967)
Ammonium chloride	40–100	1.53–1.47	Tung <i>et al.</i> (1989)
Agar-starch/water gels 3–3.5%	40–60	1.38–1.25	Tung <i>et al.</i> (1989)
Agar-water 5%	54	1.53	Evans (1958)
Bean-bentonite 75% water	115.6	1.72	Evans (1958)
Bentonite 10 Bentonite 10%	120°C	1.77	Uno and Hayakawa (1980)
Bentonite 10%	—	1.90	Bhomik and Tandon (1987)
Bentonite	—	1.86	Peterson and Adams (1983)
Ethylene glycol/water/agar 5%	—	1.11	Evans (1958)
(iii) Container materials			
Polypropylene (PP)		0.071	Shin & Bhowmik (1990, 1993)
Polycarbonate		0.013	Shin & Bhowmik (1990, 1993)
Polyvinylidene chloride (PVDC)		0.062	Shin & Bhowmik (1990, 1993)
Laminate (PP:PVDC:PP)		0.068	Shin & Bhowmik (1990, 1993)

extensive data will be found in the publications of Singh (1982), Okos (1986), Lewis (1987), George (1990), and Eszes and Rajkó (2004). The determination of physical properties from thermometric measurements and a finite element model has been reported by Nahor *et al.* (2001). A computer program, COS-THERM, was developed to predict the thermal properties of food products based on their composition (Beek & Veerkamp 1982; Miles *et al.* 1983). Many foods of high moisture content have values of α ranging from 1.4 to $1.6 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$. Palazoglu (2006) has reported an interesting study on the effect of convective heat transfer on the heating rate of materials with differing thermal diffusivities including cubic particles of potato and a polymethylpentene polymer. Using the analytical solution for heating a cube with external heat transfer it was shown that the rate of heating depended very much on the combination of heat-transfer coefficient and the thermal conductivity.

Equation (2.5) can be expressed in a variety of forms depending upon the coordinate system being used. Cartesian coordinates x, y, z – are used for heat transfer in flat plates (equation (2.4)), including slabs where the length is greater than the width, e.g. food in flexible pouches and trays, and for rectangular parallelepipeds or bricks (equation (2.6)), e.g. rectangular-shaped containers both metallic and plastic:

$$\frac{dT}{dt} = \alpha \left[\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} \right]. \quad (2.6)$$

Cylindrical coordinates – $x = r \cos b$, $y = r \sin b$, and z – where b is the angle and r the radius for transformation from a Cartesian coordinate system, are used for all containers with a cylindrical geometry, i.e. most canned foods. When transformed the previous equation becomes

$$\frac{dT}{dt} = \alpha \left[\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{1}{r^2} \frac{d^2T}{db^2} + \frac{d^2T}{dz^2} \right]. \quad (2.7)$$

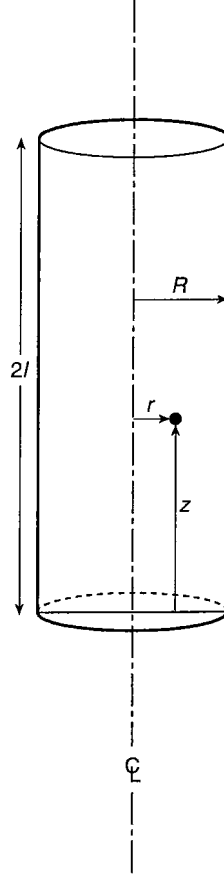
For radial flow of heat, i.e. neglecting axial heat flow, the last two terms may be neglected, so that the basic equation to be solved for radial heat transfer into a cylindrical container is (Figure 2.2):

$$\frac{dT}{dt} = \alpha \left[\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right]. \quad (2.8)$$

If the temperature is only required at the point of slowest heating, i.e. the center, where at $r = 0$, $dT/dr = 0$ and $dT/r dr = d^2T/dr^2$ (see Smith 1974), then equation (2.8) simplifies for the purposes of computation to

$$\frac{dT}{dt} = 2\alpha \left[\frac{d^2T}{dr^2} \right]. \quad (2.9)$$

While there are no containers that approximate to a spherical shape, spherical coordinates – $x = r \cos a \cos b$, $y = r \cos a \sin b$ and $z = r \sin a$ – are useful for predicting the temperature distribution in spherical-shaped food particulates, e.g.

FIGURE 2.2. Coordinate system for a cylindrical can of height $2l$ and diameter $2R$.

canned potatoes in brine. The basic equation (2.5) in spherical coordinates is

$$\frac{1}{\alpha} \frac{dT}{dt} = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{1}{r^2 \sin b} \frac{d}{db} \left(\sin b \frac{dT}{db} \right) + \frac{1}{r^2 \sin^2 b} \frac{d^2 T}{da^2}. \quad (2.10)$$

If the temperature is only required in the radial direction, the angular terms can be neglected and equation (2.10) may be simplified to give

$$\frac{1}{\alpha} \frac{dT}{dt} = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right)$$

or

$$\frac{dT}{dt} = \alpha \left[\frac{d^2 T}{dr^2} + \frac{2}{r} \frac{dT}{dr} \right]. \quad (2.11)$$

For the central point only, $r = 0$, the equation becomes

$$\frac{dT}{dt} = 3\alpha \left[\frac{d^2T}{dr^2} \right]. \quad (2.12)$$

In all these cases the problems have been simplified; for more complicated cases the reader is referred to the texts previously mentioned.

A full treatment of the term $\nabla \cdot k \nabla$ is given by Bird *et al.* (1960); for details of the equations in cylindrical and spherical coordinates, see Ruckenstein (1971).

2.2.3. Initial and Boundary Conditions

Temperature representations are often expressed simply as T . However, a more formal method is to give the coordinates of space and time in brackets. Thus a simple one-dimensional temperature distribution would be represented as $T(x, t)$ or $T(r, t)$, two-dimensional distributions as $T(x, y, t)$ and three-dimensional distributions as $T(x, y, z, t)$. For center distributions, where $x = y = z = r = 0$, $T(0, t)$ is used, and for conditions at time zero $T(x, 0)$. Since it is usually obvious what is intended from the context of the equations, this practice is often dispensed with. It will be used in the following discussion where appropriate.

There are two *initial* conditions that may apply to a particular problem:

1. The contents of the container are initially at a uniform temperature T_0 throughout, which is expressed as follows:

$$T = T_0 \quad \text{at} \quad t = 0 \quad \text{or} \quad T = T(x, y, z, 0).$$

In good canning practice this condition should be achieved, and in calculations it is nearly always assumed.

2. The contents of the container have an initial temperature distribution in space. This is usually expressed as follows:

$$T = f(x) \quad \text{at} \quad t = 0,$$

or in other suitable ways, e.g.

$$T(x, y, z, 0) = f(x, y, z) \quad T(r, 0) = f(r),$$

where $f(x)$ is some function of x . This initial condition is used at the beginning of the cooling period for canned products that have not achieved a uniform temperature distribution at the end of the heating period. It usually applies to large container sizes with conduction-heating products.

Other conditions that have to be taken into account for solving the heat transfer equations are the *boundary* or *end* conditions, the conditions to which the can is exposed during processing. The following boundary conditions are encountered in heat transfer work:

1. The surface temperature is prescribed and does not vary with time, i.e. a surface is exposed to an instantaneous change in temperature. This is referred to as a boundary condition of the first kind by some workers. It applies to steam

heating and is often assumed in heat transfer modelling work. It is the simplest condition to apply and is expressed as

$$dT(x, t)/dx = 0,$$

where x represents the external can diameter, or

$$T(x, t) = \text{constant}.$$

2. The surface temperature is governed by a convective heat coefficient, often referred to as a boundary condition of the third kind. Such a condition applies to cases where the heating medium is not condensing steam, e.g. hot water, a steam–air mixture or a cooling fluid. The surface temperature in these cases depends on the heat-transfer coefficient, which in turn depends on the velocity of the fluid over the surface (see section 2.3). This condition is expressed as follows:

$$-dT(x, t)/dx + h[T_R - T(x, t)] = 0. \quad (2.13)$$

3. The surface temperature is a function of time, i.e. the heating medium heats or cools while the containers are being processed. Three specific cases are used to illustrate this condition:

(a) Retort temperature change is a *linear function* with time: for example,

$$T_R(t) = T_0 + bt, \quad (2.14)$$

where T_0 is the initial temperature, T_R is the processing medium temperature, i.e. retort temperature, and b is a constant depending upon the magnitude of the gradient.

(b) Retort temperature change is an *exponential function* of time:

$$T_R(t) = T_\infty - (T_\infty - T_0)e^{-kt}, \quad (2.15)$$

where T_∞ is the maximum temperature reached and k a constant. This applies to the initial heating period of cans when placed in a static retort.

(c) Retort temperature a *harmonic function* of time:

$$T_R(t) = T_\infty \cos(2\pi nt), \quad (2.16)$$

where n is the frequency of oscillation.

2.2.4. Mean or Volume Average Temperatures

It is necessary to know the exact temperature distribution inside packaged foods in order to calculate the sterilization value; however, there are circumstances in which mass-average temperatures are appropriate – in particular, the determination of a heat-vulnerable component, e.g. a vitamin; for determining some cooling processes; and for determining energy changes.

The average temperature will be signified by putting a bar over the temperature term, thus the volume-average temperature is given by

$$\bar{T}(t) = \frac{1}{V} \int_0^V T(x, t) dV, \quad (2.17)$$

where V is the volume and dV is the volume element. So for a slab of dimensions $2x, 2y, 2z$, we have

$$\bar{T}(t) = \frac{1}{2x2y2z} \int_{-x}^x \int_{-y}^y \int_{-z}^z T(x, y, z, t) dx, dy, dz; \quad (2.18)$$

for a one-dimensional slab thickness $2x$:

$$\bar{T}(t) = \frac{1}{2x} \int_{-x}^x T(x, t) dx = \frac{1}{x} \int_0^x T(x, t) dx; \quad (2.19)$$

for a sphere of radius R at any point r :

$$\bar{T}(t) = \frac{3}{R^3} \int_0^R r^2 T(r, t) dr; \quad (2.20)$$

and for a cylinder of radius R :

$$\bar{T}(t) = \frac{2}{R^2} \int_0^R r^2 T(r, t) dr. \quad (2.21)$$

The mean temperature may also be used to determine the energy changes in a processing system:

$$Q = c(\bar{T}(t) - T_0) \quad (2.22)$$

where Q is the amount of energy supplied during time t , c is the specific heat and T_0 is the initial uniform temperature.

2.2.5. Summary of Basic Requirements

The following points need to be considered when attempting to formulate a model to predict the temperature distribution in a packaged food product which is being heated and cooled:

1. Is the product isotropic, i.e. does it have properties the same in all directions? If not, use k_x, k_y, k_z .
2. Do the physical properties vary with temperature, or any other prevailing condition? If so, then use $k(T)$.
3. Is the product at a uniform initial temperature? If not, use $T = f(x, y, z)$.
4. Is the product heated uniformly on all sides? Is the headspace taken into account? See special equations in section 2.5.
5. Does the container or package change shape during the processing? If so, use appropriate dimensions.
6. Is it necessary to consider the resistance to heat transfer through the container wall? If so, use x_w/k_w .

7. Does the heating or cooling medium impose a low heat-transfer coefficient? If so, use heat-transfer boundary condition equation (2.12).
8. Is the surface temperature variable? If so, use $T_R(t)$ as in equations (2.13) and (2.14).

A general rule for proceeding is to apply a simple model first, usually in one dimension, and then a more complex model if the predictions are not in agreement with the experimental results. For many practical factory applications simple models suffice.

2.2.6. Some Analytical Methods for Solving the Equations

There are many methods for solving partial differential equations, and it will suffice here to mention some of those that have been used by researchers in this subject without going into any detail. The first group are the analytical methods and the functions that they use.

2.2.6.1. Method of Separation of Variables

This method assumes that the solution to the partial differential equation, e.g. the simplest one-dimensional unsteady-state equation for the temperature distribution in a slab equation,

$$\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2}, \quad (2.23)$$

can be represented as the product of a spatial function $X(x)$ and a time function $T(t)$, viz.

$$T(x, t) = X(x) \cdot T(t). \quad (2.24)$$

Substituting the differentiated forms of (2.24) in (2.23) and separating the variables on either side of the equation results in:

$$\frac{1}{\alpha} \frac{\partial T}{T \partial t} = \frac{1}{X} \frac{\partial^2 X}{\partial x^2}. \quad (2.25)$$

Putting each side equal to a constant, e.g. $-b^2$, it is possible to obtain solutions for $T(t)$ and $X(x)$, viz.

$$T(t) = A e^{-b^2 t}$$

$$X(x) = B \cos(bx) + C \sin(bx).$$

Using equation (2.23), the general solution becomes

$$T(x, t) = [D \cos(bx) + E \sin(bx)] e^{-b^2 t}. \quad (2.26)$$

Using the initial and boundary conditions, a particular solution can then be found for the problem. This involves the use of Fourier sine series, which is discussed in 2.2.6.3.

2.2.6.2. Operational Methods: Integral Transforms and the Laplace Transform

In this method each term of the differential equation is multiplied by $e^{-\beta t}$ and integrated with respect to time from $t = 0$ to $t = \infty$. For the one-dimensional slab this results in

$$\int e^{-\beta t} \cdot \frac{\partial T}{\partial t} \cdot dt = \alpha \int e^{-\beta t} \frac{\partial^2 T}{\partial x^2} \cdot dt. \quad (2.27)$$

The Laplace transform $L[f(t)]$ is defined by

$$f(s) = L[f(t)] = \int f(t)e^{-st} dt, \quad (2.28)$$

and equation (2.27) may be written as

$$L\left[\frac{\partial T}{\partial t}(x, t)\right] = \alpha L\left[\frac{\partial^2 T}{\partial x^2}(x, t)\right]. \quad (2.29)$$

By integrating the terms in equation (2.27) from 0 to ∞ or from standard tables, the partial differential equation is transformed to a second-order differential equation, viz.

$$\alpha \frac{d^2 T}{dx^2}(x, s) - sT(x, s) = 0, \quad (2.30)$$

which has solution

$$T(x, s) = A \cosh\left(\frac{s}{\alpha}\right)^{1/2} + B \sinh\left(\frac{s}{\alpha}\right)^{1/2}. \quad (2.31)$$

This can be transformed back to the original system using the reverse transformation $f(t) = L^{-1}[f(s)]$, resulting in the Fourier series solution.

Tables of transforms are found in all standard texts on heat transfer, e.g. Carslaw and Jaeger (1959), Luikov (1968), Mickley *et al.* (1957), and Ozisik (1980).

Hayakawa (1964) made elegant use of integral transforms for handling complex boundary conditions experienced in standard canning operations. Various other types of transform are available for handling different situations. The Fourier sine transform is useful for dealing with situations with a prescribed boundary condition and the Hankel transform for dealing with cylindrical geometry (Magnus *et al.* 1966). The main problem with the integral transform method is finding the reverse function to convert the solution of the derived differential equation.

