

SOLUTIONS TO END- OF-CHAPTER PROBLEMS

for

Introduction to Planetary Science;
The Geological Perspective

by

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Published by Springer, P.O. Box 17,
3300 AA Dordrecht, The Netherlands

2007

Chapter 1: The Urge to Explore

1. Select one of the several polar explorers and write a short report about him (2 pages double-spaced). Identify what aspect of planning and execution led to success or failure of the expedition:

- | | |
|----------------------|--------------------|
| 1. Roald Amundsen | 4. Fridtjof Nansen |
| 2. Robert F. Scott | 5. Lauge Koch |
| 3. Ernest Shackleton | 6. Peter Freuchen |

Attributes that are necessary for success:

1. Adequate supplies and equipment
2. Experience gained during prior expeditions
3. Leadership qualities
4. Wide range of skills necessary for survival
5. Adaptation to the environment
6. Selection of compatible companions
7. Strong motivation
8. Careful planning based on experience
9. Realistic expectations
10. Willingness to admit defeat
11. Careful risk management
12. Good luck

2. Solve the equation

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

first for the velocity (v) and then for the orbital radius (r)

$$v^2 = \frac{GM}{r}$$

$$v = \left(\frac{GM}{r} \right)^{1/2}$$

$$r = \frac{GM}{v^2}$$

3. Calculate the velocity of the Earth in its orbit around the Sun.

$$v = \left(\frac{GM}{r} \right)^{1/2}$$

$M = 1.99 \times 10^{30}$ kg (mass of the Sun)

$r = 149.6 \times 10^9$ m (radius of the orbit of the Earth)

$G = 6.67 \times 10^{-11}$ Nm²/kg² (gravitational constant)

v = velocity

$$v = \left(\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{149.6 \times 10^9} \right)^{1/2}$$

$$v = \left(0.08872 \times 10^{10} \right)^{1/2} = 0.29785 \times 10^5 \text{ m / s}$$

$$v = \frac{0.29785 \times 10^5}{10^3} = 29.78 \text{ km / s}$$

4. What important assumption underlies the calculation in problem 3 above?

The orbit of the Earth around the Sun is a circle.

5. Calculate the average orbital velocity of the Earth given that:

$r = 149.6 \times 10^6$ km (radius of the orbit of the Earth)

$p = 365.25$ days (time required for the Earth to complete one orbit around the Sun)

$C = 2\pi r$ (circumference of a circle)

$\pi = 3.14$

Express the velocity in km/s

$$v = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{C}{P} = \frac{2\pi r}{P} = \frac{2 \times 3.14 \times 149.6 \times 10^6}{365.25}$$

$$v = 2.5721 \times 10^6 \text{ km/day}$$

$$v = \frac{2.5721 \times 10^6}{24 \times 60 \times 60} = 29.77 \text{ km / s}$$

6. Does the good agreement of the orbital velocities obtained in problems 3 and 5 above prove

that the orbit of the Earth is a circle?

No, both calculations assume that the orbit of the Earth is a circle.

Chapter 2: From Speculation to Understanding

- Express 1 kilometer in terms of millimeters

$$1 \text{ km} = 1000 \text{ m} = 1000 \times 100 \text{ cm} = 1000 \times 100 \times 10 \text{ mm}$$

$$1 \text{ km} = 10^6 \text{ mm}$$

- Convert 1 lightyear into the corresponding number of astronomical units.

$$1 \text{ AU} = 149.6 \times 10^6 \text{ km}$$

$$1 \text{ ly} = 9.46 \times 10^{12} \text{ km}$$

$$1 \text{ ly} = \frac{9.46 \times 10^{12}}{149.6 \times 10^6} = 0.06323 \times 10^6$$

$$1 \text{ ly} = 6.3235 \times 10^4 = 63,235 \text{ AU}$$

- How long would it take a spacecraft to reach the star Proxima Centauri assuming that the speed of the spacecraft is 1000 km/s and that the distance to Proxima Centauri is 4.2 ly? Express the result in sidereal years.

$$\text{velocity} = \frac{\text{distance}}{\text{time}}, \text{ time} = \frac{\text{distance}}{\text{velocity}}$$

$$\text{distance} = 4.2 \text{ ly} = 4.2 \times 9.46 \times 10^{12} \text{ km}$$

$$\text{velocity} = 1000 \text{ km/s}$$

$$\text{time} = \frac{4.2 \times 9.46 \times 10^{12}}{1000} = 39.732 \times 10^9 \text{ s}$$

$$\text{time} = \frac{39.732 \times 10^9}{60 \times 60 \times 24 \times 365.25} = 1.2590 \times 10^3 \text{ y}$$

Traveltime to Proxima Centauri: 1259 years (What does that mean?)

