Chapter 2. Water as a Natural Resource

The basic hydropower model

Some studies of hydropower at a high level of aggregation disregard the storage process and specify directly the available water within, e.g., a yearly weather cycle. The assumptions are then that there is no spill of water or binding upper reservoir constraint, and no emptying of the reservoir until the terminal period. The modelling can then be simplified by disregarding the water-accumulation relation (1.4). Another way for this specification to make sense in our framework would be for all the water to be present in the first period. The time profile of inflows should be such that the bulk of inflow comes in one period and then there is a natural seasonal precipitation cycle with little inflow until one year later. The snow, melting during a few spring and summer weeks, fills the reservoirs with about two thirds of the yearly total in Norway. This is illustrated in Figure 1.4 in Chapter 1. The inflow is low in other periods except for autumn rains. However, there are huge variations up to ± 30% from year to year in the pattern of inflow.

In the case of all water being present in the first period, utilisation of water within a horizon can be regarded as a problem of managing a resource of finite amount, just like extraction of non-renewable resources like oil.

As can be seen from Figure 1.4 the validity of the assumption of inflow in only one period depends on the length of the time period. The time periods can be arranged such that inflow occurs in the first period. The basic model is then obtained by assuming that there is inflow only in the first period, and furthermore we assume that the production of electricity is efficient, i.e., we have equality in the production function (1.2). Finally there is unlimited transferability of water to the other periods of the given total amount of water available after the first period. The sum of all releases must then equal the inflow in period 1. Using the production function (1.2) yields:

$$\sum_{t=1}^{T} r_t = w_1 \Rightarrow \sum_{t=1}^{T} a e_t^H = w_1 ,$$
$$\sum_{t=1}^{T} e_t^H = \frac{w_1}{a} = W \tag{2.1}$$

The horizon, T, is assumed to cover a seasonal cycle (one year) from spring to spring. In the first line of equation (2.1) water is measured in m^3, while in the second line of (2.1) water is measured in kWh by using the fabrication coefficient from (1.2) as deflator. Although the variable, W, representing total available inflow, is measured in energy units, kWh, we will still call W water. By assuming no wasting of water as a factor of production in producing electricity, the conversion from water to electricity does not have to be modelled as a separate relationship, but production substituted for the releases as in (2.1).

We will investigate the resource use problem as a standard social planning problem. The energy consumption in each period is evaluated by utility functions, which can be thought of as either valid for a representative consumer or constituting a welfare function. Simplifying further, there is no discounting. The horizon is at any rate usually too short for discounting to be of practical significance (however, Norway has a large proportion of multi-year reservoirs, implying that a rather long horizon, usually three to five years, is warranted). The period utility functions representing the social value of electricity consumption are:

$$U_t(e_t^H) \ , \ U_t'(e_t^H) \geq 0 \ , \ U_t''(e_t^H) < 0 \ , \quad t = 1,..,T \tag{2.2}$$

The utility functions have the standard property of concavity. The marginal utility U_t' measured in monetary units, is defined as the marginal willingness to pay, p_t, i.e., defining the *demand function* (on price form) for electricity:

$$U_t'(e_t^H) \equiv p_t(e_t^H) \tag{2.3}$$

The marginal willingness to pay for electricity is also referred to as the social price (p_t) of electricity or price for short below. We will assume that this demand function has normal properties, e.g., decreasing in quantity corresponding to the assumption about the curvature of the utility function. In light of the brief discussion in Chapter 1 about the sensitivity of demand for electricity to current price, the time period considered should not be too short.

The social optimisation problem can be formulated as follows:

$$\begin{aligned}
&\max \sum_{t=1}^{T} U_t(e_t^H) \\
&\text{subject to} \\
&\sum_{t=1}^{T} e_t^H \le W, e_t^H \ge 0, t = 1,..,T \\
&W, T \text{ given}
\end{aligned} \tag{2.4}$$

The horizon ends at T and there is no amount of water handed over to period $T + 1$. This assumption may be acceptable if the number of periods T corresponds with almost emptying the reservoir levels due to typical seasonal variation in inflows. (Introducing a lower constraint on water handed over and/or specifying a scrap-value function will be followed up in Chapter 3). The endogenous variables are the electricity production (corresponding uniquely to water use) in each period. To find a solution to the optimisation problem above, we will use a standard nonlinear programming approach (see Sydsæter et al., 1999, 2005).