2.2.6.3. Some Special Transcendental Functions Involved in the Solution of Differential Equations

Fourier series. Many of the analytical solutions to heat transfer problems involve the use of series, in particular series of trigonometrical functions. For example, a relationship $y = f(x)$ may be represented, between the limits of $x = 0$ and $x = \pi$, as follows:

$$y = f(x) = a_1 \sin(x) + a_2 \sin(2x) + \dots + a_n \sin(nx) \quad (2.32)$$

or

$$y = f(x) = a_n \sin(nx)$$

and

$$n = \frac{2}{\pi} \int f(x) \sin(mx) dx$$

If $f(x)$ is a constant, e.g. initial temperature T_0 , then

$$a_n = \frac{2}{\pi} T_0 \int \sin(mx) dx = \frac{2}{\pi n} T_0 [1 - (-1)^n]$$

If n is even, then $a_n = 0$, and if n is odd, $a_n = 4T_0/\pi n$ and

$$f(x) = 4T_0 \left(\frac{\sin(x)}{1} + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \dots \right). \quad (2.33)$$

Similar Fourier series are available with cosine terms. The standard analytical solutions for simple slab geometry involve combinations of sine and cosine series. For ease of computation it is essential that the series converge rapidly: In many cases a first-term approximation is satisfactory, especially where long times are involved.

Bessel functions. For problems involving cylindrical geometry – in particular, food and drink cans – the linear second-order equation representing the temperature distribution with time and space is known as Bessel's equation. The analytical solution of this equation requires the use of Bessel functions designated $J_v(x)$, where v is the order.

Bessel functions are defined by

$$J_v(x) = \frac{x^v}{2^v v!} \left[1 - \frac{x^2}{2(2v+2)} + \frac{x^4}{2 \times 4(2v+2)(2v+4)} - \dots \right]. \quad (2.34)$$

When $v = 0$ the function $J_0(x)$ is known as a Bessel function of the first kind and of order zero, i.e.

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \quad (2.35)$$

$J_0(x_n)$ is a continuous function of x and cuts the $y = 0$ axis at various points known as the roots, x_n :

Root number n	1	2	3	4	5
Roots of $J_0(x_n)$	2.405	5.520	8.654	11.792	14.931
Roots of $J_1(x_n)$	3.832	7.015	10.173	13.324	16.470

The first differential of $J_0(x)$ is $-J_1(x)$, known as a Bessel function of the first kind and of the first order.

Bessel functions may be calculated using the following equations:

$$J_0(x) = A \cos(x - 0.25\pi) \quad (2.36a)$$

$$J_1(x) = A \cos(x - 0.75\pi), \quad (2.36b)$$

where $A = (2/\pi x)^{1/2}$.

2.2.6.4. Duhamel's Theorem

A useful method of dealing with time-dependent boundary conditions, e.g. ramp and exponential functions, is to use Duhamel's theorem to convert the step-response solution to the required solution. This method has been widely used in the solution of canning problems (Riedel 1947; Gillespy 1953; Hayakawa 1964; Hayakawa & Giannoni-Succar 1996).

If the step-function temperature distribution is given by $\Phi(t)$ and the required solution for a time-dependent boundary condition $\Theta(t)$, then Duhamel's theorem states that the relationship between the two is given by

$$\Theta(t) = \int_0^t f(t) \cdot \frac{\partial \Phi}{\partial t}(x, t - \tau) d\tau, \quad (2.37)$$

where $f(t)$ is the temperature distribution equation for the time-dependent boundary condition and τ is the time limit for integration.

Duhamel's theorem has also been used in a direct approach to decoupling temperature data from specific boundary conditions, in order to predict data for different experimental conditions. A theoretical inverse superposition solution for the calculation of internal product temperatures in containers in retorts subjected to varying retort temperature profiles (Stoforos *et al.* 1997).

2.2.7. Some Numerical Techniques of Solution

2.2.7.1. Introduction

Many of the mathematical models for heat transfer into cylindrical containers have complex boundary conditions which do not permit simple analytical solutions to be obtained in a form which can easily be manipulated. Consequently, numerical methods have been developed and are now extensively used because of their suitability for modern computing. They require neither the solution of complex transcendental equations nor the functions outlined above. These methods are based on iterative estimations of temperatures using approximate methods. It is not possible to obtain directly solutions that show the interrelationship of the variables, and the solutions are essentially in the form of time-temperature data. In view of the fact that the temperature distributions obtained are used to determine the achieved lethality, it is necessary to check that the method used is of sufficient accuracy to prevent a sub-lethal process being recommended. There is always a possibility of cumulative error.

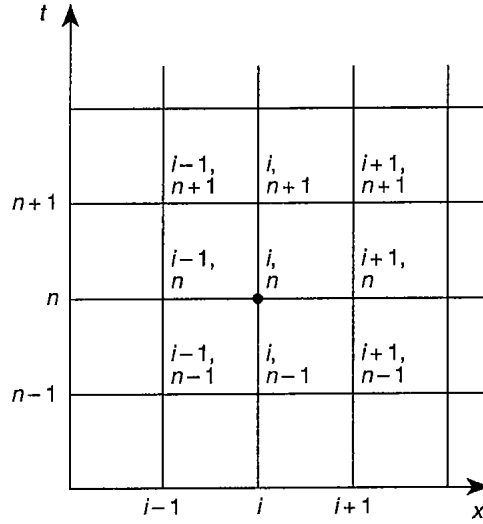


FIGURE 2.3. Temperature nodal points on space-time grid.

2.2.7.2. Finite-Difference Approximation Method

In the finite-difference methods the derivative functions are replaced by approximate values expressed by values of a function at certain discrete points known as “nodal points.” The result of this operation is to produce an equivalent finite-difference equation which may be solved by simple algebraic or arithmetic manipulation.

For unsteady-state heat transfer it is necessary to construct a space-time grid (see Figure 2.3) in which the temperatures at the nodal points are defined in terms of time (t) and space (x) coordinates.

Considering the basic one-dimensional heat transfer equation:

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}. \quad (2.38)$$

Using a Taylor series expansion the value of $T(x, t)$ may be expressed as

$$T_{(x, t+\Delta t)} = T_{(x, t)} + \Delta t \cdot \frac{\partial T}{\partial t} + \frac{\Delta t^2}{2} \cdot \frac{\partial^2 T}{\partial t^2} + \frac{\Delta t^3}{3} \cdot \frac{\partial^3 T}{\partial t^3}. \quad (2.39)$$

If the increment is sufficiently small then the terms higher than T may be neglected, thus

$$\frac{\partial T}{\partial t} = \frac{T_{(x, t+\Delta t)} - T_{(x, t)}}{\Delta t} = \frac{T_{i, n+1} - T_{i, n}}{\Delta t}. \quad (2.40)$$

Two series are required for $\partial^2 T / \partial x^2$: these are the expansion of $T_{(x+\Delta x, t)}$ and $T_{(x-\Delta x, t)}$, from which

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{(x+\Delta x, t)} - 2T_{(x, t)} + T_{(x-\Delta x, t)}}{\Delta x^2} = \frac{T_{i+1, n} - T_{i, n} + T_{i-1, n}}{\Delta x^2}. \quad (2.41)$$

Equation (2.39) then becomes

$$T_{i, n+1} = T_{i, n} + \frac{\alpha \Delta t}{\Delta x^2} [T_{i+1, n} - 2T_{i, n} + T_{i-1, n}]. \quad (2.42)$$

For a better approximation the term $T_{(x, t+2\Delta T)}$ can be expanded in the same way.

Two important aspects of this method of solution are the convergence and the stability criteria, without which full confidence in the solution cannot be guaranteed. Convergence implies that the finite-difference solution will reduce to the exact solution when the size increments, e.g. Δx and ΔT , are infinitesimally small. Stability implies that the errors associated with the use of increments of finite size, round-off errors and numerical mistakes will not increase as the calculations proceed. Tests for these are given in the standard books on numerical analysis.

This method is usually referred to as the *explicit* method. The time step t has to be kept very small and the method is only valid for $0 < t / \Delta x^2 \leq \frac{1}{2}$, i.e. $\Delta t \leq \frac{1}{2} \Delta x^2$, and Δx has to be kept small in order to obtain reasonable accuracy.

A more suitable, but computationally more demanding, method is the *implicit* method in which the nodal points are replaced by the values taken at $n+1$ rather than n , although this results in three unknown temperatures, and N equations, where N is the number of spatial nodes representing the conductive body. The nodal temperatures are obtained by simultaneous solution of the equations. The basic equation for the relationship between the nodal points is given by

$$T_{i, n+1} = T_{i, n} + \frac{\alpha \Delta t}{\Delta x^2} [T_{i+1, n+1} - 2T_{i, n+1} + T_{i-1, n+1}]. \quad (2.43)$$

Several other methods, which are intermediate between the explicit and implicit methods, have been developed, e.g. the Crank–Nicholson method, Jacobi method, Gauss–Seidel method and “over-relaxation” methods. Discussions of the application finite-difference methods are available in a large number of text-books: See Smith (1974), Adams and Rogers (1973), Croft and Lilley (1977), Carnahan *et al.* (1969), and Minkowycz *et al.* (1988).

The finite-difference technique has been applied to a wide range of canning problems (see Tables 2.3, 2.4 and 2.5). Tucker and Badley (1990) have developed a commercial center temperature prediction program for heat sterilization processes, known as CTemp.

Welt *et al.* (1997) have developed a no-capacitance surface node NCSN procedure for heat transfer simulation and, which can be used for process design. This method when used with simulation steps of 10 s was found to provide a better fit to the experimental data compared with capacitance surface node technique (CNS).

TABLE 2.3. Some conduction-heating models for predicting temperatures in cylindrical cans of food.

Form of solution	Product/container	Process conditions	Comments	Reference
Analytical equations	Geometrical objects	Linear surface heating & step-change	Any position & center	Williamson and Adams (1919)
Analytical equations	Metal cans & glass jars, fruit & vegetables	Linear surface heating & step-change	Determination of effective thermal diffusivity	Thompson (1919)
Analytical equations	Cans: fruit & vegetables	Variable surface temperature	Duhamel's theorem	Thompson (1922)
Analytical equations	Cans: various fish	Step-change	Thermal properties	Langstroth (1931)
Analytical equations	Cans: fish	Step-change	Based on Langstroth (1931) first-term approximation	Cooper (1937)
Analytical equations	Cans: fish	Step-change	Cooling	Tani (1938a)
Analytical equations	Cans: fish	Step-change	Based on Williamson and Adams (1919)	Okada (1940a)
Analytical equations	Cans: food	Step-change	Very useful method	Taggart and Farrow (1941, 1942)
Tables of numerical values	Geometrical objects	Step-change		Olson and Schultz (1942)
Analytical equations	Cans: food	Step-change & initial temperature distribution	Classical equation for cylindrical can	Olson and Jackson (1942)
Analytical equations	Cans: food	Various heating profiles	Duhamel's theorem	Riedel (1947)
Analytical equations	Cans: food	Step-change & initial temperature distribution	Heating & cooling Duhamel's theorem	Hicks (1951)
Analytical equations with numerical tables	Cans: food	Step-change & variable surface temperature	Duhamel's theorem	Gillespy (1951, 1953)
Analytical equations	Cans: meat	Step-change & initial temperature distribution	Heating & cooling derivation given	Hurwicz and Fischer (1952)
Analytical equations	Cans: food	Step-change & finite-surface heat-transfer coefficient	Effect of headspace on temperature distribution	Evans and Board (1954)
Analytical equations	Cans: food	Step-change	Hyperbolic secant	Jakobsen (1954)

Analytical equations	Cans: food	Variable surface temperature profiles	A major contribution to the theoretical analysis	Hayakawa (1964, 1969, 1970)
Analytical equations	Cans: food	Step-change	f_h/j analysis	Hayakawa and Ball (1968)
Analytical equations	Cans: food	Step-change	Heating & cooling	Hayakawa & Ball (1969a)
Analytical equations	Cans: food	Step-change	Cooling curve	Hayakawa & Ball (1969b)
Numerical solution	Cans: food	Step-change	2D finite-difference equation	Teixeira <i>et al.</i> (1969)
Analytical equations	Cans: food	Step-change	Jakobsen's equation (1954)	Shiga (1970)
Analytical equations	Cans: food	Variable surface temperature profiles	Based on Hayakawa (1964)	Hayakawa and Ball (1971)
Analytical equations	Cans: food	Variable surface temperature profiles	Duhamel's theorem	Hayakawa (1971)
Analytical equations	Cans: food	Step-change	Heating & cooling	Flambert & Deltour (1971, 1973a, b)
Analytical equations	Cans: food	Multiple step-changes	Duhamel's theorem	Wang <i>et al.</i> (1972)
Response charts	Cans: simulant & food	Variable surface temperature	Comparison with Gillespy (1953); mass average temps	Hayakawa (1974)
Analytical equations & response charts	Cans: food	Step-change	Central & average temperatures	Leonhardt (1976a, b)
Analytical & numerical solutions	Cans: food	Step-change	Lethality-Fourier number method	Lenz and Lund (1977)
Analytical equations	Cans: food	Variable surface temperature	f_h/j concept	Ikegami (1978)
Finite-element solution	Cans: model	Surface heat-transfer coefficient	Galerkin residual method of transform used	De Baerdemaeker <i>et al.</i> (1977)
Analytical equations	Cans: food	Step-change, center temperature	Second-order linear system	Skinner (1979)
Analytical equations	Cans: food	Surface heat-transfer coefficient	Simplified equations	Ramaswamy <i>et al.</i> (1982)
Analytical equations	Cans: food	Surface heat-transfer coefficients	Thermocouple errors	Larkin and Steffe (1982)
Finite-element solution	Glass jar: apple sauce, can: salmon	Surface heat-transfer coefficient	Cooling effects	Naveh <i>et al.</i> (1983b)
Analytical equations	Cans: food	Heating & cooling	Lethality-Fourier number method	Lund and Norback (1983)

(cont.)