4. Derive an equation for the conversion of temperatures on the Fahrenheit to the Kelvin scale

$$C = \frac{(F - 32) 5}{9} \quad (1)$$

$$C = K - 273.15 \quad (2)$$

From equation (1);

$$F = \frac{9C}{5} + 32$$

Substitute equation (2) for C

$$F = \frac{9 (K - 273.15)}{5} + 32$$

$$F = \frac{9}{5} K - \frac{9 \times 273.15}{5} + 32$$

$$F = \frac{9K}{5} - 491.67 + 32 = \frac{9K}{5} - 459.67$$

$$\frac{9K}{5} = F + 459.67$$

$$K = \frac{(F + 459.67) 5}{9}$$

Test: Let $T = 32^{\circ}\text{F}$

$$K = \frac{(32 + 459.67) 5}{9} = 273.15$$

5. Write a brief essay about the life and scientific contributions of one of the following pioneers of astronomy.

- | | |
|--------------------|--------------------|
| a. Galileo Galilei | d. Albert Einstein |
| b. Johannes Kepler | e. Edwin Hubble |
| c. Isaac Newton | f. Carl Sagan |

Consider these and other aspects:

1. Education

2. Evidence of unusual talent as a child
3. Support from colleagues
4. Support from contemporary society
5. Motivation
6. Problem to be solved
7. Available data
8. Reasoning to reach conclusions
9. Recognition for achievement
10. Perseverance in the face of adversity
11. Vindication or despair

Chapter 3: The Planets of the Solar System

1. Calculate the volume of the Earth (V_E) and the Sun (V_S) and compare them to each other by dividing the volume of the Sun by the volume of the Earth. Express the result in words.

Radius of the Sun: 695,700 km

Radius of the Earth: 6378 km

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Volume of the Sun } V_s = \frac{4 \times 3.14 \times (695,700)^3}{3}$$

$$V_s = 1.4097 \times 10^{18} \text{ km}^3$$

$$\text{Volume of the Earth } V_E = \frac{4 \times 3.14 \times (6378)^3}{3}$$

$$V_E = 1.0862 \times 10^{12} \text{ km}^3$$

$$\frac{V_S}{V_E} = \frac{1.4097 \times 10^{18}}{1.0862 \times 10^{12}} = 1.297 \times 10^6 \text{ (dimensionless)}$$

$$V_S = 1.297 \times 10^6 V_E$$

The Volume of the Sun is one million two hundred ninety seven times larger than the volume of the Earth.

2. Express the average distance between the Sun and the Earth as a percent of the average distance between the Sun and Pluto. State the results in words.

Orbital radii: Earth = 149.6×10^6 km
 Pluto = 39.53 AU

Orbital radius of the Earth: 149.6×10^6 km = 1 AU

Orbital radius of the Earth: $= \left(\frac{1}{39.53} \right) 100 = 2.52\%$

The radius of the orbit of the Earth is only 2.52% of the radius of the orbit of Pluto.

3. Calculate the time for light emitted by the Sun to reach the Earth. Express the results in seconds, hours, minutes, and days. The speed of light (c) is 2.99×10^{10} cm/s. Look up the length of one AU in Appendix 1.

$$\text{speed} = \frac{\text{distance}}{\text{time}}; \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{149.6 \times 10^6 \times 1000 \times 100}{2.99 \times 10^{10}} = 500.33 \text{ s}$$

$$t = \frac{500.33}{60} = 8.33 \text{ min}$$

$$t = \frac{8.33}{60} = 0.138 \text{ h}$$

$$t = \frac{0.138}{24} = 0.00579 \text{ d}$$

4. Halley's comet was first observed in 240 BC by Chinese astronomers and was also in the sky in the year 1066 AD at the Battle of Hastings. Calculate the number of years that elapsed between these dates and determine how many times this comet reappeared in this interval of time, given that its period of revolution is 76 years but excluding the appearance at 240 BC.

Years between 240 BC and 1066 AD = $240 + 1066 = 1306$ y

Number of reappearances of Halley's comet: 240 BC, 164 BC, 88 BC, 12 BC, 64 AD, 140 AD, 216 AD, 292 AD, 368 AD, 444 AD, 520 AD, 596 AD, 672 AD, 748 AD, 824 AD, 900 AD, 976 AD, 1052 AD

16 reappearances excluding 240 BC and 1066 AD

17 reappearances excluding only 240 BC

Note: The period of Halley's comet cannot be 76.0 years because it arrived in 1052 AD or 14 years before 1066 AD. Therefore, the period of Halley's comet must be increased by an

$$\text{increment } \Delta t = \frac{14}{17} = 0.82 \text{ y}$$

$$\text{Check: } 17 \times 76.82 = 1305.94 \text{ y}$$

$$1305.94 - 240 = 1065.9 \text{ AD}$$

5. Calculate the density of a rock composed of the four minerals listed below together with their abundances and densities.