The Lagrangian function for problem (2.4) is:

$$L = \sum_{t=1}^{T} U_t(e_t^H) - \lambda(\sum_{t=1}^{T} e_t^H - W) , \tag{2.5}$$

where λ is the Lagrangian parameter. Necessary first-order conditions for this problem, where all the variables are non-negative, are:

$$\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = U_t'(e_t^H) - \lambda \le 0 \; (= 0 \text{ for } e_t^H > 0), t = 1,..,T \\
&\lambda \ge 0 \; (= 0 \text{ for} \sum_{t=1}^{T} e_t^H < W)
\end{aligned} \tag{2.6}$$

The endogenous variables are e_t^H, λ ($t = 1,\dots,T$), $T + 1$ variables in all, and exogenous variables are W, T , two in all. The number of equations is the T first-order conditions in (2.6) and the resource constraint from (2.4), yielding as many endogenous variables as equations. We are conducting a qualitative analysis assuming that a unique solution to problem (2.4) exists. A sufficient condition for a solution to problem (2.4) is that the Lagrangian (2.5) is concave, which is satisfied under our assumptions. Therefore we focus our attention on interpreting the first-order conditions (2.6).

From the Kuhn – Tucker conditions (2.6) we know that the marginal utility of electricity consumption is equal to the shadow price on the resource constraint if we have an interior solution for the energy consumption for period t, i.e., $e_t^H > 0$. The shadow price on the resource constraint is zero if the constraint is not binding. The general interpretation of a shadow price on a constraint is that it shows the change in the objective function of a marginal change of the constraint. In our case the shadow price shows the increase in the sum of utilities over all periods of a marginal increase in stored water, W.

In such a highly stylised model as above it is reasonable to assume that there is positive consumption of electricity in each period and that consumption is not satiated, i.e., that marginal utility is positive in all periods. It then follows that the shadow price on the resource constraint must be positive. The typical conclusion in this basic model with a given amount of resources is that the marginal utility of electricity is constant and equal for all periods:

$$U_t'(e_t^H) = \lambda \text{ for all } t = 1,..,T \tag{2.7}$$

As mentioned above when measuring utility in money, marginal utility may be interpreted as the demand function for electricity on price form. The result of the basic model can then be equivalently stated as the price of electricity being the same for all periods. This is *Hotelling's rule* for the resource price for our model. We do not discount, and by arbitrage of the water asset the social price must be the same for all periods. If prices were different, then, by the assumption of unlimited transferability of water between the periods, transferring water to high-price periods will increase welfare until the prices are equalised in the optimal solution. The shadow price on the water resource constraint measures the increase in the sum of utilities of a marginal increase in the resource, and due to perfect transferability between periods there is only one shadow price.

The typical solution for both periods is illustrated in Figure 2.1 in the case of two periods via a *bathtub* diagram. The two marginal-willingness-to-pay-functions are measured along the left- and right-hand vertical axes for period 1 and period 2, respectively. Total available electrical energy in kWh for the two periods corresponds to the horizontal length of the bathtub. The economic interpretation of the solution to the allocation problem is that electricity should be allocated between the periods in such a way that the shadow price of electricity (i.e., the increase in the objective function of a marginal increase in the given amount of total energy) is equal to the marginal utility of energy in each period, and thus the marginal utilities become equal. In Figure 2.1, if period 1 is summer and period 2 winter, the marginal utility should be equal. Although the marginal utility of energy