TABLE 2.3. (continued)

Form of solution	Product/container	Process conditions	Comments	Reference
Analytical equations, computer programs	Geometrical objects	Heating & cooling	Based on Olson and Schultz (1942)	Newman and Holdsworth (1989)
Analytical equations	Cans: food	Surface heat-transfer coefficient	Cooling effects	Datta <i>et al.</i> (1984)
Analytical equations	Cans: food	Non-homogeneous food	Effective thermal diffusivity	Olivares <i>et al.</i> (1986)
Analytical solution	Cans: food	Surface heat-transfer coefficient	Effect of air on the can base coefficient	Tan and Ling (1988)
Finite difference	Cans: food	Surface heat-transfer coefficient	Applicable for process deviations	Mohamed (2003)
Numerical solution	Cans: food	Surface heat-transfer coefficient	Finite-difference model	Richardson and Holdsworth (1989)
Analytical equations, computer programs	Geometrical objects, cans: potato purée	Surface heat-transfer coefficient	Effect of l/d ratio	Thorne (1989)
Finite difference	Cans: simulant	Surface heat-transfer coefficient	Cooling	Tucker & Clark (1989, 1990)
Finite element	Cans: fish/oil	Step-change	Anisotropic model, solid & liquid layer	Perez-Martin <i>et al.</i> (1990)
Finite difference	Plastic: 8% bentonite	Wall resistance	ADI technique	Shin and Bhowmik (1990)
Finite difference	Cans: model	Surface heat-transfer coefficient	Step process	Silva <i>et al.</i> (1992)
Finite element	Cans: fish/oil	Step-change, solid & liquid layer	Anisotropic model	Banga <i>et al.</i> (1993)
Analytical equations	Cans: sea food	Surface heat-transfer coefficient	z-transfer function	Salvadori <i>et al.</i> (1994a, b)
Analytical	Model	Finite surface resistance	3D model	Cuesta and Lamu (1995)
Analytical equations	Cans: mashed potato	Step change	Linear recursive model	Lanoiselle <i>et al.</i> (1995); Chiheb <i>et al.</i> (1994)
Numerical	Cans: fruit/syrup	Variable boundary conditions	z-transfer function	Marquez <i>et al.</i> (1998)
Zone Modeling	Cans: baked beans	Heating & cooling	Uncertain data	Johns (1992a, b)
Zone Modeling	Cans	Heating & cooling	Simple model	Tucker <i>et al.</i> (1992)
Finite element	Cans: tomato concentrate	Surface heat-transfer coefficient	Stochastic boundary conditions	Nicolai & De Baerdemaeker (1992)
Finite element/Monte Carlo	AI can: tomato concentrate	Surface heat-transfer coefficient	Parameter fluctuations	Nicolai & De Baerdemaeker (1997); Nicolai <i>et al.</i> (1998)
Various	Cans: model	Various	Comparison of techniques	Noronha <i>et al.</i> (1995)

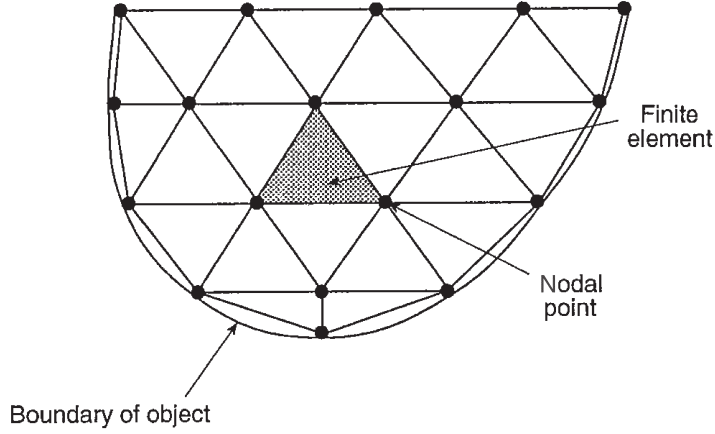


FIGURE 2.4. Discretization of an object into finite elements.

Chen and Ramaswamy (2002a, b, c) have also developed a method of modelling and optimization based neural networks and genetic algorithms.

2.2.7.3. The Finite-Element Method

With this method the body under investigation is divided up into an assembly of subdivisions called *elements* which are interconnected at *nodes* (see Figure 2.4). This stage is called *discretization*. Each element then has an equation governing the transfer of heat, and system equations are developed for the whole assembly. These take the form

$$q = kT \quad (2.44)$$

where k is a square matrix, known as the stiffness or conductance matrix, q is the vector of applied nodal forces, i.e. heat flows, and T is the vector of (unknown) nodal temperatures.

In the case of one-dimensional heat conduction, the heat in is given by

$$q_i = -\frac{kA}{L}(T_j - T_i)$$

and the heat out by

$$q_j = -\frac{kA}{L}(T_i - T_j)$$

which in matrix form is

$$\begin{bmatrix} q_i \\ q_j \end{bmatrix} = \frac{kA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ T_j \end{bmatrix}. \quad (2.45)$$

The stages in developing a finite-element model are as follows:

1. *Discretization.* For two-dimensional solid bodies that are axisymmetrical, triangular or rectangular elements may be used; whereas for three-dimensional objects, cubes or prisms may be used. In problems involving curved areas, shells of appropriate curved geometry are chosen.
2. *Size and Number of Elements.* These are inversely related: As the number of elements increases, the accuracy increases. It is generally more useful to have a higher density of mesh elements, especially where temperatures are changing rapidly. This requires careful planning at the outset of the analysis. It is important that all the nodes are connected at the end of the mesh.
3. *Location of Nodes.* It is essential that where there is a discontinuity in the material, the nodes should join the two areas.
4. *Node and Element Numbering.* Two different methods are used, either horizontal or vertical numbering for the nodes; the elements are given numbers in brackets.
5. *Method of Solution.* Most finite-element computer programs use wavefront analysis; however, the Gaussian elimination technique may also be used. The reader is recommended to consult the texts by Segerlind (1984) and Fagan (1992) for further information in relation to the application of the method. The ANSYS (1988) computer software package is very useful for problem-solving.

Relatively few applications of the technique to food processing problems have been reported. General discussions and overviews have been given by Singh and Segerlind (1974), De Baerdemaeker *et al.* (1977), Naveh (1982), Naveh *et al.* (1983), Puri and Anantheswaran (1993), and Nicolai *et al.* (2001).

The method has been applied to the heating of baby foods in glass jars (Naveh 1982) and the heating of irregular-shaped objects, e.g. canned mushrooms in brine (Sastry *et al.* 1985). The temperature distribution during the cooling of canned foods has been analyzed by Naveh *et al.* (1983b), and Nicolai and De Baerdemaeker (1992) have modelled the heat transfer into foods, with stochastic initial and final boundary conditions. Nicolai *et al.* (1995) have determined the temperature distribution in lasagne during heating.

2.2.7.4. Some Other Methods

Hendrickx (1988) applied transmission line matrix (TLM) modelling to food engineering problems. This method, like the finite-difference model, operates on a mesh structure, but the computation is not directly in terms of approximate field quantities at the nodes. The method operates on numbers, called pulses, which are incident upon and reflected from the nodes. The approximate temperatures at the nodes are expressed in terms of pulses. A pulse is injected into the network and the response of the system determined at the nodes.

For systems with complex boundary conditions, such as those found in canning operations, the equations have been solved using the response of the linear system to a disturbance in the system, in this case a triangular or double ramp pulse. The solution uses both Laplace transforms and *z*-transfer functions. The formal

solution of the heat-transfer equation for a finite cylinder with complex boundary conditions was derived by Salvadori *et al.* (1994). Márquez *et al.* (1998) have applied the technique to the study of a particulate/liquid system, viz., pasteurizing fruit in syrup.

2.2.8. Some Analytical Solutions of the Heat Transfer Equation

2.2.8.1. Simple Geometrical Shapes

The simplest cases are the temperature distributions in one dimension for an infinite slab, an infinite cylinder and a sphere. From the first, two more complex solutions can be obtained (see Section 2.2.8.2).

2.2.8.1.1. The Infinite Slab

The one-dimensional flow of heat in a slab is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (2.46)$$

(see equation (2.38)). The general form of the solution $F(t)$ is given by

$$T = T_R - (T_R - T_0)F(t), \quad (2.47)$$

where T is the temperature distribution at space coordinate x and time t , often written as $T(x, t)$; T_R is the retort temperature and T_0 is the initial temperature of the solid body, at time $t = 0$.

For a slab of thickness $2X$, the solution at any point is

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos\left((2n-1)\frac{\pi x}{2X}\right) e^{[-(2n-1)^2 \pi^2 \alpha t / 4X^2]}, \quad (2.48)$$

which, at the center, becomes

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} e^{[-(2n-1)^2 \pi^2 \alpha t / 4X^2]}. \quad (2.49)$$

It is often conveniently designated by $S(\theta)$, where θ is the dimensionless Fourier number $\alpha t / l^2$ where l is the thickness of the body, i.e. $2X$:

$$S(\theta) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} e^{[-(2n-1)^2 \pi^2 \theta]} \quad (2.50)$$

$$= \frac{4}{\pi} \left[e^{-\pi^2 \theta} - \frac{1}{3} e^{-9\pi^2 \theta} + \frac{1}{5} e^{-25\pi^2 \theta} - \dots \right]. \quad (2.51)$$

This subject is discussed in detail by Ingersoll *et al.* (1953), Olson and Schultz (1942), and Newman and Holdsworth (1989). The latter includes a range of useful computer programs in BASIC.

If the body has an initial temperature distribution $f(\lambda)$, then the temperature distribution is given by

$$T_R - T = \frac{2}{l} \sum_{n=1}^{\infty} (n\pi x/l) \int_0^{\lambda} f(\lambda) \sin(n\pi \lambda/l) d\lambda \cdot e^{-n^2 \pi^2 \theta}. \quad (2.52)$$

This equation ultimately reverts to (2.50), when $f(\lambda)$ is a uniform temperature $T_0 - T_R$.

If the body has a surface heat-transfer coefficient, h , then the temperature distribution is given by

$$F(t) = 2 \sum_{n=1}^{\infty} \frac{\sin M_n \cos(M_n(x/X))}{M_n + \sin M_n \cos M_n} e^{-M_n^2 \theta} \quad (2.53)$$

and for the center temperature, this becomes

$$F(t) = 2 \sum_{n=1}^{\infty} \frac{\sin M_n}{M_n + \sin M_n \cos M_n} e^{-M_n^2 \theta} \quad (2.54)$$

where M_n is obtained from the solution of $M_n = B_i \cot M_n$ and B_i is the Biot number hX/k . This is also known as the Nusselt number in heat-transfer correlations.

2.2.8.1.2. Infinite Cylinder

The basic one-dimensional heat transfer equation is given by

$$\frac{dT}{dt} = \alpha \left[\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right]. \quad (2.55)$$

The solution for a rod of radius $2a$, at any radial point r , for a constant retort temperature T_R and a uniform initial temperature distribution T_0 is given by

$$F(t) = \sum_{i=1}^n A_i J_0(R_i r/a) e^{-R_i^2 \theta}, \quad (2.56)$$

where $A_n = 2/[R_n J_1(R_n)]$, $J_0(x)$ is a Bessel function of zero order, $J_1(x)$ a Bessel function of the first order (see Section 2.2.6.3), R_n is the n th root of the characteristic equation $J_0(x) = 0$, and $\theta = \alpha t/a^2$ is the dimensionless Fourier number.

For the center point it is possible to define $C(\theta)$, similarly to $S(\theta)$ in equation (2.50):

$$C(\theta) = 2 \sum_{i=1}^n A_i e^{-R_i^2 \theta}, \quad (2.57)$$

i.e.

$$C(\theta) = 2[A_1 e^{-R_1^2 \theta} + A_2 e^{-R_2^2 \theta} + A_3 e^{-R_3^2 \theta} + \dots].$$

If the initial temperature distribution at time $t = 0$ is $f(r)$, then the temperature distribution after time t is given by

$$T_R - T = \sum_{i=1}^n A'_i J_0(R_i r/a) e^{-R_i^2 \theta} \cdot \int_0^r r f(r) J_0(R_n r/a) dr, \quad (2.58)$$

where $A'_n = 2/[a^2 J_1(R_n)]$.

If there is a finite surface heat coefficient on the outside, then the temperature distribution is given by

$$F(t) = A''_n J_0(R_n \cdot r/a) e^{-[R_n^2 \theta]}, \quad (2.59)$$

where

$$\begin{aligned} A''_n &= [2J_1(R_n)/R_n][J_0^2(R_n) + J_1^2(R_n)], \\ R_n J_1(R_n) &= J_0(R_n) Bi, \\ Bi &= hr/k. \end{aligned}$$

2.2.8.1.3. A Spherical Object

The basic heat transfer equation for determining the temperature distribution in a spherical object of radius a is given by

$$\frac{dT}{dt} = \alpha \left[\frac{d^2 T}{dr^2} + \frac{2}{r} \frac{dT}{dr} \right]. \quad (2.60)$$

The solution for the simplest case of a step function is given by

$$F(t) = (2a/\pi r) \sum_{n=1}^{\infty} (-1)^{n+1} \sin(n\pi r/a) e^{-n^2 \pi^2 \theta}. \quad (2.61)$$

where $\theta = \pi t/a^2$.

For the central temperature only the equation reduces to

$$F(t) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n^2 \pi^2 \theta}. \quad (2.62)$$

A function $B(x)$ is defined in terms of $F(t)$ for later use as follows:

$$F(t) = B(x), \quad (2.63)$$

where $x = \pi^2 \theta$.

For the case of external heat transfer the following equation is applicable:

$$F(t) = \frac{2r}{a} \sum_{n=1}^{\infty} \frac{\sin M_n - M_n \cos M_n}{M_n - \sin M_n - \cos M_n} \cdot \frac{\sin(M_n(r/a))}{M_n} e^{-[M_n^2 \theta]}, \quad (2.64)$$

where $\tan M_n = M_n/(1 - Bi)$.

2.2.8.2. More Complex Geometries

2.2.8.2.1. Rectangular Parallelepiped or Brick

The simple analytical solution for this case is obtained using the $S(\theta)$ function given in equation (2.50). The temperature distribution is given by

$$F(t) = S(\theta_x)S(\theta_y)S(\theta_z), \quad (2.65)$$

where $S(\theta_n) = \alpha t / n^2$ and n is the overall side dimension. For a cube of side a , the temperature is given by

$$F(t) = [S(\theta)]^3. \quad (2.66)$$

Applications of these formulae to various conditions for heating canned foods in rectangular metallic or plastic containers are presented in Table 2.4.

2.2.8.2.2. Finite Cylinder

The simple analytical solution for a cylinder of radius r and length l is given by

$$F(t) = S(\alpha t / l^2) C(\alpha t / r^2). \quad (2.67)$$

Tables for $S(\theta)$, $C(\theta)$ and $B(x)$ are given in Ingersoll *et al.* (1953), Olson and Schultz (1942), and Newman and Holdsworth (1989).

The complex analytical equations that are required to interpret heat penetration data are as follows. For a can with infinite surface heat-transfer coefficient and uniform initial temperature, radius a and length $2l$ (Cowell & Evans 1961)

$$F(t) = \frac{8}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^n}{2n+1} \cos(2n+1) \frac{\pi x}{2l} \frac{J_0(R_m r / a)}{R_m J_1(R_m)} e^{-\psi \alpha t}, \quad (2.68)$$

where

$$\psi = \frac{(2n+1)^2 \pi^2}{4l^2} + \frac{R_m^2}{a^2}.$$

At the center point

$$F(t) = \frac{8}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^n}{2n+1} \frac{1}{R_m J_1(R_m)} e^{-\psi \alpha t}. \quad (2.69)$$

A slightly different form of this equation is given by Hurwicz and Tischer (1952).