Mineral	Abundance by volume, %	Density, g/cm ³
Olivine	15	3.30
Augite	50	3.20
Plagioclase	25	2.68
Magnetite	10	5.18

Hint: The total mass of the rock is the sum of the masses of its minerals

$$\text{density} = \frac{\text{mass}}{\text{volume}}; \text{mass} = \text{volume} \times \text{density}$$

Density of the rock (d_R)

$$d_R = 3.30 \times 0.15 + 3.20 \times 0.50 + 2.68 \times 0.25 + 5.18 \times 0.10$$

$$d_R = 3.28 \text{ g/cm}^3$$

Chapter 4: Life and Death of Stars

1. Calculate the diameter of the Milky Way galaxy in kilometers assuming that its radius is 50 ly.

Diameter of the Milky Way galaxy (D):

$$D = 2 \times 50 \times 9.46 \times 10^{12} = 9.46 \times 10^{14} \text{ km}$$

2. Calculate the period of revolution of a star that is located 35 ly from the center of the galaxy assuming that its radial velocity is 150 km/s.

$$\text{velocity} = \frac{\text{distance}}{\text{time}}; \text{time} = \frac{\text{distance}}{\text{velocity}}$$

$$\text{period} = \frac{\text{circumference of the orbit}}{\text{velocity}} = \frac{2\pi r}{\text{velocity}}$$

$$\text{period} = \frac{2 \times 35 \times 9.46 \times 10^{12} \times 3.14}{150} = 13.862 \times 10^{12} \text{ s}$$

$$\text{period} = \frac{13.862 \times 10^{12}}{60 \times 60 \times 24 \times 365.25} = 0.4392 \times 10^6 \text{ y}$$

$$\text{period} = 439,200 \text{ y}$$

3. Calculate the age of an expanding universe characterized by a Hubble constant $H = 15 \text{ km/s/}10^6 \text{ ly}$.

$$\text{Hubble equation: } v = H \times d \quad (1)$$

$$\text{Substituting: } v = \frac{d}{t} \quad (2)$$

$$\frac{d}{t} = H \times d \text{ or } t = \frac{1}{H} \quad (3)$$

Age of the expanding Universe (t)

$$t = \frac{9.46 \times 10^{12} \times 10^6}{15} = 0.6306 \times 10^{18} \text{ s}$$

$$t = \frac{0.6306 \times 10^{18}}{60 \times 60 \times 24 \times 365.25} = 19.98 \times 10^9 \text{ y}$$

4. According to the Oddo-Harkins rule, chemical elements with even atomic numbers are more abundant than elements that have odd atomic numbers. Examine Figure 4.5 and note the elements Al, Si, and P. Determine whether these elements obey the Oddo-Harkins rule and suggest an explanation for this phenomenon. (See Faure, 1998. Principles and applications of geochemistry, 2nd edn., Prentice Hall, Upper Saddle River, NJ).

Let Z = atomic number = number of protons in the nucleus of an atom

$$Z(\text{Al}) = 13, Z(\text{Si}) = 14, Z(\text{P}) = 15$$

Figure 4.5 indicates that Si ($Z = 14$) is more abundant than Al ($Z = 13$) and P ($Z = 15$). Therefore, these elements obey the Oddo-Harkins rule.

The cause of this relationship is that nuclei having an even number of protons are more stable than atoms that contain an odd number of protons. During nucleosynthesis in stars, atoms that are especially stable have a better chance to survive than atoms whose nuclei are less stable.

5. Write an essay about one of the following scientists who contributed significantly to the theory of cosmology and stellar evolution.
 - a. George Gamow
 - b. Fred Hoyle
 - c. Hans Bethe
 - d. Edwin Hubble
- a. George Gamow: Nucleosynthesis by neutron capture and the hydrogen-fusion reaction in stellar cores.
 - b. Fred Hoyle: Steady-state cosmology and spontaneous creation of hydrogen nuclei in intergalactic space.
 - c. Hans Bethe: CNO-cycle for hydrogen fusion in the Sun catalyzed by $^{12}_6\text{C}$.
 - d. Edwin Hubble: Expansion of the Universe indicated by the red-shift of the light received from distance galaxies.

Chapter 5: Origin of the Solar System

1. The average speed of the solar wind is 850,000 miles per hour. Convert that speed to kilometers per second.

$$1 \text{ mile} = 1.608 \text{ km}$$

$$\begin{aligned} \text{Speed of the solar wind} &= \frac{850,000 \times 1.608}{60 \times 60} \\ &= 379.6 \text{ km/s} \end{aligned}$$

2. Interstellar space contains about one atom (or ion) per cubic centimeter of space. Calculate the number of atoms (or ions) in one cubic kilometer of space.

$$1 \text{ km} = 1000 \times 100 = 10^5 \text{ cm}$$

$$1 \text{ km}^3 = (10^5)^3 = 10^{15} \text{ cm}^3$$

$$\text{Number of atoms in } 1 \text{ km}^3 \text{ of space} = 10^{15} \text{ atoms}$$