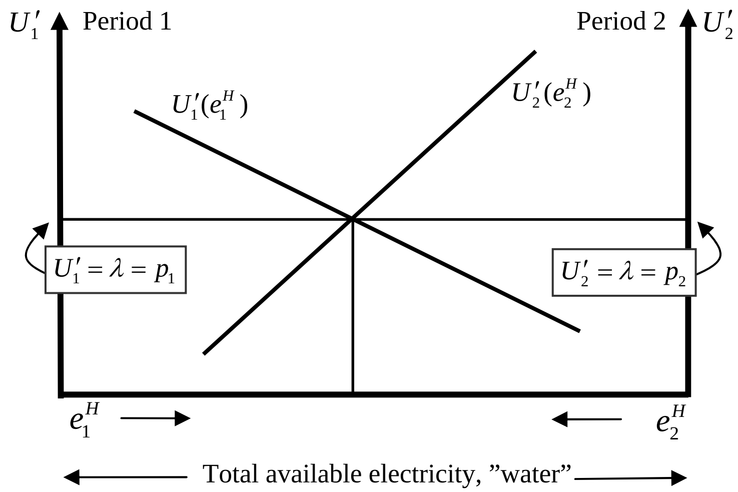

Figure 2.1. Bathtub illustration of optimal allocation of electricity between two periods.

consumption may be higher in winter than in summer for the same level of consumption, marginal utility in the winter should not become greater than in summer in the optimal solution. The consumption in the winter may be substantially higher than in the summer, just as we saw in Figure 1.2 in Chapter 1 for summer and winter days and in Figure 1.4 for weekly periods. The solution for the shadow price is such that all available water is just used up for electricity production.

Water as a non-renewable resource: Hotelling revisited

In the problem (2.4) above water appears as if it is a non-renewable resource with a known initial deposit like oil or minerals since the horizon ends at T. The Hotelling rule for a change in the price of a non-renewable resource is usually stated as requiring the resource price to increase with the discount rate. We introduce discounting in our model to show how the familiar form of the Hotelling rule can be derived. Denoting the discount factor β_t we have the following optimisation problem:

$$\max \sum_{t=1}^{T} U_t(e_t^H)\beta_t$$

$$\text{subject to} \qquad \sum_{t=1}^{T} e_t^H \leq W \tag{2.8}$$

The discount factor is in discrete time specified as

$$\beta_t = (1+r)^{-(t-1)},\ t = 1,..,T\ , \tag{2.9}$$

where r is the rate of discount, assumed to be the same for all periods. The utilities are discounted to period 1, so the discount factor for this period is 1. Notice that the discount rate must correspond to the period length in question, e.g., if a yearly rate is 5%, then if the time period is a week, using the rule for compound interest rate, the weekly discount rate is $r = 0.0009$ and $\beta_2 = 0.999$.

The first-order conditions are straightforward extensions of (2.6):

$$\begin{aligned} &\frac{\partial L}{\partial e_t^H} = U_t'(e_t^H)\beta_t - \lambda \leq 0\ (= 0 \text{ for } e_t^H > 0),\ t = 1,..,T \\ &\lambda \geq 0\ (= 0 \text{ for} \sum_{t=1}^{T} e_t^H < W) \end{aligned} \tag{2.10}$$

The discounted marginal utilities shall be set equal for all periods and equal to the shadow price on the water resource constraint. The shadow price now measures the change in the *discounted* sum of utilities of a marginal change in the amount of the resource.

The growth rate in marginal utility is found by first using the first-order condition (2.10) for period t and $t + 1$ substituting for the discount factor from (2.9):

$$\begin{aligned} &U_t'(e_t^H)\beta_t = U_{t+1}'(e_{t+1}^H)\beta_{t+1} \\ &\Rightarrow U_{t+1}'(e_{t+1}^H) = U_t'(e_t^H)\frac{\beta_t}{\beta_{t+1}} = U_t'(e_t^H)(1+r) \end{aligned} \tag{2.11}$$