For a can with infinite surface heat-transfer coefficient and initial temperature distribution $f(r, \theta, z)$, radius a and height $2b$, where z is any point on the height axis and r is any point on the radial axis, the equation takes the form given by Ball and Olson (1957), based on Olson and Jackson (1942) (see also Carslaw & Jaeger 1959):

$$F(t) = \sum_{j=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [A_{jmn} \cos(n\theta) + B_{jmn} \sin(n\theta)]$$

$$\times J_n(R_j r/a) \sin\left(\frac{m\pi}{2b}(z+b)\right) e^{-\psi' \pi t}, \quad (2.70)$$

where

$$\psi' = \frac{m^2 \pi^2}{b^2} + \frac{R_j^2}{a^2}$$

and A_{jmn} and B_{jmn} are factors depending upon the initial temperature distribution. For the center point $r = 0$

$$F(t) = A_{110} \sin\left(\frac{\pi(z+b)}{2b}\right) J_0(R_1 r/a) e^{-\psi'' \alpha t}, \quad (2.71)$$

where

$$\psi'' = \frac{\pi^2}{4b^2} + \frac{R_1^2}{a^2}$$

and

$$\begin{aligned} A_{110} = & \frac{2}{\pi^2 a^2 l [J'_0(R_n)]^2} \int_0^a R J_0(R_1 r/a) dr \int_{-b}^{+b} \sin\left(\frac{\pi(z+b)}{2b}\right) dz \\ & \times \int_{-\pi}^{+\pi} (\cos \theta) f(r, \theta, z) d\theta. \end{aligned} \quad (2.72)$$

For a can with finite surface heat-transfer coefficient and an initial constant temperature,

$$F(t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{n,1} A_{m,2} J_0(R_{n,1} r/a) \cos(R_{m,2} z/l) e^{-\psi''' \alpha t}, \quad (2.73)$$

where

$$A_{n,1} = \frac{2Bi_1}{J_0(R_{n,1})(R_{n,1}^2 + Bi_1^2)}$$

(for tables, see Luikov 1968: 273),

$$A_{m,2} = (-1)^{n+1} \frac{2Bi_2(Bi_2^2 + R_{m,2}^2)^{1/2}}{R_{m,2}(Bi_2^2 + Bi_2 + R_{m,2}^2)}$$

(for tables, see Luikov 1968: 224), $R_{n,1}$ and $R_{m,2}$ are the roots of the corresponding characteristic equations (for extended tables, see Hayakawa 1975; Peralta Rodriguez 1987b), and

$$\psi''' = \frac{R_{n,1}^2}{a^2} + \frac{R_{m,2}^2}{l^2}.$$

For cans with other boundary conditions, Table 2.4 lists a number of models which have been used to determine the temperature distribution in canned foods with a wide range of boundary conditions. The effect of variable surface temperature profiles, including various combinations of exponential and linear heating profiles

TABLE 2.4. Some conduction-heating models for predicting temperatures in slab- or brick-shaped containers of food.

Form of solution	Product/container	Process conditions	Comments	Reference
Analytical equations	Cans: fish	Finite surface resistance	3D model	Okada (1940c)
Analytical equations	Cans: fish	Thermal conductivity varies with temperature	ID nonlinear model	Fujita (1952)
Analytical equations	Pouches: food	Finite surface resistance	3D model	Chapman and McKernan (1963)
Analytical/numerical	Model	Time-variable boundary conditions	ID model	Mirespassi (1965)
Finite difference	Model	Step-change	3D model	Manson <i>et al.</i> (1970)
Analytical equations	Model	Step-change	3D model	Alles and Cowell (1971)
Analytical equations	Model	Centre & average temperatures	3D Model	Leonhardt (1976a, b)
Analytical equations	Model	Finite surface resistance	ID non-symmetric model	Uno and Hayakawa (1979)
Analytical equations	Pouches: simulant	Step-change	3D model	Castillo <i>et al.</i> (1980)
Analytical equations	Metal brick	Retort profile	3D model	Ramaswamy <i>et al.</i> (1983)
Finite difference	Pouches: bentonite	Finite surface resistance	3D model	McGinnis (1986)
Finite difference	Pouches: bentonite	Finite surface resistance	3D model	Bhowmik and Tandon (1987)
Finite difference	Cans: food	Finite surface resistance & wall thickness	3D model	Tucker & Holdsworth (1991a, b)
Finite difference	Model	Ramp/hold/step-cool	ID model	Hendrickx <i>et al.</i> (1993)
Analytical equations	Model	Finite surface resistance	z-transfer function	Salvadori <i>et al.</i> (1994)
Finite difference	Pouches: model	Step-change	2- & 3D model	Silva <i>et al.</i> (1994)
Analytical	Model	Finite surface resistance	3D model	Cuesta and Lamua (1995)
Finite element	Lasagne: brick	Finite surface resistance	3D model	Nicolai <i>et al.</i> (1995); Nicolai & De Baerdemaeker (1996)

for heating and cooling phases, has been dealt with analytically by Hayakawa (1964). The effect of air in the headspace has been analyzed by Evans and Board (1954).

In many solutions to heat transfer problems it has often been assumed that the solution for a solid body may be reasonably approximated by an infinite body. For example, Olson and Schultz (1942) considered that a length $4\times$ the diameter of a cylinder or $4\times$ the shortest dimension of a square rod would be sufficient to use the infinite geometry approximation. Erdogdu and coworkers have shown that the infinite assumption ratio depends on the Biot number (hd/k) and the above assumptions do not always hold, Turnham and Erdogdu (2003, 2004) and Erdogdu and Turnham (2006).

2.2.8.2.3. Other Geometrical Shapes of Container

A number of other shapes for containers are used, e.g. pear- or oval-shaped cans for fish products and tapered cans for corned beef and fish products. Table 2.5 lists the solutions to the heat transfer equation which have been obtained.

Smith *et al.* (1967) used a generalized temperature distribution model,

$$F(t) = C e^{-M^2\theta}, \quad (2.74)$$

where C is a pre-exponential factor and M a general shape modulus given by

$$M^2 = G\pi^2, \quad (2.75)$$

in which G is the geometry index, which takes the value 1.000 for a sphere, 0.836 for a cylinder and 0.750 for a cube. A general formula for G is

$$G = 0.25 + 3C^{-2}/8 + 3E^{-2}/8, \quad (2.76)$$

where $C = R_s/(\pi a^2)$ and $E = R_l/(\pi a^2)$, in which R_s is the smallest cross-sectional area of the body which includes the line segment a , R_l is the largest cross-sectional area of the body which is orthogonal to R_s . This approach has been discussed in detail by Hayakawa and Villalobos (1989) and Heldman and Singh (1980).

2.2.8.3. Heating and Cooling

While it is important to know the temperature distribution during heating and to establish an adequate *process* on this basis, it is also important to know what contribution the cooling phase makes in order to prevent overheating and to optimize the process for maximum quality retention. For smaller-sized cans the contents of the can reach the processing temperature during the heating period; however, for larger cans there is a temperature distribution within the can at the onset of cooling. In fact the temperature of the center point continues to rise for some time before the effect of the cooling of the outer layers is felt. This being so, it is necessary to know at what stage to commence cooling to achieve a satisfactory process. This can best be done by studying the equations which govern the heating and cooling process. The technique used is to derive

TABLE 2.5. Some conduction-heating models for predicting temperatures in arbitrary-shaped products & containers.

Geometry	Product/container	Process conditions	Comments	Reference
Any shape	Model	Finite surface resistance	Analytical solution	Smith <i>et al.</i> (1967)
Any shape	Model	Series of step-changes, finite surface resistance	Analytical solution	Wang <i>et al.</i> (1972)
Any shape	Model	Finite surface resistance	Analytical solution	Thijssen <i>et al.</i> (1978), Thijssen and Kochen (1980)
Any shape	Model	Finite surface resistance	Analytical solution	Ramaswamy <i>et al.</i> (1983)
Any shape	Model, particulates in cans	Finite surface resistance	Numerical solution	Lekauwa and Hayakawa (1986)
Any shape	Model	Finite surface resistance	Analytical solution based on Smith <i>et al.</i> (1968)	Hayakawa and Villalobos (1989)
Any shape	Model	Finite surface resistance	Finite element	Akterian and Fikini (1994)
Any shape	Model	Finite surface resistance	Analytical solution	Cuesta and Lamua (1995)
Cubes	Polycarbonate/heated in water	Finite surface resistance	Finite difference	Kim and Teixeira (1997)
Cylinders	Polycarbonate/heated in water	Step-change	Finite difference	Kim and Teixeira (1997)
Ellipsoidal	Sweet potato	Finite surface resistance	ADI: finite difference	Wadsworth and Spirado (1970)
Ellipsoidal	Plastic model, processed ham	Finite surface resistance	Analytical solution	Smith (1966); Smith <i>et al.</i> (1967, 1968), Clary and Nelson (1970), Clary <i>et al.</i> (1971)
Elliptical cross-section	Shrimp	Finite surface resistance	Heat flow lines	Erdogdu <i>et al.</i> (1998a, b)
Oval-shaped	Cans	Step-change	Analytical solution	Iwata (1940)
Oval-shaped	Cans	Step-change	Finite difference	Simpson <i>et al.</i> (1989)
Oval-shaped	Solid shape	Surface heat transfer	volume element	Erdogdu <i>et al.</i> (2001a)
Oval-shaped	Polycarbonate/heated in water	Step-change	Finite difference	Kim and Teixeira (1997)
Pear-shaped	Cans	Step-change	Based on Smith <i>et al.</i> (1966)	Manson <i>et al.</i> (1974)
Mushroom-shaped	Model	Finite surface resistance	Finite difference	Sastry <i>et al.</i> (1985)
Mushroom-shaped	Model	Finite surface resistance	Finite difference	Akterian (1995)
Shrimp-shaped	Shrimp	Finite surface resistance	Finite difference	Chau and Snyder (1988)
Elliptical cross-section	Shrimp	Finite surface resistance	Heat flow lines	Erdogdu <i>et al.</i> (1998a, b)
Elliptical cylinders	Solid shape	Infinite surface resistance	Volume element	Erdogdu <i>et al.</i> (2001a)
Conical shape	Acrylic cones	Finite surface resistance	Volume element	Pornchaloempong <i>et al.</i> (2001, 2003)
Cone frustrums	Fish/pouch	Finite surface resistance	Finite difference	Simpson <i>et al.</i> (2004)
Bowl-shaped, plastic	Apple sauce, bentonite	Finite surface resistance	ADI: finite difference	Sheen <i>et al.</i> (1993)
Irregular shaped particles/liquid	Model	Finite surface resistance	Numerical	Calafiano and Zaritzky (1993)

an equation for the cooling period with an initial temperature distribution, and substitute the temperature distribution at the end of heating in this equation. A rigorous derivation of the equation for a cylindrical container has been given by Hurwicz and Tischer (1952).

The heating stage is represented by the equation

$$T = T_R - (T_R - T_0)F(t_h), \quad (2.76a)$$

where t_h is the time for heating, T the temperature at time t at the center (or spatially distributed), T_R the retort temperature, i.e. process temperature, and T_0 the initial temperature of can contents. Heating followed by cooling is given by

$$T = T_R - (T_R - T_0)F(t) + (T_C - T_R)(1 - F(t - t_h)), \quad (2.77)$$

where t is the total heating and cooling time, i.e. $t_h + t_c$, and T_C the temperature of the cooling water, assumed to be constant at the surface of the container. This equation has been used to study the location of the point of slowest heating by various workers: see Hicks (1951), Hayakawa and Ball (1969a), and Flambert and Deltour (1971, 1973a, b). It should be noted that if first-term approximation of the heat-transfer equation for the heating effect is used, it is less reliable to use the same for the cooling period. The first-term approximation is only applicable for determining the temperature after the long heating times and is not applicable to estimating the temperatures during the early stages of the cooling period. The latter require many terms in the summation series of the solution in order to obtain convergence.

In general, and especially for large sized containers, the temperature distribution at the end of heating is not uniform and therefore the condition of a uniform initial temperature does not apply.

2.2.8.4. Computer Programs for Analytical Heat Transfer Calculations

Newman and Holdsworth (1989) presented a number of computer programs in BASIC for determining the temperature distributions in objects of various geometric shapes. These were based on the analytical solutions for the case of infinite surface heat-transfer coefficients and applied essentially to the case of a finite cylinder and a parallelepiped. The programs also calculate the lethality of the process (see Chapter 6). Thorne (1989) extended the range of available computer programs to the case of external heat-transfer coefficients. These programs operated under MS-DOS. A number of computer simulation programs used for engineering applications are given in 6.9.

2.2.9. *Heat Transfer in Packaged Foods by Microwave Heating*

The applications of microwave heating in the food industry are numerous, and several processes, including tempering and thawing, have been developed commercially (Metaxas & Meredith 1983; Decareau 1985; Decareau & Peterson

1986). However, applications to the pasteurization and sterilization of food products are at present in the early stages of development.

The interaction of microwave energy and food products causes internal heat generation. The rapidly alternating electromagnetic field produces intraparticle collisions in the material, and the translational kinetic energy is converted into heat. For many food products the heating is uneven; the outer layers heat most rapidly, depending on the depth of penetration of the energy, and the heat is subsequently conducted into the body of the food. Current research is concerned with achieving uniform heating, especially in relation to pasteurization and sterilization of foods, where non-uniform heating could result in a failure to achieve a safe process. For materials that are electrical conductors – e.g. metals, which have a very low resistivity – microwave energy is not absorbed but reflected, and heating does not occur. Short-circuiting may result unless the container is suitably designed and positioned. Metallic containers and trays can effectively improve the uniformity of heating (George 1993; George & Campbell 1994). Currently most packages are made of plastic materials which are transparent to microwave energy.

The amount of heat generated in microwave heating depends upon the dielectric properties of the food and the loss factor (see below), which are affected by the food composition, the temperature and the frequency of the microwave energy. For tables of electrical properties of food, and discussion of their application, see Bengtsson and Risman (1971), Ohlsson and Bengtsson (1975), Mudgett (1986a, b), Kent (1987), Ryyänen (1995), and Calay *et al.* (1995).

Many mathematical models have been developed for microwave heating. The most basic are Maxwell's electromagnetic wave propagation equations. These are difficult to solve for many applications, and a simpler volumetric heating model involving the exponential decrease of the rate of heat generation in the product is used. The basic equation for this model is

$$q = q_0 e^{-x/\delta}, \quad (2.78)$$

where q is volumetric heat generation (W m^{-3}), q_0 heat generated in the surface (W m^{-3}), x the position coordinate in the product, and δ the penetration depth based on the decay of the heating rate.

The general equation for the temperature distribution based on microwave heating and conduction into the product is

$$\partial T / \partial t = \nabla^2 T + q_0 e^{-x/\delta}. \quad (2.79)$$

The rate of heat generation is given by

$$q_0 = \pi f \varepsilon_0 \varepsilon'' |E^2|, \quad (2.80)$$

where f is frequency (Hz), ε_0 the permittivity of free space (F m^{-1}), ε'' the dielectric loss, and $|E^2|$ the root-mean-square value of the electric field (V m^{-1}).

The penetration depth x in the product is obtained from the equation

$$x = \lambda_0 / [2\pi(\varepsilon' \tan \delta)^{1/2}]. \quad (2.81)$$

The term $\tan \delta$ is the ratio of the dielectric loss ε'' to the dielectric constant ε' , and is known variously as the loss tangent, loss factor or dissipation constant. It is listed in tables of physical property data, e.g. Kent (1987).