3. The fusion of four protons into one nucleus of ${}^4_2\text{He}$ releases energy in the amount of 19.794 MeV. Convert this amount of energy into the equivalent energy expressed in joules ($1 \text{ eV} = 1.6020 \times 10^{-19} \text{ J}$).

$$1 \text{ MeV} = 1.6020 \times 10^{-19} \text{ J} \times 10^6 = 1.6020 \times 10^{-13} \text{ J}$$

$$\begin{aligned} 19.794 \text{ MeV} &= 19.794 \times 1.6020 \times 10^{-13} \text{ J} \\ &= 31.709 \times 10^{-13} \text{ J} \end{aligned}$$

4. Calculate the number of ${}^4_2\text{He}$ atoms that must be produced in the Sun each second to provide enough energy to equal the solar luminosity of $3.9 \times 10^{26} \text{ J/s}$.

Let x be the number of ${}^4_2\text{He}$ atoms that are produced per second

$$x = \frac{3.9 \times 10^{26}}{31.709 \times 10^{-13}} = 0.1229 \times 10^{39} \text{ atoms of } {}^4_2\text{He}$$

$$x = 1.229 \times 10^{38} \text{ atoms of } {}^4_2\text{He} \text{ per second.}$$

5. Calculate the number of hydrogen nuclei that exist in the Sun based on the following information:

$$\text{Mass of the Sun} = 1.99 \times 10^{30} \text{ kg}$$

$$\text{Concentration of hydrogen} = 74\% \text{ by mass}$$

$$\text{Atomic weight of hydrogen} = 1.00797$$

$$\text{Number of atoms per mole} = 6.022 \times 10^{23}$$

$$1 \text{ mole of hydrogen weighs } 1.00797 \text{ g}$$

$$1 \text{ mole of hydrogen contains } 6.022 \times 10^{23} \text{ atoms}$$

$$\text{Mass of 1 atom of H} = \frac{1.00797}{6.022 \times 10^{23}} \text{ g}$$

$$= \frac{1.00797}{6.022 \times 10^{23} \times 10^3} \text{ kg}$$

$$= 0.1673 \times 10^{-26} \text{ kg}$$

Total mass of hydrogen in the Sun = $1.99 \times 10^{30} \times 0.74 \text{ kg}$

$$\text{Number of H atoms in the Sun} = \frac{1.99 \times 10^{30} \times 0.74}{0.1673 \times 10^{-26}} = 8.797 \times 10^{56} \text{ atoms}$$

6. Given that the rate of helium production is $1.229 \times 10^{38} \text{ atom/s}$ and that the number of hydrogen nuclei in the Sun is $8.79 \times 10^{56} \text{ atoms}$, estimate how much time in years is required to convert all of the hydrogen in the Sun into helium. Remember that four hydrogen atoms form one helium atom.

Life expectance of the Sun (t)

$$t = \frac{8.79 \times 10^{56}}{1.229 \times 10^{38} \times 4} = 1.7880 \times 10^{18} \text{ s}$$

$$t = \frac{1.7880 \times 10^{18}}{60 \times 60 \times 24 \times 365.25} = 56.6 \times 10^9 \text{ y}$$

7. Refine the calculation of the life expectancy of the Sun by considering that hydrogen fusion in the Sun occurs only in its core and that the density of the core is greater than the bulk density of the Sun as a whole.

- a) Calculate the mass of the core of the Sun given that its radius is 140,000 km and that its density is 150 g/cm^3 .

$$\text{Volume of the core } V_c = \frac{4}{3} \pi r^3$$

$$V_c = \frac{4 \times 3.14 \times (140,000)^3}{3} = 1.148 \times 10^{16} \text{ km}^3$$

Mass of the core $M_c = \text{density} \times \text{volume}$

$$M_c = 150 \times 1.148 \times 10^{16} \times 10^{15} = 172.2 \times 10^{31} \text{ g}$$

$$M_C = 1.72 \times 10^{33} \text{ g} = 1.72 \times 10^{30} \text{ kg}$$

- b) Calculate the number of hydrogen atoms in the core of the Sun assuming that its concentration is 74% by weight. (See problem 5).

$$\text{Mass of one hydrogen atoms} = 0.1673 \times 10^{-26} \text{ kg}$$

$$\text{Mass of the core of the Sun} = 1.72 \times 10^{30} \text{ kg}$$

Number of hydrogen atoms in the core:

$$N = \frac{1.72 \times 10^{30} \times 0.74}{0.1673 \times 10^{-26}} = 7.607 \times 10^{56} \text{ atoms}$$

- c) Estimate how much time in years is required to convert all of the hydrogen in the core of the Sun into helium.