The growth rate in marginal utility from period t to period $t + 1$ is then:

$$\frac{U_{t+1}'(e_{t+1}^H) - U_t'(e_t^H)}{U_t'(e_t^H)} = \frac{U_t'(e_t^H)(1+r) - U_t'(e_t^H)}{U_t'(e_t^H)} = r \tag{2.12}$$

The growth rate is the rate of discount, just as the Hotelling rule tells us about the resource price. Remembering that the marginal utilities by definition (2.3) are interpreted as prices, we have established the Hotelling rule:

$$\frac{p_{t+1}(e_{t+1}^H) - p_t(e_t^H)}{p_t(e_t^H)} = r \tag{2.13}$$

In light of the results of the previous section it should be emphasised that without discounting the fundamental insight of the Hotelling rule for the asset equilibrium, at least for time spans of restricted length, is not really the price growth, but the *level* of the prices. Empirical investigations of resource price development that only check the rate of growth are not so interesting unless the optimal level of prices is checked, too.

An illustration of the consequence of discounting is set out in Figure 2.2 for two periods. The optimal situations without discounting from Figure 2.1 are shown by the dotted lines. The discount factor is one in period 1. In period 2 the discount factor means that the discounted demand curve constitutes a downward vertical shift of the demand curve with the distance

$$U_2'(e_2^H) - U_2'(e_2^H)\beta_2 = U_2'(e_2^H)(1-(1+r)^{-1}) = U_2'(e_2^H)\frac{r}{1+r} \tag{2.14}$$

This curve is shown as the solid curve in Figure 2.2 for period 2. For period 1 the marginal utility and the price are equal to the shadow price on the total water resource. The allocation of electricity in the two periods is determined by the intersection of the demand curve for period 1 and the shifted demand curve for period 2. We see that discounting implies that

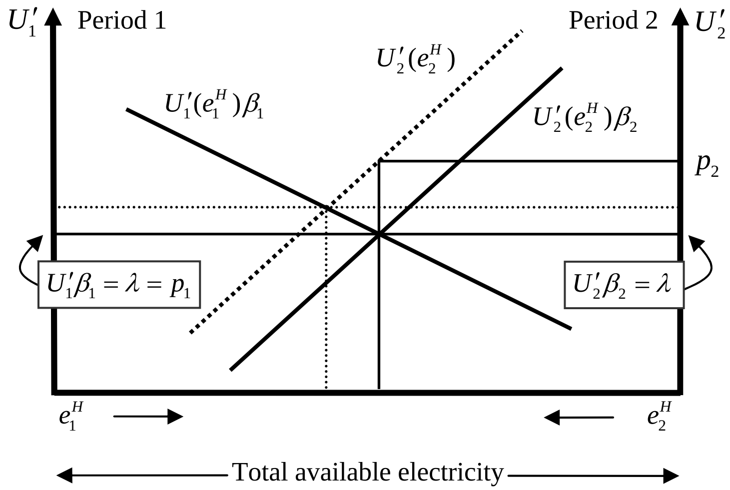

Figure 2.2. Bathtub illustration of the Hotelling rule with discounting. Situation without discounting shown by thin dotted lines.

more is consumed in the first period and less in the second compared with a situation without discounting. The shadow price on the water resource is lower with discounting. This reflects the fact that discounting means more of the resource is preferred to be consumed earlier, and to realise this, prices in earlier periods must be decreased. The price for period 2 is found by going up to the period 2 demand curve. The period 2 price is higher than the period 1 price in accordance with the Hotelling rule.