The main frequency bands used are 2450 and 896 MHz in Europe and 915 MHz in the USA. Greater penetration and more uniform heating are obtained at the longer wavelengths for food products with low loss factors.

Datta and Liu (1992) have compared microwave and conventional heating of foods and concluded that microwave heating is not always the most effective method, especially for nutrient preservation. The effect depends on a variety of properties of the system.

Burfoot *et al.* (1988) examined the microwave pasteurization of prepared meals using a continuous tunnel device. The product was heated to 80–85°C for a few minutes, sufficient to inactivate vegetative pathogenic bacteria, e.g. *Salmonella* and *Campylobacter*, but not bacterial spores. The latter are controlled by storing the product below 10°C. This type of product is not shelf-stable at room temperature and a full sterilization process would be necessary with low-acid products of this type to obtain a stable product. Microwave tunnels for this purpose would have to be pressurized to maintain the integrity of the package when sterilizing temperatures (121°C) had been achieved. A general-purpose plant known as Multitherm has been developed by AlfaStar Ab, Tumba, Sweden (Hallström *et al.* 1988). Burfoot *et al.* (1996) have modeled the pasteurization of simulated prepared meals in plastic trays with microwaves. Large differences between actual and predicted temperatures were found at some points.

For measuring temperatures in microwave systems an invasive fibre-optic probe has been developed, which uses the change in color with temperature of a crystal situated at the end of a glass fibre. Fluoroptic probes are manufactured by Luxtron Corp., CA, USA.

2.2.10. Dielectric Heating

Dielectric heating is performed at radio frequencies of 13.56, 27.12, or 40.68 MHz. This distinguishes from microwave heating, which uses $\cong 900$ or 2450 MHz. (Rowley 2001). Wang *et al.* (2004) have emphasized the advantage of the volumetric heating characteristics of dielectric heating, especially for heating sealed polymeric and pouches. In a study on the heating of 6 lb. capacity polymeric trays filled with whey protein gels, the production of M-1 (see §6) was used as a chemical marker to determine the degree of lethality and cooking value achieved by RF dielectric heating and steam.

2.3. Heat Transfer by Convection

2.3.1. Introduction

Convective heat transfer inside containers results either from the natural effects of changes in density in the liquid induced by changes in temperature at the container

walls (free or natural convection) or by creating motion in the container contents by axial or end-over-end rotation (forced convection).

The process of natural convection initially involves heat transferred by conduction into the outer layers of fluid adjacent to the heated wall; this results in a decrease in the density, and the heated fluid layer rises. When it reaches the top of the liquid at the headspace, the induced fluid motion causes it to fall in the central core, the displaced hot fluid at the wall being replaced by colder fluid in the core. As the temperature of the can contents becomes more uniform and the driving force smaller, the fluid velocity tends to decrease and eventually, when the fluid becomes uniformly heated, the motion ceases.

The mechanism can also be represented by an initially thin boundary layer of liquid in which the fluid is rising, the thickness of which increases as the process continues. According to experimental investigations by Datta and Teixeira (1988), after 30 seconds heating the thickness of this layer was 2 mm and the temperature drop from the can wall to the boundary layer edge was from 121 to 45°C.

With more viscous fluids the process is somewhat slower, although the viscosity and the density are, in general, reduced by a rise in temperature. There will also be an increase in the conductive component with thicker fluids. Ball and Olson (1957) devised a qualitative method of determining the intensity of convection currents in containers based on the ratio of the conductive to convective heat transfer components. This situation should not be confused with the following phenomenon.

For both pure convection and pure conduction heating in canned foods the “heating curve” plot of the logarithm of the reduced temperature against time is more or less linear, after an initial lag period (see Chapter 5). However, for some products the plot shows two lines with differing slope, known as a “broken heating curve”. Products showing this type of effect usually start as convective heating packs, but due to physico-chemical changes, e.g. starch gelation, they finally heat by a highly conductive mechanism.

Convection currents inside a container can be visualized, in an idealized form, as shown in Figure 2.5. In the upper part of the container the hot liquid is being pumped by the heated fluid rising in the boundary layer, and being placed on the cold liquid in the core. Simultaneously, in the case of a vertical container, there is also heat rising from the bottom end of the can, which produces mixing eddies in the bulk of the fluid. Datta (1985) has shown that as result of instabilities in the temperature distribution, regular bursts of hot liquid occur on the base of the container. This phenomenon is known as Bernard convection.

The mechanism of unsteady-state convection is very complex and varies with time of heating and/or cooling; consequently it is very difficult to model precisely. Hiddink (1975) used a flow visualizing technique with metallic powders in liquids in containers with light-transmitting walls, to highlight the streamlines in the bulk of the fluid. Other workers, e.g. Blaisdell (1963), have used thermocouples at several points in the containers to plot the temperature profiles.

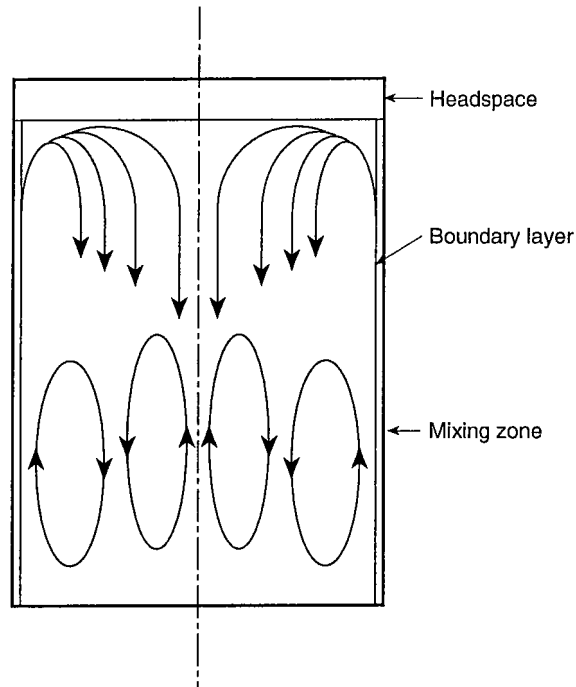


FIGURE 2.5. Idealized representation of convection currents in a can of liquid.

Guerrei (1993) has discussed the internal flow patterns in convection-heating bottles of cylindrical and square shape, using the concept of a double-layer system of rising hot liquid on the wall of the container which discharge into a central volume. The point-of-slowest-heating was shown to be 0.006 m from the wall of an 0.06 m internal diameter container.

The most important point, from a practical safety point of view, is where the slowest heating point is situated. This subject is dealt with in Chapter 5.

While the discussion has been concerned with convection heating/cooling inside containers, there is also the problem of convection heating on the outside, from the heating or cooling medium to the container wall. In the case of pure steam heating, condensation of steam on the container wall surface raises the temperature of the surface almost immediately to that of the steam and, consequently, no problem arises. If, however, the steam contains air, either as an adulterant or intentionally, then the velocity of the mixture over the surface will affect the temperature of the surface and a more complex situation will arise. Similarly, if water is used for heating, and also for cooling, then the wall temperature will be affected by the velocity of the heat transfer medium. In all cases, except for condensing pure steam, it is necessary to consider external convective heat transfer to the outer surface of the container.

2.3.2. Basic Concepts in Convection Heat Transfer

Mathematical models for the prediction of temperatures in the heating of canned foods by convection are necessary in order to determine the process requirements. However, they are much less easy to obtain than in the case of heating by conduction. There are three approaches to convective heat transfer: the film theory; the use of dimensionless numbers; and the more rigorous mathematical treatment of the basic fluid dynamic and heat transfer models. Reviews of this subject have been presented by Ball and Olson (1957), Holdsworth and Overington (1975), Rao and Anantheswaran (1988), as well as in various theses, e.g. Blaisdell (1963), Stevens (1972), Hiddink (1975), and Zechman (1983).

2.3.2.1. Film Theory

The basic heat transfer equation for convection is

$$Q = h_s A (T_b - T_s), \quad (2.82)$$

where Q is the quantity of heat flowing (J s^{-1}), h_s is the surface heat-transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$), A is the surface area (m^2), T_b is the bulk fluid temperature (K) and T_s is the surface temperature.

The idealized temperature profile for a material being heated by a fluid and separated by a container wall is shown in Figure 2.6. On either side of the wall boundary layers or films may be visualized, through which the bulk of the temperature drop takes place. Thus if T_1 is the temperature of the heating medium and T_4 is the temperature in the bulk of the fluid being heated, and the

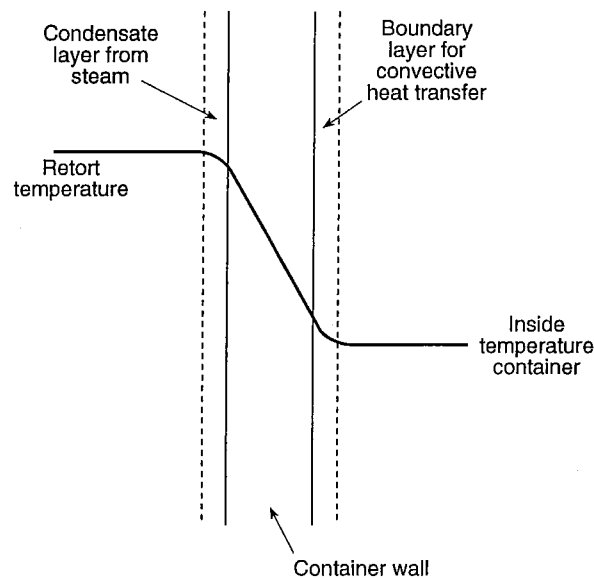


FIGURE 2.6. Temperature profile across container wall.

average film temperatures are T_2 and T_3 , then an overall heat-transfer coefficient U (alternatively denoted by H) can be defined as follows

$$Q = UA(T_1 - T_4), \quad (2.83)$$

where

$$\frac{1}{U} = \frac{1}{h_s} + \frac{x_w}{k_w} + \frac{1}{h_b}, \quad (2.84)$$

and h_s and h_b are the heat-transfer coefficients for the surface being heated and the bulk of the fluid, x_w is the thickness of the wall (m), and k_w is the thermal conductivity of the wall material ($\text{W m}^{-1} \text{K}^{-1}$).

Some typical values for h_s ($\text{W m}^{-2} \text{K}^{-1}$) are: steam, 12 000; steam + 3% air, 3500; steam + 6% air, 1200; cold water, 500; air moving at 3 m s^{-1} , 30; still air, 6. From these values it can be seen that the resistance to heat transfer, the reciprocal of the heat transfer coefficient, is extremely small for condensing steam; consequently this can be ignored in all heat transfer calculations.

Some values for the thermal conductivities of container materials ($\text{W m}^{-1} \text{K}^{-1}$) are: ferrous metal containers, 40–400; aluminum, 220; glass, 1–2; polyvinylchloride, 0.29; polyethylene, 0.55; water, 0.57. From these values it can be seen that it may be necessary, when using container materials other than metals, to consider the x_w/k_w term in the overall heat transfer equation.

2.3.2.2. Correlations for Predicting Heat-Transfer Coefficients

Engineering practice makes use of dimensionless numbers for correlating heat-transfer coefficients with the physical circumstances of heat transfer and the physical properties and flow conditions of the fluids involved. The four dimensionless numbers used in heat transfer studies are:

- (a) Reynolds number, used for flow characterization,

$$Re = \nu d \rho / \mu;$$

- (b) Nusselt number, ratio of the heat transferred by convection to that transferred by conduction,

$$Nu = h d / k;$$

- (c) Prandtl number, influence of physical properties

$$Pr = c \rho / k;$$

- (d) Grashof number, influence of buoyancy forces in natural convection

$$Gr = d^3 \rho^2 \beta g \Delta T / \mu^2;$$

- (e) Rayleigh number, product of Grashof and Prandtl numbers

$$Ra = Gr \cdot Pr$$

where

c = specific heat at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$);
 d = characteristic linear dimension – thickness, length or diameter, as appropriate (m);
 g = gravitation constant (m s^{-2});
 h = heat-transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$);
 k = thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$);
 T = temperature difference (K);
 v = velocity (m s^{-1});
 β = coefficient of thermal expansion (K^{-1});
 μ = dynamic viscosity (Pa s);
 ρ = density (kg m^{-3}).

For example, for forced convection we have the correlation

$$Nu = A(Re)^a(Pr)^b.$$

For gas flow across cylinders, with Re in the range 10^3 – 10^5 , values of $A = 0.26$, $a = 0.60$ and $b = 0.30$ have been obtained. For liquids, with Re between 4×10^3 and 4×10^4 , we have $A = 0.193$, $a = 0.618$ and $b = 0.333$.

For natural convection, the relationship is

$$Nu = A(Pr)^c(Gr)^d.$$

For horizontal or vertical cylinders with streamline flow and $Re < 2100$, values of $A = 0.47$ and $c = d = 0.25$ have been obtained. With turbulent flow and $Re > 10000$, we have $A = 0.10$, and $c = d = 0.33$. All the fluid properties are measured at the mean film temperature $\frac{1}{2}(T_w + T_b)$, where the subscripts b and w to the temperature refer to the bulk fluid and the wall respectively.

Several other dimensionless numbers are encountered in heat transfer studies and these will be defined where appropriate.

Since the numerical values for the indices and coefficients are determined from experimental results, it is unwise to use dimensionless correlations outside the ranges for which the experimental results have been obtained. Extrapolation of the data requires careful consideration.

For methods of deriving dimensionless correlations and for further information on different applications, the standard engineering texts should be consulted, e.g. Burmeister (1983), Coulson and Richardson (1984), Hallström *et al.* (1988), Jaluria (1980), and Perry and Green (1984).

2.3.3. Models for Convection Heat Transfer

Some of the models that have been used to predict temperature distributions and velocity profiles in heated and cooled can liquid products are shown in Tables 2.6–2.9.

The models may be classified as follows:

2.3.3.1. Energy Balance Model

This is the simplest of the models; it was first proposed by Jones (1931) and subsequently used by many other workers. It is often known as the Schultz–Olson model in recognition of the contribution of the American Can Company workers (Schultz & Olson 1938).

By equating the overall rate of heat transfer into the can with the rate of accumulation of heat inside the can, an energy balance equation can be established. For the heat transfer into the can,

$$Q = UA(T_s - T_m), \quad (2.85)$$

and the rate of accumulation of heat

$$Q = mc \frac{dT}{dt}, \quad (2.86)$$

where

- Q = rate of heat transfer (W);
- U = overall heat-transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$);
- A = surface area (m^2);
- T_s = temperature of the heating medium (K);
- T_m = temperature of the product (mass average) (K);
- c = specific heat ($\text{J kg}^{-1} \text{K}^{-1}$);
- t = time (s);
- m = mass (kg).