$$\text{Number of hydrogen atoms: } 7.607 \times 10^{56}$$

$$\text{Rate of production of helium } 1.229 \times 10^{38} \text{ atom/s}$$

Life expectancy of the Sun (t)

$$t = \frac{7.607 \times 10^{56}}{1.229 \times 10^{38} \times 4} = 1.5473 \times 10^{18} \text{ s}$$

$$t = \frac{1.5473 \times 10^{18}}{60 \times 60 \times 24 \times 365.25} = 49.0 \times 10^9 \text{ y}$$

- d) How does this result compare to the accepted life expectancy of the Sun and consider possible reasons for the discrepancy?

$$\text{Life expectancy of the Sun} \sim 10 \times 10^9 \text{ y}$$

The life expectancy is shorter than the estimate in part c of problem 7

Reasons:

1. The rate of conversion of hydrogen into helium is higher than estimated
2. The amount of H that is available for conversion to He is less than estimated
3. The conversion of H to He does not go to completion. If only 20% of the H in the core is available for conversion to He, the estimated life expectancy of the Sun (t)

$$t = \frac{7.607 \times 10^{56} \times 0.2}{1.229 \times 10^{38} \times 4} = 0.3094 \times 10^{18} \text{ s}$$

$$t = \frac{0.3094 \times 10^{18}}{60 \times 60 \times 24 \times 365.25} = 9.8 \times 10^9 \text{ y} \approx 10 \text{ billion years}$$

Chapter 6: Earth, Model of Planetary Evolution

1. Calculate the surface area (A_1) of a disk having a radius of 50 AU and compare it to the area (A_2) of a disk whose radius is only 5 AU. Express the areas in units of square kilometers and state the results in words with reference to Section 6.1

Area of a circle $A = r^2 \pi$

$$A_1 = (50 \times 149.6 \times 10^6)^2 \times 3.14 = 1.756 \times 10^{20} \text{ km}^2$$

$$A_2 = (5 \times 149.6 \times 10^6)^2 \times 3.14 = 1.756 \times 10^{18} \text{ km}^2$$

Outer region of the solar system:

$$A_1 - A_2 = (175.6 - 1.756) \times 10^{18} = 173.84 \times 10^{18} \text{ km}^2$$

Inner region of the solar system:

$$A_2 = 1.756 \times 10^{18} \text{ km}^2$$

$$\frac{\text{Inner region}}{\text{Outer region}} = \frac{1.756 \times 10^{18}}{173.84 \times 10^{18}} = 0.0101$$

Inner region = 0.0101 Outer region

2. Compare the volume (V_1) of the solid part of the core of the Earth to the volume (V_2) of the whole core both by means of the percent difference and by a factor (e.g., the volume of the inner core is “x” % of the volume of the whole core and is “y” times larger than the inner core.).

Radius of the inner core = 1329 km

Radius of the whole core = 3471 km

Volume of a sphere: $v = \frac{4}{3} \pi r^3$

$$V_1 = \frac{4 \times 3.14 \times (1329)^3}{3} = 98.275 \times 10^8 \text{ km}^3$$

$$V_2 = \frac{4 \times 3.14 \times (3471)^3}{3} = 1.7507 \times 10^{11} \text{ km}^3$$

The volume of the inner core is:

$$= \frac{98.275 \times 10^8 \times 100}{1.7507 \times 10^{11}} = 56 \times 10^{-1} = 5.6 \%$$

The volume of the inner core is 5.6 % of the volume of the whole core

$$\frac{V_2}{V_1} = \frac{1.7507 \times 10^{11}}{98.275 \times 10^8} = 0.01781 \times 10^3$$

$$V_2 = 17.81 V_1 \quad \text{or} \\ V_1 = 0.0561 V_2$$

3. Calculate the mass of the core of the Earth and of its mantle (without the core) and interpret the results by expressing their masses as a percent of the total mass of the Earth.

Radius of the whole core	3471 km
Radius of the Earth	6378 km
Density of iron	7.87 g/cm ³
Density of peridotite	3.2 g/cm ³

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

Volume of the whole Earth (V_E):

$$V_E = \frac{4 \times 3.14 \times (6378)^3}{3} = 1.086 \times 10^{12} \text{ km}^3$$

Volume of the core (V_C):

$$V_E = \frac{4 \times 3.14 \times (3471)^3}{3} = 1.750 \times 10^{11} \text{ km}^3$$

Volume of the mantle (V_M):

$$V_M = V_E - V_C = 1.086 \times 10^{12} - 1.750 \times 10^{11}$$

$$V_M = 9.11 \times 10^{11} \text{ km}^3$$

Mass of the mantle (M_m):

$$M_M = \frac{9.11 \times 10^{11} \times 3.2 \times 10^{15}}{10^3} = 29.152 \times 10^{23} \text{ kg}$$

Mass of the core (M_C):

$$M_C = \frac{1.750 \times 10^{11} \times 7.87 \times 10^{15}}{10^3} = 13.772 \times 10^{23} \text{ kg}$$