An interesting economic question is how the endogenous variables change in response to changes in exogenous variables. The consequence of a change in the rate of discount can be found by differentiating the discount factor (2.9) with respect to the rate of discount:

$$\frac{\partial \beta_t}{\partial r} = \frac{\partial (1+r)^{-(t-1)}}{\partial r} = -(t-1)(1+r)^{-t} < 0 \ (t = 2,..,T) \tag{2.15}$$

The reduction in the discount factor (increase in the rate of discount) means that future periods count less in the objective function in the optimisation problem (2.8). The effect is illustrated in Figure 2.3, based on Figure 2.2. The dotted lines represent the situation before an increase in the rate of discount and the solid lines the situation after the increase. The dotted demand curve for period 2 reflects the value of the discount factor before the change and the solid demand curve reflects the value of the discount factor after the change. With less emphasis on the future more will be consumed in the first period. The price then has to go down in the first

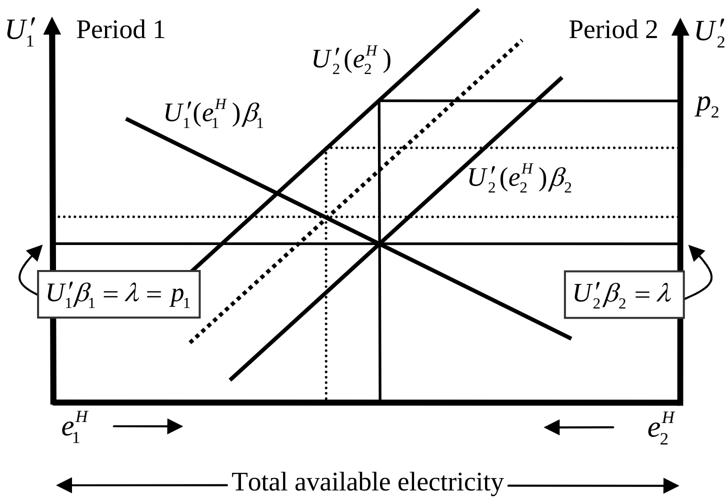

Figure 2.3. An increase in the rate of discount. Situation before change shown by dotted lines.

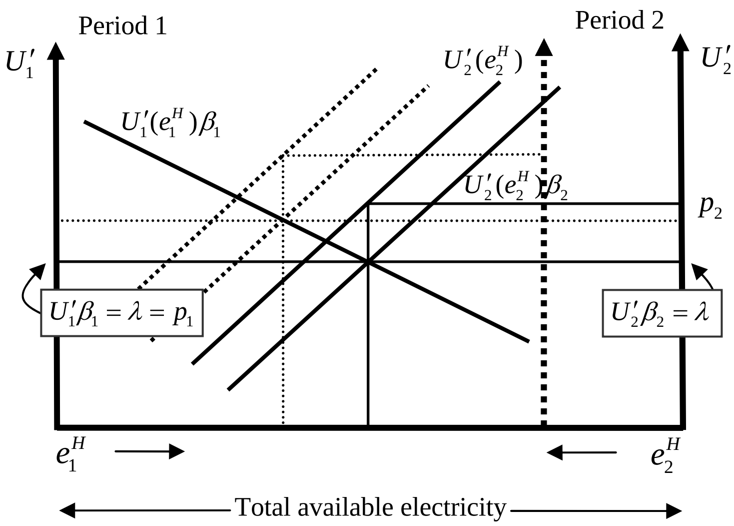

Figure 2.4. An increase in the amount of the resource.
Situation before increase shown by thin dotted lines.

period, and the price is increased in the second period to match the decease in the availability of electricity. There is a downward vertical shift in the discounted demand curve for period 2, as is also evident from the expression (2.14) for the distance between the marginal-willingness-to-pay curve and the discounted curve for period 2.

An increase in the availability of water can in the two-period case be illustrated by letting the bathtub wall for period 2 shift outwards to the right in Figure 2.2 as done in Figure 2.4. The dotted curves illustrate the situation in Figure 2.2 before the change in the availability of water, while the two solid demand curves for period 2 represent the situation after the increase in the resource. The shadow price on the resource decreases, as do both period prices. Consumption in each period correspondingly increases.