Equating the left-hand sides of (2.85) and rearranging we obtain:

$$\frac{mc}{A} \frac{dT}{T_s - T_m} = U dt. \quad (2.87)$$

Integrating,

$$\int_{T_0}^{T_m} \frac{mc}{A} \frac{dT}{T_s - T_m} = \int_0^t U dt. \quad (2.88)$$

leads to

$$\log_e \frac{(T_s - T_m)}{(T_s - T_0)} = \frac{UA}{mc} t \quad (2.89)$$

or

$$T_m = T_s - (T_s - T_0) e^{-[(UA/mc)t]} \quad (2.90)$$

This equation shows that the rate of heating is an exponential function, which depends on the overall heat-transfer coefficient U (or the internal heat-transfer coefficient h_{int} when steam is used as the heating medium with metallic cans), the surface area A , the mass of the contents and their specific heat, as well as the temperature of the heating medium and the initial temperature of the contents T_0 .

This approach has been modified by Videv (1972) and applied by Akterian (1995) to a range of canned and bottled foods heating by convection. Their approach uses the concept of thermal inertia E for the term mc_p/UA . This characterizes the temperature lag between the liquid temperature and the heating medium. The theoretical treatment has been extended to the case of an external linear heating profile (Videv 1972). The variation in heat transfer rate into canned products has been discussed theoretically by Dincer *et al.* (1993).

2.3.3.1.1. Slow Come-Up Time with Perfect Mixing

Equation (2.91) has been derived from equation (2.87), solving an ordinary differential equation and assuming a linear retort temperature profile (i.e. simulating temperature profile during come-up time) (Simpson *et al.* 2003).

$$T_m = a \left(1 - \exp \left(-\frac{UA}{mc} t \right) \right) + b \left(t - \frac{mc}{UA} \right) + b \frac{mc}{UA} \exp \left(-\frac{UA}{mc} t \right) + T_0 \exp \left(-\frac{UA}{mc} t \right) \quad (2.91)$$

Where retort temperature is time dependent and expressed as: $T_S(t) = a + bt$ and equation (2.91) is valid for: $0 < t \leq CUT$. For $t > CUT$, temperature T (or T_m) can be expressed by equation (2.89) using an appropriate initial temperature (constant T_S).

Provided that f_h is defined as $\ln 10 \times [MCp/UA]$ (Merson *et al.* 1978), equation (2.91) can be re-arranged and expressed as:

$$T_m = (a + bt) - b \frac{f_h}{\ln 10} + \left(-a + b \frac{f_h}{\ln 10} + T_0 \right) \exp \frac{-t \ln 10}{f_h} \quad (2.92)$$

Further working on equation (2.92) renders:

$$\frac{T_m - (a + bt) + b \frac{f_h}{\ln 10}}{\left(-a + b \frac{f_h}{\ln 10} + T_0 \right)} = \exp \frac{-t \ln 10}{f_h} \quad (2.93)$$

From equation (2.93), the dimensionless temperature ratio can be expressed as:

$$\frac{T_m - (a + bt) + b \left(\frac{f_h}{\ln 10} \right)}{T_0 - a + b \left(\frac{f_h}{\ln 10} \right)} = \frac{T'_m - (a' + b't) + b' \left(\frac{f_h}{\ln 10} \right)}{T'_0 - a' + b' \left(\frac{f_h}{\ln 10} \right)} = \text{Constant} \quad (2.94)$$

2.3.3.2. Effective Thermal Diffusivity Model

This model, first discussed by Thompson (1919), makes use of the unsteady-state conduction model solutions and an apparent or effective thermal diffusivity. Thompson found that this depended on the ratio of solids to liquid in the container. Tani (1938b) also used this approach to studying the temperature profiles in heated 1 lb tall cans filled with water. More recently, Teixeira *et al.* (1992) have used a

similar method with the j -value concept (see chapter 5). Early Japanese work was discussed by Olson (1947).

2.3.3.3. Transport Equation Model

This is the most rigorous approach to determining the temperature distributions and the velocity profiles in containers filled with liquids. The early work in this area was reviewed by Ede (1967) and particularly useful work was reported by Evans and Stefany (1966) and Evans *et al.* (1968). The form of the equations depends on the geometry of the container; care should be taken to use the most appropriate coordinate system (Bird *et al.* 1960; Ruckenstein 1971).

The equations which have to be solved in relation to the container boundaries are the equation of continuity:

$$\frac{1}{r} \frac{d}{dr}(r\rho v) + \frac{d}{dz}(\rho u) = 0; \quad (2.95)$$

the equation of momentum in the radial direction:

$$\rho \left(\frac{du}{dt} + v \frac{du}{dr} + u \frac{du}{dz} \right) = -\frac{dP}{dz} + \mu \left\{ \frac{1}{r} \frac{d}{dr} \left(r \frac{d\mu}{dr} \right) + \frac{d^2\mu}{dz^2} \right\} + \rho g; \quad (2.96)$$

the equation of momentum in the axial direction:

$$\rho \left(\frac{dv}{dt} + v \frac{dv}{dr} + u \frac{dv}{dz} \right) = -\frac{dP}{dr} + \mu \left\{ \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr}(rv) \right) + \frac{d^2v}{dz^2} \right\}; \quad (2.97)$$

and the energy equation to determine the temperature distribution:

$$\frac{dT}{dt} + v \frac{dT}{dr} + u \frac{dT}{dz} = \frac{k}{\rho c_p} \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{d^2T}{dz^2}. \quad (2.98)$$

The boundary conditions required for the solution are, for the side wall,

$$r = R, \quad 0 < z < H, \quad T = T_1, \quad u = 0, \quad v = 0;$$

for the center line,

$$r = 0, \quad 0 < z < H, \quad dT/dr = 0, \quad du/dr = 0;$$

for the bottom wall,

$$0 < r < R, \quad z = 0, \quad T = T_1, \quad u = 0, \quad v = 0;$$

and for the top wall,

$$0 < r < R, \quad z = H, \quad T = T_1, \quad u = 0, \quad v = 0.$$

The initial conditions are that the fluid is at rest and at a uniform temperature:

$$0 < r < R, \quad 0 < z < H, \quad T = T_0, \quad u = 0, \quad v = 0.$$

Numerical solutions for these equations have been attempted (see Table 2.6). The work of Datta (1985) and Datta and Teixeira (1987, 1988) represents the most advanced approach at the present time.

2.3.4. *Some Experimental Work and Correlations*

Here some of the experimental work to determine internal heat-transfer coefficients will be considered. Both homogeneous and heterogeneous products heated in static and rotary retorts will be mentioned. Tables 2.6 to 2.9 give further references to work in this area.

2.3.4.1. Homogeneous Products Heated in Batch Systems

Much of the early work on heat penetration into cans of liquid products was aimed at determining the time – temperature profiles. Jones (1931) was one of the first workers to apply a simple model to determine temperature profiles in canned liquid products. Blaisdell (1963) made an extensive survey of the literature and carried out experimental work on the heating of a 50% sucrose solution in glass jars of varying sizes. The results were correlated using the heat penetration parameters f_h and j (see Chapter 5). Evans and Stefany (1966), using a range of liquids, obtained a simple correlation for predicting internal heat-transfer coefficients,

$$Nu = 0.55(Gr \cdot Pr)^{0.25},$$

using the length of the container as the characteristic length. Hiddink (1975), in a similar study, obtained a correlation

$$Nu = 0.31(Gr \cdot Pr)^{0.268}.$$

Verboven *et al.* (2004) have described various applications of CFD to optimizing thermal processes and have given details of the techniques used in various applications.

Further work is listed in Table 2.6.

2.3.4.2. Homogenous Systems Heated with Agitation

Many products are now processed in retorts (see Chapter 8) that impart either axial or end-over-end motion to the container contents. By choosing a suitable headspace it is possible to increase the rate of heat transfer and thereby reduce the time required for processing. The success of this type of process depends on the suitability of the product and careful control of the headspace during the filling operation. Table 2.7 gives some examples of the types of correlation which have been applied to the estimation of heat-transfer coefficients. Some important studies have been carried out, from which quantitative correlations for heat-transfer coefficients have been obtained. Some of these will now be discussed briefly; however, other matters relating to external heat-transfer coefficients from the heating medium will be dealt with in Chapter 8.

TABLE 2.6. Some studies on convection heat transfer for non-agitated homogeneous canned foods.

Product	Process	Model	Comments	Reference
Water	100°C: No. 2 & No. 3 cans	Energy balance	Film model	Jones (1931)
Water	Cooling: No. 2 & No. 3 cans	Energy balance	Newton's law	Jones (1932)
Water	100°C: 1 lb tall glass	Conduction (modified)	Apparent thermal diffusivity	Tani (1938b)
Water	100°C: 1 lb tall glass	Navier-Stokes equation	Dimensionless; data from Tani (1938)	Okada (1940b)
Water/aluminum powder	100°C: 1 lb glass, two sizes	Energy balance	Flow visualization	Ban and Kaziwara (1941)
Ideal fluid ($Pr = 1$)	Heating & cooling	Navier-Stokes equation	Analytical solution	Hammitt and Chu (1962)
Water/glycerol, butanol, methanol	100°C: 2" & 2.5" d cylinders	Energy balance	Dimensionless groups, heat transfer coefficients	Evans and Stefany (1966)
Water	Heating: closed container	Semi-empirical model	Turbulent free convection	Tatom and Carlson (1966)
Ideal fluid	Heating: closed container	Navier-Stokes equation	Laminar natural convection	Barakat and Clark (1966)
Water & glycerol	Heating: cylinder	Energy balance	Mixing/main core study	Evans <i>et al.</i> (1968)
Water/sucrose solution	< 83°C: 8-22 oz glass jars	Navier-Stokes equation	Dimensionless groups, flow visualization	Blaisdell (1963)
Ethylene glycol	100°C: 401 × 411 can	Navier-Stokes equation	Velocity & temperature profiles	Stevens (1972)
Water & sucrose solution	100°C: 300 × 410 can	Energy balance	Rayleigh nos v. temperature	Jowitt and Mynott (1974)
Water & silicone fluid	115°C: half cylinder	Energy & stratification	Flow visualization	Hiddink (1975); Hiddink <i>et al.</i> (1976)
Water	100°C: 3 kg can 156 × 150 mm	Energy balance	Temperature distribution	Miglioli <i>et al.</i> (1983)
Beer	100°C: bottle	Finite-element model (CFD)	Temperature/velocity profiles	Engleman and Sani (1983)
Water	121°C: 303 × 406 can	Navier-Stokes equation	Temperature/velocity profiles	Datta & Teixeira (1987, 1988)
Water, xanthan gum, oils	120°C: 300 × 401 can	Conduction (modified)	Apparent thermal diffusivity	Bera (1988)
Water/CMC	121°C: 81 × 111 mm can	Finite volume model (CFD)	Temperature distributions	Abdul Ghani <i>et al.</i> (1999a, b)
Liquid	121°C: 4 × 12 × 22 cm pouch	Finite volume model (CFD)	Temperature distributions	Abdul Ghani <i>et al.</i> (2002a)
Starch/water	121°C: 303 × 406 can	Fluid dynamics analysis	Viscosity study of gelation	Yang and Rao (1998)
3.5% corn starch/water	121°C: 303 × 406 can	2D-finite element	Energy balance	Tattiyakul <i>et al.</i> (2001, 2002a)
3.5% starch/water	110-121°C: 162 × 111 mm can	2D-finite element	Energy balance	Tattiyakul <i>et al.</i> (2002b)

(cont.)

TABLE 2.6. (Continued)

Product	Process	Model	Comments	Reference
5% waxy maize starch/water	110–121°C: 162 × 111 mm can	2D-finite element	Energy balance	Tattiyakul <i>et al.</i> (2002b)
5.6° Brix tomato juice	110–121°C: 162 × 111 mm can	2D-finite element	Energy balance	Tattiyakul <i>et al.</i> (2002b)
5% Tomato paste solids	121°C: cans	Distributed parameter model	Energy balance	Bichier <i>et al.</i> (1995)
Carrot/orange soup	121°C: can	Finite volume model (CFD)	Velocity/temperature profiles	Abdul Ghani <i>et al.</i> (2002a)
Carrot/orange soup	121°C: 4 × 12 × 22 cm pouch	Finite volume model (CFD)	Temperature distributions	Abdul Ghani <i>et al.</i> (2001)
Carrot/orange soup	121°C: 72 × 106 mm can	CFD PHOENICS	Temperature/flow profile	Ghani <i>et al.</i> (2001); Abdul Ghani <i>et al.</i> (2002b)
Tomato dip product	Holding 90°C & cooling 70, 50, 20°C: 690 gm. jars	Finite difference	Temperature distribution	Plazl <i>et al.</i> (2006)
Non-Newtonian simulant	121°C: 303 × 406 can	Finite-element model (CFD)	Temperature/velocity profiles	Kumar <i>et al.</i> (1990) Kumar and Battacharya (1991)
Non-newtonian, CMC/water	125°C: 75 × 115 mm	CFD; CFX-4	Transient natural convection	Quarini and Scott (1997)
CMC, 0.85% w/w	121°C: 307 × 408 & conical can	Finite element model (CFD)	Temperature/velocity profiles	Varma and Kannan (2006)

TABLE 2.7. Some studies on convection heat transfer for agitated homogeneous canned foods.

Product	Process	Model	Comments	Reference
Sucrose solution, juices & fruit purées	Spin-cooker/cooler 98°C; axial rotation 300 × 406; 401 × 411; 604 × 614 cans	Energy balance	Heat transfer coefficients	Quast and Siozawa (1974)
Silicone oil	Laboratory device 120°C; axial & end-over-end	Energy balance	Heat-transfer coefficients – duration of a transfer unit	Bimbenet and Michiels (1974)
Water & sucrose solution	<p>FMC Steritort 121°C; axial rotation 303 × 406; 608 × 700</p> <p>Hydrolock pilot sterilizer 130°C; end-over-end 54 × 90; 73 × 52; 73 × 107; 99.4 × 111.2 mm cans</p>	Energy balance	Temperature profiles, heat-transfer coefficients	Lenz and Lund (1978)
Aqueous & organic liquids	Stork simulator; various modes 125°C; 105 × 112 mm can	Energy balance	Heat-transfer coefficients, surface tension consideration	Duquenoy (1980); Duquenoy (1984)
Glucose syrup 84° brix	Stork simulator; various modes 125°C; 105 × 112 mm can	Energy balance modified for process deviations	Heat-transfer coefficients	Naveh and Kopelman (1980)
Water, glycerol & sucrose solutions 30, 50 & 60%	Agitating sterilizer 100°C; 303 × 406 can; end-over-end	Energy balance	Heat-transfer coefficients, dimensionless analysis	Anantheswaran & Rao (1985a)
3.5% corn starch/water	FMC Steritort. Intermittent agitation 121°C; 303 × 406 can	Energy balance	2D-finite element	Tattiyakul <i>et al.</i> (2001, 2002a)
3.5% starch/water	FMC Steritort: 110–121°C; 162 × 111 mm can	Energy balance	2D-finite element	Tattiyakul <i>et al.</i> (2002b)
5% waxy maize starch/water	FMC Steritort: 110–121°C; 162 × 111 mm can	Energy balance	2D-finite element	Tattiyakul <i>et al.</i> (2002b)
5.6° Brix tomato juice	FMC Steritort: 110–121°C; 162 × 111 mm can	Energy balance	2D-finite element	Tattiyakul <i>et al.</i> (2002b)
5% Tomato paste solids	FMC Steritort: 5 rpm; 121°C; cans	Energy balance	Distributed parameter model	Bichier <i>et al.</i> (1995)
Aqueous solution guar gum, various conc.	Agitating sterilizer 100°C; 303 × 406 can; end-over-end	Energy balance	Heat-transfer coefficients, dimensionless analysis	Anantheswaran & Rao (1985b)
Water, silicone oil	Rotary simulator 100°C; axial rotation 303 × 406; 401 × 411; 404 × 700 cans	Energy balance	Heat-transfer coefficients	Soule and Merson (1985)
Aqueous solution guar gum	Rotary device/batch retort 121°C; end-over-end; 303 × 406 can	Energy balance	Heat-transfer coefficients	Price and Bhowmik (1994)

Naveh and Kopelman (1980) studied a variety of rotational modes, based on end-over-end and axial rotation modes, some in off-set positions, using 105×112 mm containers of glucose syrup 84° brix. A modified energy balance model was used to determine the overall heat-transfer coefficient, using a modified energy balance model, as follows:

$$U = \frac{1}{n} \sum_{i=1}^n \frac{mc_p}{At_i} \log \left(\frac{T_R - T}{T_R - T_0} \right)_i \quad (2.99)$$

where U is the average momentary overall heat-transfer coefficient, taken at intervals of time t_i , m is the mass of can and contents, c_p is the specific heat of the product, A is the surface area for heat transfer, T is the temperature of the food, T_R is the retort temperature and T_0 is the initial temperature of the product.