Mass of the whole Earth (M_E):

$$M_E = M_C + M_M = 13.772 \times 10^{23} + 29.152 \times 10^{23}$$

$$M_E = 42.924 \times 10^{23} \text{ kg}$$

$$\frac{M_C}{M_E} = \left(\frac{13.772 \times 10^{23}}{42.924 \times 10^{23}} \right) 10^2 = 32.1 \%$$

$$\frac{M_M}{M_E} = \left(\frac{29.152 \times 10^{23}}{42.924 \times 10^{23}} \right) 10^2 = 67.9 \%$$

Note: The validity of this result can be questioned on the grounds that the density of iron in the core of the Earth is greater than 7.87 g/cm^3 . A better way to phrase this problem would be to use the bulk density of the Earth (5.52 g/cm^3) and the density of the mantle (3.2 g/cm^3) to determine the mass of the core by difference. In that case:

$$M_C = M_E - M_M$$

$$M_E = \frac{1.086 \times 10^{12} \times 5.52 \times 10^{15}}{10^3} = 5.994 \times 10^{24} \text{ kg}$$

$$M_M = 2.915 \times 10^{24} \text{ kg}$$

$$M_C = (5.994 - 2.915) \times 10^{24} = 3.079 \times 10^{24} \text{ kg}$$

$$\frac{M_C}{M_E} = \left(\frac{3.079 \times 10^{24}}{5.994 \times 10^{24}} \right) 10^2 = 51.3 \%$$

Density of the core: $D_c = \frac{M_c}{V_c}$

$$D_c = \frac{3.079 \times 10^{24} \times 10^3}{1.750 \times 10^{11} \times 10^{15}} = 1.759 \times 10$$

$D_c = 17.6 \text{ g/cm}^3$ (More than 2.2 times the density of iron at 1 atm).

4. Estimate the depth of the hypothetical global ocean in kilometers before any continents or oceanic islands had formed. Assume that the total volume of water on the surface of the Earth was $1.65 \times 10^9 \text{ km}^3$

Area of a sphere (A) = $4 \pi r^2$

Area of the Earth $A_E = 4 \times 3.14 \times (6378)^2$

$$A_E = 5.109 \times 10^8 \text{ km}^2$$

Volume of water $V_O = 1.65 \times 10^9 \text{ km}^3$

$V_O = d \times A_E$, where d = depth of the ocean in kilometers.

$$d = \frac{V_O}{A_E} = \frac{1.65 \times 10^9}{5.109 \times 10^8} = 3.23 \text{ km}$$

Chapter 7: The Clockwerk of the Solar System

1. Calculate the distance from the center of the Sun to the center of the Earth when the Earth is at perihelion (q) and aphelion (Q) of its orbit, given that the eccentricity is 0.017. Express the results in kilometers.

Average distance between the Sun and the Earth: $a = 149.6 \times 10^6 \text{ km}$

Eccentricity $e = \frac{f}{a} \therefore f = e \times a$

$$f = 0.0017 \times 149.6 \times 10^6 = 2.54 \times 10^6 \text{ km}$$

At perihelion: $q = a - f = (149.6 - 2.54) \times 10^6 = 147.06 \times 10^6 \text{ km}$

At aphelion: $Q = a + f = (149.6 + 2.54) \times 10^6 = 152.14 \times 10^6 \text{ km}$

2. Calculate the number of jovian days in one jovian year using the data in Table 7.1.

Period of revolution of Jupiter = 11.856 y

Period of rotation of Jupiter = 9.936 h

Number of days in one jovian year (N)

$$N = \frac{11.856 \times 365.25 \times 24}{9.936} = 10,459.9$$

N = 10,460 jovian days

3. Calculate the escape velocity of an object located on the surface of Mercury using data in Appendices 1 and 2 and express the results in km/s.

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

M = 3.30×10^{23} kg, r = 2439 km, G = 6.67×10^{-11} N/m²/kg²

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 3.30 \times 10^{23}}{2439 \times 10^3} \right)^{1/2}$$

$$v_e = (0.01804 \times 10^9)^{1/2} = 0.4247 \times 10^4 \text{ m/s}$$

$$v_e = \frac{0.4247 \times 10^4}{10^3} = 4.25 \text{ km / s}$$

4. Verify by calculation the magnitude of the synodic period of Pluto using data in Table 7.1.

Period of revolution (sidereal) of Earth: 1.0 y

Period of revolution (sidereal) of Pluto: 248.60 y

For superior planets

$$\frac{1}{P} = \frac{1}{E} - \frac{1}{S}$$

$$\frac{1}{248.6} = 1 - \frac{1}{S}$$

$$0.004022 = 1 - \frac{1}{S}$$

$$\frac{1}{S} = 1 - 0.004022 = 0.9959$$

$$1 = 0.9959 S$$

$$S = \frac{1}{0.9959} = 1.004 \text{ ly}$$

$$S = 366.7 \text{ d}$$

5. Use equation 7.3 to calculate the average distance from the surface of the Earth at the equator of a satellite in “geostationary orbit” (i.e., whose period of revolution is equal to the period of rotation of the Earth).