Several user groups

In the model the water is used to (costless) produce electricity. But we may also consider preferences for water directly by substituting the release r_t for electricity in the utility functions. Water resources are also of interest for activities other than producing electricity. An interesting problem is then how to allocate water if there is competing interest for the water resource. Broad water use groups may be households, industry, agriculture and hydropower. In the case of drinking water for households the interest may lie in the utility of different groups of households, for instance representing different income groups or living within specific locations. As to farmers, industry and hydropower plants, it may be more appropriate to operate with profit functions. However, we will use utility functions for water user groups without being more specific. The G groups are indexed with a superscript g:

$$U_t^g(r_t^g)\ ,\ U_t^{g\prime} \geq 0, U_t^{g\prime\prime} \leq 0,\ \ g = 1,..,G,\ t = 1,..,T \tag{2.16}$$

The release of water, r_t^g, to each group is drawn from a common reservoir. Utility function can vary over time periods because households' utility of water may vary with outdoor temperature and for agriculture utility may vary with growth season. Industry demand may be more neutral as to time periods.

We will still use the reservoir model (1.4) in Chapter 1, and either assume that all inflows of water occur in the first period or that the upper constraint on the reservoir is never binding and that the reservoir is not emptied until the terminal period. The water constraint can be aggregated into a single one and expressed analogously to (2.1):

$$\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g \leq W \tag{2.17}$$

Both the total water resource W and the release r_t^g from the reservoir are now measured directly in m^3. The user groups draw water from the same source. The priority given to different user groups is taken care of by specifying a social benefit or welfare function, $B(.)$, constant over time for simplicity, in the utilities of the user groups. This benefit function has the traditional properties from welfare theory, i.e., it is increasing at a decreasing rate in all the utilities. The social planning problem can then be formulated as:

$$\begin{aligned}
&\max \sum_{t=1}^{T} B(U_t^1(r_t^1),..,U_t^n(r_t^G)) \\
&\text{subject to} \\
&\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g \leq W \\
&r_t^g \geq 0, g = 1,..,G, t = 1,..,T \\
&T, W \text{ given}
\end{aligned} \tag{2.18}$$

It is straightforward to introduce discounting in the model using discount factors such as β_t in the previous section.

The Lagrangian is:

$$L = \sum_{t=1}^{T} B(U_t^1(r_t^1),..,U_t^G(r_t^G)) - \lambda(\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g - W) \tag{2.19}$$

The necessary first-order conditions are:

$$\begin{aligned} &\frac{\partial L}{\partial r_t^g} = B_g' U_t^{g\prime}(r_t^g) - \lambda \le 0 \; (= 0 \text{ for } r_t^g > 0), t = 1,..,T, g = 1,..,G \\ &\lambda \ge 0 \; (= 0 \text{ for} \sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g < W) \end{aligned} \tag{2.20}$$

The shadow price, λ, on the water constraint may now properly be termed the *water value* since it measures the change in the objective function of a marginal increase in the amount of water measured in m^3. Assuming that water is to be consumed for each group in each period we have that the discounted socially weighted marginal utilities of water consumption should all be equal between different user groups and equal over time,[1] and equal to the water value. The water value is the crucial equilibrating variable telling us that the socially weighted value of the marginal utility of drinking water should be set equal to the socially weighted value of marginal utility of irrigation water, equal to the socially weighted marginal utility of industry consumption and equal to the socially weighted marginal utility of hydropower water use.

If the distributional objective expressed by the benefit function is dropped, e.g., by specifying the benefit function as a pure summation of utilities, and in addition assuming that utilities are measured in money, then a total demand function (on price form) can be formed by adding (horizontally) the individual demands. Each group's marginal willingness to pay is now measured in the same unit, money:

$$\sum_{g=1}^{G} U_t^{g\prime}(r_t^g) = \sum_{g=1}^{G} D_t^g(r_t^g) = D_t(r_t) = p_t , \sum_{g=1}^{G} r_t^g = r_t \tag{2.21}$$

An optimal allocation of water between groups for a time period can be illustrated as in Figure 2.5, specifying three groups. The group demand curves derived from the marginal utilities measured in money are drawn as straight lines sloping downwards starting at finite levels at zero consump-

[1] If a discount factor is used, then the socially weighted marginal utilities will change correspondingly over time.