Duquenois (1980, 1984) determined heat-transfer coefficients for cans containing a range of aqueous and organic liquids agitated in end-over-end mode. The correlation obtained included a dependence on surface tension of the liquid as well as filling ratio.

Anantheswaran and Rao (1985a) determined heat-transfer coefficients for end-over-end can rotation for a range of Newtonian fluids in 303×406 cans. They obtained the correlation

$$Nu = 2.9Re^{0.436}Pr^{0.287}, \quad (2.100)$$

where $Nu = h_id_r/k$, $Re = d_r^2 N \rho / \mu$ (rotational), and $Pr = \mu/k$, in which d_r is the diameter of rotation, and N the speed of rotation (s^{-1}).

The correlation was not improved by trying to incorporate natural convection effects, length to diameter ratio, or headspace volume. The arithmetic average heat-transfer coefficient was determined from instantaneous heat-transfer coefficients by integration of the basic energy equation.

Subsequently these workers (Anantheswaran & Rao 1985b) reported experimental work using non-Newtonian fluids (guar gum of various concentrations) and obtained a general correlation:

$$Nu = 1.41GRe^{0.482}GPr^{0.355}, \quad (2.101)$$

where the generalized Reynolds GRe and Prandtl GPr numbers were obtained using the non-Newtonian apparent viscosity term $(8/N)^{n-1}K[(3n+1)4n]^n$ in place of the viscosity term. N is the rotational speed (s^{-1}). It was found that other factors, e.g. can length to diameter ratio and the flow index, had no statistically significant influence on the heat-transfer coefficient.

Soule and Merson (1985) obtained an overall heat transfer correlation for axial rotation of cans filled with water and silicone oils of varying viscosity. They found a dependency on both the can dimension ratio L/D and the viscosity ratio μ_b/μ_w , where the subscripts b and w refer to the viscosities evaluated at the average bulk temperature and can wall temperature, respectively, taken over the whole process. Their correlation is:

$$Nu = 0.434Re^{0.571}Pr^{0.278}(L/D)^{0.3565}(\mu_b/\mu_w)^{0.154}. \quad (2.102)$$

The average internal heat-transfer coefficient, h_i , was estimated from the overall coefficient, U , from the following equation:

$$h_i = (1.07 \pm 0.04)U. \quad (2.103)$$

Datta and Teixeira (1987, 1988) solved the basic fluid flow and heat transfer equations numerically. The solution compared well, both qualitatively and quantitatively, with the available experimental data. The fluid flow patterns revealed liquid rising at the wall because of the buoyancy of the heated boundary layer, radial flow and mixing at the top of the liquid and uniform core flow near the axis.

Kumar *et al.* (1990) used a finite-element technique and computational fluid dynamics to study the flow of a thick viscous liquid heated in cans. The liquid had non-Newtonian flow characteristics and obeyed a simple power law model. A flow model with the viscosity dependent on temperature was used.

Further examples are referred to in Table 2.7

2.3.4.3. Heterogeneous Products Heated in Batch Retorts

To estimate the ratio of conduction/convection heat transfer within the container, the following equation has been proposed (Ball & Olson 1957):

$$K_{va} = \frac{K_v f_h - f_{h'}}{f_h K_v - f_h K_a} \quad (2.104)$$

Where:

K_{va} : Conduction-convection heat transfer index (dimensionless)

K_v : Can conduction index (dimensionless)

K_a : Can convection index (dimensionless)

f_h : slope index of the first can (min)

$f_{h'}$: slope index of the second can (min)

According to equation (2.104), when K_{va} index tends to 1 the heat transfer is mainly carried out by conduction. On the other hand, if K_{va} index tends to 0 the main heat transfer mechanism is convection.

2.3.4.4. Heterogenous Products Heated in Batch Retorts

Most workers, dealing with convection packs consisting of particulates in a covering liquid, carry out experimental trials to determine process safety with a thermocouple placed at a convenient point, often considered to be the point of slowest heating in the liquid. While this is adequate for particulates of small size, larger-sized constituents may not receive an adequate process at their centers. For some vegetable products this could result in a sub-botulinum process (see Chapter 4). Potatoes have been shown to receive inadequate processes in their centers (Thorpe & Atherton 1969; Atherton *et al.* 1970). While, in practice, it is necessary to make a judgement on the efficacy of a given process, it may well be, as in the case of canned carrots, that many years of industrial practice dictate that this criterion is not always essential. However, with many other products,

e.g. artichoke hearts, microbiological challenge tests are required to establish the safety of the process.

Models for predicting the temperature distributions in both the particulates and the covering liquid have been developed by relatively few workers. However, some of the work will be briefly mentioned here; other work is listed in Table 2.8.

Ikegami (1977) developed an equation based on heat transfer parameters f_h and j for both solid and liquid phases, which could be used to predict the temperature profile:

$$\frac{T_s - T_R}{T_0 - T_R} = \frac{jj_2 f_2}{f_2 - f} 10^{-t/f_2}, \quad (2.105)$$

where j and f are the heat transfer parameters for the solid, and j_2 and f_2 for the liquid, and T_s , T_R and T_0 are, respectively, the solid food, retort and initial food temperatures.

Rumsey (1984) used a numerical approach to solve an energy balance model, which had been modified by the addition of a term representing the temperature gradient in the particulate:

$$m_1 c_1 \frac{dT_1}{dt} + n_p m_p c_p \frac{dT_p}{dt} = U_c A_c (T_R - T_1), \quad (2.106)$$

where m is the mass, c is the specific heat, t the time, T the temperature, n the number of particulates; the subscripts referring to 1 the liquid, p the particulate, R the retort and c the container. The temperature distribution in the particulate, T_p , is obtained using the solution of the unsteady-state heat conduction equation for the particular geometry.

One of the more important studies was due to Lekwauwa and Hayakawa (1986), who developed a model assuming an initially uniform temperature in the particulates, which was not necessarily the same as the covering liquid; the solid and liquid components had homogeneous, isotropic and temperature-independent properties; the particulate sizes were statistically distributed (approximately gamma); the heat-transfer coefficient between the solid surface and the bulk liquid was constant; and the heat-transfer coefficients for cooling conditions were different from those for heating. The technique used for determining the particle temperature distributions was to apply Duhamel's theorem (Section 2.2.6.4) to the conduction step response (modeled in terms of f_h and j) with the change in liquid temperature. The energy balance on the whole system was established considering the heat transfer between the external heating medium and the liquid in the container, and also between the liquid and the particulates. A computer algorithm was presented for this model. Experimental results for the heating of aspheroidally shaped white potatoes in distilled water were in good agreement with those from the theoretical models, for both analytical and numerical solutions. Hayakawa *et al.* (1997) have used Duhamel's theorem to determine the temperature distribution in irregular shaped objects, subjected to an external heat transfer coefficient. This involved using the experimentally determined values of f_h and j (see Chapter 3) The method was also applicable to deviant processes.

TABLE 2.8. Some studies on convection heat transfer for non-agitated heterogeneous canned foods.

Product	Process	Model	Comments	Reference
Glass spheres/water/silicone fluid	115°C; half cylinder	Fluid dynamic model	Temperature distribution	Hiddink (1975)
Bamboo shoots, sausages, baby clams, mushrooms/liquid	JCS Nos 1, 2, 4 & 7 cans	f_h & j model	Heat penetration data	Ikegami (1977)
Mushroom-shaped aluminum, mushrooms	115.6–126.7°C; 211 × 212 can	Energy balance	Heat transfer coefficients, effect of temperature & particle size	Sastry (1984)
Aluminum, PTFE & potato spheres/liquids	100°C; see Hassan (1984)	Energy balance–particles/liquids	Heat transfer coefficients	Rumsey (1984)
Mushroom-shaped aluminum in water	80°C; heating bath	Energy balance	Heat transfer coefficients	AlhAMDAN <i>et al.</i> (1990)
Mushroom-shaped aluminum in carboxymethylcellulose	80°C; heating bath	Energy balance	Dimensionless correlations	AlhAMDAN and Sastry (1990)
Mushrooms/brine	121°C; pilot retort Omnia jars 0–8	Energy balance–thermal inertia	Temperature profile	Akterian (1995)
Fruits in syrup	93.3°C; retort; Omnia jars 105 mm diam	Empirical model	f_h & heat-transfer coefficients	Akterian (1996)
Vegetables in brine	121°C; retort; Omnia 105 mm diam	Empirical model	f_h & heat-transfer coefficients	Akterian (1996)
Food particles/CMC	cans	Energy balance	Surface heat-transfer coefficients	AwuH <i>et al.</i> (1993)

The energy balance model was also used by Sastry (1984) to determine heat-transfer coefficients to canned aluminum mushroom-shaped particles packed in water. The heat-transfer coefficients were correlated using the relationship:

$$Nu = 0.01561(Gr \cdot Pr)^{0.529}.$$

The convective heat-transfer coefficient was found to be time-dependent, increasing to a maximum during the early stages of heating and steadily declining. The mean heat-transfer coefficients varied between 396 and 593 W m⁻² K⁻¹.

Another application of the energy balance model was to determine the heat-transfer coefficients between irregular-shaped particulates and Newtonian as well as non-Newtonian covering liquids (Alhamdan *et al.* 1990; Alhamdan and Sastry 1990). For water, dimensionless correlations were, for heating,

$$Nu = 5.53(Gr \cdot Pr)^{0.21},$$

and for cooling,

$$Nu = 0.08(Gr \cdot Pr)^{0.27}.$$

The average heat-transfer coefficients ranged between 652 and 850 W m⁻² K⁻¹ for heating and 384–616 W m⁻² K⁻¹. For a non-Newtonian liquid, carboxymethyl cellulose solution, the heat-transfer coefficients were for heating 75–310 W m⁻² K⁻¹ and for cooling 22–153 W m⁻² K⁻¹. The heat-transfer coefficients decreased with increasing apparent viscosity.

Stoforos and Merson (1990) developed a technique for estimating heat-transfer coefficients in liquid/particulate canned foods using only liquid temperature data. They solved the energy balance equation, modified for particulates, analytically using Laplace transformations.

2.3.4.5. Heterogeneous Products Heated with Agitation

In this category there are a large number of products containing particulates and covering liquid, which can benefit from being processed in rotary retorts (see Chapter 8). Processes are shorter, energy can be saved, and textural and other quality attributes can be improved. Using rotary processes requires the contents of each container to receive the same rotary motion; this means that the ratio of solids to liquids and the headspace in each container have to be carefully controlled. Table 2.9 gives some examples of experimental work that has been done to investigate the heat transfer into rotating and agitated containers. Relatively little work has been done on the quantitative aspects of the subject: three theses by Hassan (1984), Deniston (1984), and Stoforos (1988) and an overview by Stoforos and Merson (1990) cover what has been done. The technology has received a thorough treatment in the work of Manfred Eisner (1988). Some of the more important contributions will now be discussed.

Lenz and Lund (1978) derived a useful heat-transfer correlation for spherical particulates 9.65–38.1 mm in diameter and void fraction 0.32–0.41 in covering liquids of both water and 60 wt% sucrose solution using 303 × 406 and

TABLE 2.9. Some studies on convection heat transfer for agitated heterogeneous canned foods.

Product	Process	Model	Comments	Reference
Silicone oil, sausage	Laboratory device, 120°C; axial & end-over-end	Energy balance & particle conduction	Heat-transfer coefficients – duration of transfer unit	Bimbenet and Michiels (1974)
Water & sucrose solution, peas & lead shot	FMC Steritort, 121°C; axial rotation 303 × 406; 608 × 700 can	Energy balance & particle conduction	Temperature profiles – heat-transfer coefficients	Lenz and Lund (1978)
Potatoes/water	Stock Pilot Rotor-900, 121°C; end-over-end 307 × 409 cans	Energy balance & particle conduction	Effect of size distribution & surface heat transfer	Lekwauwa and Hayakawa (1986)
Water, silicone fluid, PTFE, aluminum, potato	Rotary single can simulator, 50°C; axial rotation	Energy balance & particle conduction	Analytical solution, heat-transfer coefficients	Deniston <i>et al.</i> (1987), Stoforos (1988), Merson and Stoforos (1990)
Spheres	303 × 406 can			Stoforos & Merson (1990, 1992)
Water, sucrose solution, raw snap beans & aluminum	FMC Steritort, 115.6°C; axial rotation 303 × 406	Energy balance & particle conduction	Heat-transfer coefficients – dimensionless correlation	Fernandez <i>et al.</i> (1988)
Sucrose syrup/particles	100 × 119 mm cans; axial rotation; ambient temperature	Position emission particle tracking	Head space & mixing study	Cox <i>et al.</i> (2003)
Spheres, cylinders, cubes/liquid	Stock Rotomat-PR 900	Flow visualization	Heat-transfer coefficients-mixing studies	Sablani and Ramaswamy (1998)
Nylon spheres/water & /oil	Stock Rotomat-PR 900	Experimental correlations	Heat-transfer coefficients–influence of particle size & concentration	Sablani and Ramaswamy (1997)
Poly-propylene spheres/water	110–130°C; full immersion water sterilizer	Experimental correlations	Heat-transfer coefficients; effect of head space	Sablani and Ramaswamy (1996)
Spheres, cylinders, cubes/liquid	Stock Rotomat- PR 900	Flow visualization	Heat-transfer coefficients & mixing studies	Sablani & Ramaswamy (1993, 1998)
Potato shape 7 aluminum spheres	Axial rotation	Energy balance & particle conduction	Heat-transfer coefficients & 4th order Runge-Kutta model	Stoforos & Merson (1992, 1995)

608 × 700 cans. These were processed in an FMC Steritort pilot simulator. The correlations were, for liquids only

$$Nu = 115 + 15Re^{0.3}Pr^{0.08}, \quad (2.107)$$

for liquids and particulates,

$$Nu = -33 + 53Re^{0.28}Pr^{0.14}X, \quad (2.108)$$

where $X = D_p/[S(1 - \varepsilon)]$ and D_p is the particle diameter, S is the radius of the reel in the simulator, and ε is the fraction of the container volume occupied by the fluid. Values for the particulate–liquid heat-transfer coefficients ranged from 180 to 970 W m⁻² K⁻¹.