Express the results in kilometers.

Use Newton’s form of Kepler’s third law:

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

$$\begin{aligned} \text{Mass of the Earth (M)} &= 5.98 \times 10^{24} \text{ kg} \\ G &= 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \\ p &= 24 \times 60 \times 60 \text{ s} = 8.64 \times 10^4 \text{ s} \\ m &\ll M, M + m = M \end{aligned}$$

$$a = \left(\frac{p^2 GM}{4\pi^2} \right)^{1/3}$$

$$a = \left[\frac{(8.64 \times 10^4)^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4 (3.14)^2} \right]^{1/3}$$

$$a = (75.497 \times 10^{21})^{1/3}$$

$$\frac{\log 75.497 \times 10^{21}}{3} = 7.6259$$

$$a = 42.257 \times 10^6 \text{ m} = 42.257 \times 10^3 \text{ km}$$

Height of satellite above the surface (h)

$$h = 42,257 - 6378 = 35,879 \text{ km}$$

Chapter 8: Meteorites and Impact Craters

1. Calculate the density (d) of a spherical meteoroid having a diameter of 15m and a mass of 6×10^9 kg. Express the result in g/cm³.

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

Radius $r = 7.5 \text{ m} = 750 \text{ cm}$

$$V = \frac{4}{3} \times 3.14 \times (750)^3 = 17.66 \times 10^8 \text{ cm}^3$$

$$\text{Density } d = \frac{6 \times 10^9 \times 10^3}{17.66 \times 10^8} = 3.39 \text{ g/cm}^3$$

2. Use equation 8.5 to calculate a Rb-Sr date (t) of a lunar meteorite given that its measured daughter/parent (D/P) ratio is 0.04649 and that the decay constant (λ) of ⁸⁷Rb is $1.42 \times 10^{-11} \text{ y}^{-1}$.

Equation 8.5: $D = P (e^{\lambda t} - 1)$

$$\frac{D}{P} + 1 = e^{\lambda t}$$

$$\ln\left(\frac{D}{P} + 1\right) = \lambda t$$

$$t = \frac{1}{\lambda} \ln\left(\frac{D}{P} + 1\right)$$

$$t = \frac{\ln 1.04649}{1.42 \times 10^{-11}} = \frac{0.04544}{1.42 \times 10^{-11}} = 0.0320 \times 10^{11}$$

$$t = 3.20 \times 10^9$$

3. The diameter of the meteorite impact crater at Sudbury, Ontario, is approximately 250 km. Calculate the amount of energy (E) released by this impact (Consult Science Brief 8.11.4). Express the energy in terms of tonnes of TNT where 1 kilotonne = 4.2×10^{19} erg.

Equation 8.12: $D = 1.96 \times 10^{-5} E^{0.294}$

$$\log D = \log (1.96 \times 10^{-5}) + 0.294 \log E$$

D is in kilometers, E is in joules where $1\text{J} = 10^7 \text{ erg}$

$$D = 250 \text{ km}$$

$$\log 250 = 4.707 + 0.294 \log E$$

$$2.3979 + 4.707 = 0.294 \log E$$

$$\log E = \frac{7.1049}{0.294} = 24.166$$

$$E = 1.4655 \times 10^{24} \text{ J} = 1.4655 \times 10^{31} \text{ erg}$$

$$E = \frac{1.4655 \times 10^{31}}{4.2 \times 10^{19}} = 0.3489 \times 10^{12}$$

$$E = 34.89 \times 10^{10} \text{ tonnes of TNT}$$

4. Calculate the diameter (D) of the crater formed by the explosive impact of a meteoroid having a mass of $6 \times 10^9 \text{ kg}$ and traveling with a velocity of 20 km/s .

$$\log D = -4.707 + 0.294 \log E$$

Kinetic energy: $E = \frac{1}{2} m v^2$

$$E = \frac{1}{2} \times 6 \times 10^9 \times 10^3 (20 \times 10^5)^2$$

$$E = 1200 \times 10^{22} \text{ erg} = \frac{1200 \times 10^{22}}{10^7} \text{ J}$$

$$E = 1200 \times 10^{15} \text{ J}$$

$$\log D = -4.707 + 0.294 (1200 \times 10^{15})$$

$$\log D = -4.707 + 5.315 = 0.6082$$

$$D = 4.05 \text{ km}$$

Chapter 9: The Earth-Moon System

1. Write an essay to explain why the polar regions of the Moon are the most favorable sites for the establishment of a lunar base.