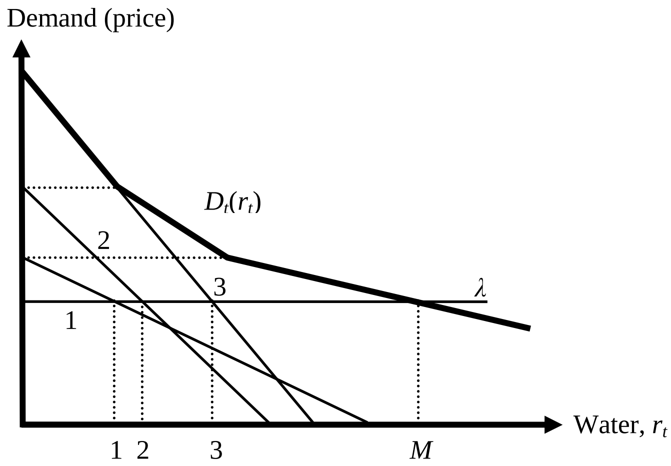

Figure 2.5. Aggregation of individual demand curves. Equilibrium water shares for period *t*.

tion of water. The individual group allocations are found by the intersections of the demand curves with the common horizontal shadow price line of the water resource. The levels are indicated by 1, 2, and 3 on the horizontal axis. If the shadow price is higher than the choke price, then no water is allocated to this group. The aggregated total demand curve is $D_t(r_t)$ and the total consumption is indicated by the point *M*.

As to the time allocation problem we could use the bathtub construction for two periods and extend Figure 2.5 to a figure like 2.1. The point of intersection of the aggregate demand curves will coincide with the value of the horizontal line for the shadow value of water. The social prices will be equal for each group for all time periods. The quantities allocated to the groups may vary with the time period, but the social price remains the same. (If discounting is introduced we get the same change in focus to discounted prices being equal as in the previous section.)

The allocation over time is illustrated in a bathtub diagram in Figure 2.6 for two periods. The allocation between the two periods is given by the intersection of the total demand curves and shown by the point *M* on the horizontal total water axis. The equilibrium price is given by λ and is equal both across user groups and periods. The three groups get the allocation of water in period 1 indicated by the vertical dotted lines marked 1, 2, and 3. The demand structure, keeping roughly the order of period 1, is such that group 1 now does not get any water in period 2. The willingness to pay is not high enough. This may be the case of irrigation water in the rainy sea-

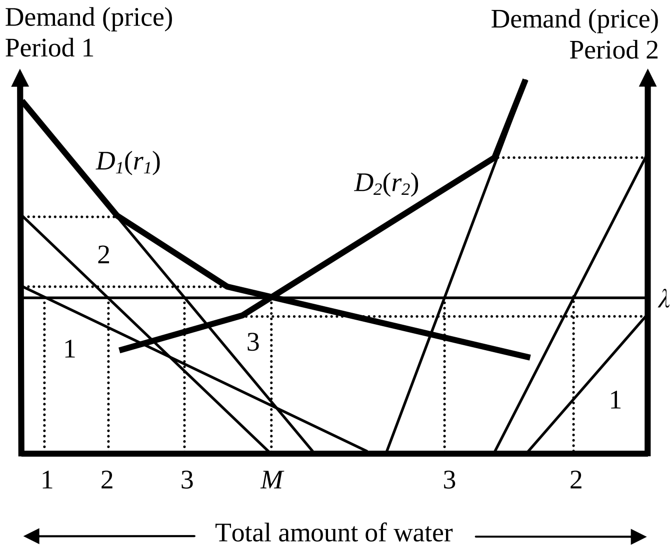

Figure 2.6. Allocation across groups and over time.

son. Groups 2 and 3 get the allocations indicated by the vertical dotted lines for period 2. Notice that the price is the same even if one period is a drought period and the other is a rainy season. Without uncertainty the water that is collected during the first period is always shared in such a way that the price is the same over time.

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