Fernandez *et al.* (1988) used the simple energy balance model to determine the heat-transfer coefficients between canned snap beans and the covering liquid in 303 × 406 cans. The best correlation obtained for the experimental data was

$$Nu = 2.7 \times 10^4 Re^{0.294} Pr^{0.33} (S_f)^{6.98}, \quad (2.109)$$

where the Reynolds number was in a rotational form $D^2 N \rho / \mu$, in which D is the can diameter and N the number of rotations per minute, and S_f is a shape factor given by $\pi(6V_p/\pi)^{2/3} A_p^{-1}$, with V_p and A_p being, respectively, the particulate volume and area.

Merson and Stoforos (1990) studied the effect of axial rotation on heat-transfer coefficient and the effect of the relative velocity between the particle and the liquid. They obtained a more complex correlation, in which the first term on the right includes particulate conduction, the second, heat transfer in the laminar boundary layer on the upstream side of the sphere, and the third term is the heat transfer in the turbulent wake.

$$Nu = 2 + 0.4Re^{0.5}Pr^{0.4}(\mu/\mu_s)^{0.25} + 0.06Re^{0.66}Pr^{0.4}(\mu/\mu_s)^{0.25}. \quad (2.110)$$

A semi-analytical solution for determining particle temperature and a semi-numerical solution for determining fluid temperature has been applied to the solution of heat transfer in axially rotating liquid/particle canned foods. The time dependent boundary condition was modeled using Duhamel's theorem and a 4th-order Runge-Kutta technique was used for the solution of the finite difference mode. (Storofos & Merson 1995).

This measurement of particulate temperatures is difficult. The presence of a thinwire thermocouple prevents free motion in the particle, and consequently could seriously affect the actual value of the heat-transfer coefficients. Liquid crystal sensors incorporated in the particulates have been used, which allows unrestricted movement of the monitored particulates (Storofos and Merson 1991, 1992). For a critical overview concerning heat-transfer coefficients to particulates, see Maesmans *et al.* (1992).

Sablani *et al.* (1997a) have used dimensionless correlations to determine the heat-transfer coefficients for a range of different plastic particles in both oil and water undergoing end-over-end agitation. The correlation for multiparticles

involves the ratio of the particle diameter (d_e) to that of the can diameter (D_c) see equation (2.111).

$$Nu = 0.71Re^{0.44}Pr^{0.36}(\varepsilon/100 - \varepsilon)^{-0.37}(d_e/D_c)^{-0.11}\Psi^{0.24} \quad (2.111)$$

where ε is the particle concentration and Ψ is the particle sphericity.

2.3.5. Conclusions

While a considerable amount of effort has been put into modelling convective heating of liquids and liquids/particulates in containers, there are wide discrepancies in the results obtained by different workers. With the development of more advanced techniques for monitoring temperatures inside containers and with the ability to solve coupled hydrodynamic and heat transfer problems using finite-element techniques and computational fluid dynamics, it should be possible in the future to obtain more reliable models capable of extrapolation to a range of can sizes. At the present time the work on convective heating has led to a better understanding of the heat-transfer regimes inside canned foods. This in turn has given us a better understanding of the factors that must be considered in order to be assured of the microbiological safety of canned foods. This is particularly important in reducing the severity of processes to give better-quality products, where this is appropriate, and to reduce the energy requirements. It is important to consider carefully the meaning of heat-transfer coefficients; whereas it is relatively easy to obtain an overall coefficient, which includes the internal coefficient from the wall to the covering liquid, the average heat transfer from the liquid to the particulate, the container wall resistance and the external heat-transfer coefficient from the heating medium to the external wall, it is more difficult to quantify the instantaneous local distribution of heat transfer depending upon the particle motion relative to the fluid motion. It may be, because of the unpredictability of the relative liquid/particulate motion inside cans, that we have reached the limit of the usefulness of this approach.

2.4. Radiation Heating

Thermal radiation has a wavelength from 0.8 to 400 μm and requires no medium to transmit its energy. The transfer of energy by radiation involves three processes: first, the conversion of the thermal energy of a hot source into electromagnetic waves; second, the passage of the waves from the hot source to the cold receiver; and third, the absorption and reconversion of the electromagnetic waves into thermal energy.

The quantity of energy radiated from the surface per unit time is the emissive power E of the surface. For a perfect radiator, known as a black body, the emissive power is proportional to the fourth power of the temperature T :

$$E = \sigma T^4, \quad (2.112)$$

where σ is the Stefan–Boltzmann constant, $5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$, and the heat-transfer equation for the net exchange of heat between two bodies at temperatures T_1 and T_2 , respectively, is given by

$$Q = \varepsilon \sigma (T_1^4 - T_2^4), \quad (2.113)$$

where Q is the net radiation per unit area, and ε is the emissivity, i.e. the ratio of the energy emitted by a grey body to that emitted by a black body. For a highly polished surface ε can be as low as 0.03, and for a black body ε is 1.

The heat-transfer coefficient for radiation can be obtained from equation (2.113) as follows:

$$\begin{aligned} Q &= h_r (T_1 - T_2) \\ h_r &= \frac{Q}{T_1 - T_2} = \frac{\varepsilon \sigma}{T_1 - T_2} (T_1^4 - T_2^4). \end{aligned} \quad (2.114)$$

The main application for radiation theory is in relation to can sterilization using gaseous flames. The cooling of cans to the atmosphere also involves loss of heat by radiation as well as convection.

A more complex heat-transfer model for canned foods heated in a flame sterilizer was developed by Peralta Rodriguez and Merson (1983). The following assumptions were made in developing the model:

1. The flow of combustion gases past the can surface is uniform.
2. A single average heat-transfer coefficient applies to the whole of the can surface.
3. Heat transfer through the end of the can is neglected.
4. The flame is considered to be a regular-shaped gaseous body.
5. Radiation from the hot gases after they have left the flame is neglected.
6. Radiation exchange only occurs between the flame and the can, and from the can to the surroundings.
7. The temperature of the flame is the adiabatic temperature calculated on the dissociation of the combustion species.
8. The dissociated species do not combine on the can surface.

The heat balance equation is

$$Q_T = Q_C + Q_R, \quad (2.115)$$

where Q is the rate of heat transfer and the subscripts T , C , R refer to total, convection and radiation, respectively.

The convective contribution is given by

$$Q_C = h_c A_c (T_g - T_{\text{sur}}), \quad (2.116)$$

where h_c is the average film heat-transfer coefficient for the surface, A_c is the surface area of the can, T_g is the gas temperature, and T_{sur} is the temperature of the surface of the can.

The radiative contribution is given by

$$Q_R = Q_i - Q_e - Q_r, \quad (2.117)$$

where Q_i is the radial energy reaching the surface, Q_e is the energy re-emitted, and Q_r is the reflected energy. These three terms are given by the following equations:

$$Q_i = \varepsilon_f A F_{fc} \sigma T_f^4, \quad (2.118)$$

where the subscript f refers to the flame properties and F_{fc} is the dimensionless view factor from the flame to the can;

$$Q_e = \varepsilon_c A_c \sigma T_{sur}^4, \quad (2.119)$$

where the subscript c refers to the can; and

$$Q_r = \rho_c \varepsilon_f A_f F_{fc} \sigma T_f^4, \quad (2.120)$$

where ρ_c is the reflectivity of the can. Combining these equations we obtain

$$Q_R = (1 - \rho_c) \varepsilon_f A_f F_{fc} \sigma T_f^4 - \varepsilon_c A_c \sigma T_{sur}^4. \quad (2.121)$$

From Kirchhoff's laws of radiation, the absorptivity α_c is equal to the emissivity ε_c . This, and using the reciprocity relation, $A_f F_{fc} = A_c F_{cf}$, give

$$Q_R = \alpha_c A_c \sigma (\varepsilon_f F_{cf} T_f^4 - T_c^4). \quad (2.122)$$

Using these equations and other basic heat-transfer equations, the temperature of the liquid T_1 was derived:

$$\frac{T_1 - T_g}{T_0 - T_g} = \exp(-Xt) - \Psi[1 - \exp(-Xt)], \quad (2.123)$$

where T_0 is the initial temperature of the fluid, T_g is the temperature of the radiating gas, and

$$X = A_c U_c / Mc$$

$$\Psi = [\alpha_c \sigma (\varepsilon_f F_{cf} T_f^4 - T_{sur}^4)] / [h_c (T_g - T_0)],$$

in which M is the mass of fluid, c is the specific heat, and U_c is the overall heat-transfer coefficient, based on the log mean cylindrical area.

For a 60% sucrose solution in a 304×406 can and using an average flame temperature up to 1000°C , the external heat-transfer coefficient was $26.38 \text{ W m}^{-2} \text{ K}^{-1}$ and the internal coefficient ranged from 220 to $621 \text{ W m}^{-2} \text{ K}^{-1}$, depending on the rotational speed of the can. The overall heat-transfer coefficient, U_c , varied from 23.86 to $25.12 \text{ W m}^{-2} \text{ K}^{-1}$ and did not increase with increasing rotational speeds above 60 rpm. The flame emissivity was taken as 0.0241 and the absorptivity of the can body 0.055. From this work it was concluded that the external heat-transfer coefficient, unlike steam sterilization, was the rate-controlling step. Table 2.10 gives some information about other work

TABLE 2.10. Some heat transfer studies for radiation heating from flames.

Product	Container size	Process conditions	Comments	Reference
Water	$401 \times 411 (A2 \frac{1}{2})$	Flame sterilization Single can	Heating curves External temperature Profile	Paulus and Ojo (1974)
Bentonite/water 4 & 10%		Axial rotation 0–60 rpm		
Bentonite 1%	$303 \times 406 (\frac{1}{2} \text{ kg})$	Flame sterilization Single can	Energy balance model Internal heat transfer Coefficients	Merson <i>et al.</i> (1981)
Sucrose syrup 60%		Axial rotation 30 & 50 rpm		
Sucrose syrup 60%	$303 \times 406 (\frac{1}{2} \text{ kg})$	Flame sterilization Single can	Radiation & convective Convective model	Peralta Rodriguez (1982, 1987a) Peralta Rodriguez & Merson (1982, 1983)
Silicone fluids		Axial rotation 10–200 rpm	Heat-transfer coefficients	
Water		Flame sterilization Single can	Energy balance model	Teixeira Neto (1982)
Carboxymethylcellulose/water 1.5–2.5%	$303 \times 406 (\frac{1}{2} \text{ kg})$	Flame sterilization Single can	Heat-transfer coefficients	
		Axial rotation 50–200 rpm		
Bentonite/water 1%	209×413 (aluminum)	Flame pasteurization Single can	Energy balance model	Noh <i>et al.</i> (1986)
		Axial rotation 40 rpm	Temperature profiles Heat-transfer coefficients	

on heat transfer to cans being sterilized by flame heating. Further aspects of flame sterilization are dealt with in chapter 8.

2.5. Some Computer Programs

One of the most important developments in the study of thermal processing has been the application of computer simulation programs. This has enabled the operation of processing equipment to be improved by elucidating the mechanisms of heat transfer and fluid flow in processing systems. The following selected programs have been applied, not only to process evaluation but also to control problems.

2.5.1. Conduction Heat Transfer Analysis Programs

(i) **ANSYS Finite Element Software** (deSalvo & Gorman 1989). Used by a number of workers, including, Hendrickx *et al.* (1992) to determine the center sterilization value in a food can and to optimize the nutrient retention, and Christianini and Massaguer (2002) for evaluating the thermal process for tuna/brine in institutional retort pouches. The latter work is interesting because of a comparison between the analytical solution given in §2.2.8.2. and both a 1-D and a 3-D Finite element solution. All three models give good agreement for the heating phase; however, for the cooling phase the finite-element models were better than the analytical solution.

(ii) **CHAMPSPACK** Uses Matlab and Fortran routines for finite element computations of coupled heat and mass transfer problems with internal heat generation and convective boundary conditions. It was developed for the design and simulation of thermal process applications for foods and agricultural products (Scheerlinck & Nicolai 1999). The package includes facilities for insertion of kinetic data for microbial destruction and quality factor inactivation. For application to enzyme inactivation in vegetables see Martens *et al.* (2001).

(iii) **SPEEDUP™** (Simulating Program for Evaluating and Evolutionary Design of Unsteady State Process) Alwis *et al.* (1992) used the SPEEDUP™ system developed by Sargent *et al.* (1982) for optimizing quality and processing effects of a conduction-heating meat product. This showed the need for higher temperatures and shorter times for obtaining better meat quality.

(iv) **NEURAL Networks** Neural networks have been used to predict optimum sterilization temperatures and corresponding process time and quality factors. The method has the advantage over conventional regression models that all three parameters can be determined simultaneously (Sablini *et al.* 1995). Neural network modeling has also been used to study heat transfer to liquid particle mixtures in cans undergoing end-over-end processing (Sablini *et al.* 1997b).

2.5.1.1. Computational Fluid Dynamics (CFD) programs

(i) **PHOENICS** (Cham Ltd., London, UK) Simulations using a PHOENICS package were used to determine the relative concentration profiles of the spores and also the temperature profiles. An application to microbial inactivation is discussed by Ghani *et al.* (1999a, b; 2002a, c), Ghani *et al.* (2001b).

(ii) **CFX/TASCflow** (CFX International, AEA Technology, Harwell, UK) This has been used to simulate a range of canning applications, e.g., transient natural convection in containers filled with non-Newtonian CMC solution (Quarini & Scott 1997), the axial and end-over-end rotation in canned foods (Emond & Tucker 2001; James 2001; Hughes *et al.* 2003; Tucker 2004) and also natural convection heating of canned foods in conical and cylindrical containers (Varma *et al.* 2006).

(iii) **FLOWPACK** (ICI, Ltd., UK) – Zone Modeling Tucker *et al.* (1992) have developed zone modeling as a method of determining temperature profile in simple geometrically shaped canned food. Best *et al.* (1994a, b) have presented a steady-state and a dynamic simulation of the processing of cans in a hydrolock sterilizer using zone modeling and a computer program “Flowpack.”

(iv) **Others STAR-CD** (Computational Dynamics Ltd. London UK).

Fluent/FIDAP (Lebanon, NH, USA).

Fluent v. 6.1 has been used by Jun and Sastry (2005) to study the effect pulsed ohmic heating has on food in flexible pouches.

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