Points to consider:

- a. The polar regions of the Moon receive almost continuous sunshine because the inclination of its axis of rotation (i.e., its axial obliquity) is only 6.68° compared to 23.45° for the Earth.
 - b. Sunlight is an important source of heat, light, and energy.
 - c. Deep impact craters in the polar regions are in perpetual shade which may have allowed water ice released by impacting comets to have accumulated there.
2. Calculate the escape velocity of the Moon in units of km/s (Appendix 1 and Table 9.3)

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

$$r = 1738 \text{ km}, M = 7.349 \times 10^{22} \text{ kg}, G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 7.349 \times 10^{22}}{1738 \times 10^3} \right)^{1/2} \text{ m / s}$$

$$v_e = (0.05640 \times 10^8)^{1/2} = 0.237 \times 10^4 \text{ m/s}$$

$$v_e = \frac{0.237 \times 10^4}{10^3} = 2.37 \text{ km / s}$$

3. Calculate the distance between the apogee and the perigee of the orbit of the Moon measured along a straight line through the center of the ellipse.

$$a = 384.4 \times 10^3 \text{ km}$$

$$\text{Distance (d) between apogee and perigee} = 2a$$

$$d = 384.4 \times 10^3 \times 2 = 768.8 \times 10^3 \text{ km}$$

4. Several subsamples of a lunar meteorite collected in Antarctica were analyzed for dating by the Rb-Sr method. The analytical results for two subsamples are:

Subsample	$^{87}\text{Sr}/^{86}\text{Sr}$	$^{87}\text{Rb}/^{86}\text{Sr}$
PX1,R	0.703415	0.07779
PI2,R	0.699193	0.001330

The values define two points in the x-y plane where $^{87}\text{Rb}/^{86}\text{Sr} = x$ and $^{87}\text{Sr}/^{86}\text{Sr} = y$

- a. Calculate the slope m of a straight line connecting the two points:

$$m = \frac{0.703415 - 0.699193}{0.07779 - 0.001330} = \frac{0.004222}{0.07646}$$

$$m = 0.055218$$

- b. Calculate a Rb-Sr date from the slope m using equation 8.11 in Science Brief 8.11.2

$$t = \frac{\ln(m + 1)}{\lambda}$$

where $\lambda = 1.42 \times 10^{-11} \text{ y}^{-1}$ (decay constant of ^{87}Rb) and \ln is the natural logarithm.

$$t = \frac{\ln(0.055218 + 1)}{1.42 \times 10^{-11}} = \frac{0.053747}{1.42 \times 10^{-11}} = 0.03785 \times 10^{-11}$$

$$t = 3.785 \times 10^9 \text{ y}$$

Chapter 10: Mercury: Too Hot for Comfort

1. The map of a small area on Mercury in Figure 10.7 contains numerous impact craters having a range of diameters.
 - a. Measure the diameters of each crater in this map in millimeters until close to 100 separate craters have been measured.
 - b. Record these measurements in a table under the headings:

Range	Count	Total	Percentage
0-2			
2-4			
4-6			
6-8			
8-10			
10-12			
12-14			
 - c. Sum the number of craters in each size range and record these totals in the appropriate column. Add up all of the totals and express the number of craters in each category as a percentage of the total number of craters you measured.
 - d. Plot the size ranges along the x-axis and the percentage of each category along the y-axis. Fit a smooth curve by eye to the resulting array of points. Make it neat.

- e. Interpret the graph you have constructed and explain your conclusions in a short technical report under the subheadings: Objective, Procedure, Presentation of Data, Interpretation, and Conclusions. Compose a cover sheet with the title of your project, your name, and the date. The text must be neatly typed on high-quality white paper, the histogram must be neatly drafted, and all pages must be numbered consecutively and stapled or bound attractively.

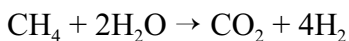
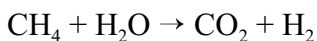
Range	Count	Total	Percent
0-2	too small to record		
2-4		35	41.7
4-6		24	28.6
6-8		10	11.9
8-10		7	8.3
10-12		8	8.3
12-14		<u>1</u>	<u>1.2</u>
Totals		84	100.00

Observations:

1. Small craters are more numerous than large ones.
2. The abundance of craters decreases exponentially with increasing diameter
3. The frequency distribution of the craters reflects the size distribution of the impactors
4. Small impactors were more numerous than large ones
5. The impactors may have formed from large bodies that broke up as a result of collisions that produced a large number of small fragments and a small number of large ones.

Chapter 11: Venus, Planetary Evolution Gone Bad

1. Balance the equation representing the reaction between methane gas (CH_4) and water vapor (H_2O) to form carbon dioxide (CO_2) and molecular hydrogen:



2. Write a brief essay explaining the potential significance of this reaction for the chemical evolution of the atmosphere of Venus.

Points to consider:

1. The problem with the atmosphere of Venus is to explain why it does not contain a lot more water vapor than it actually does.

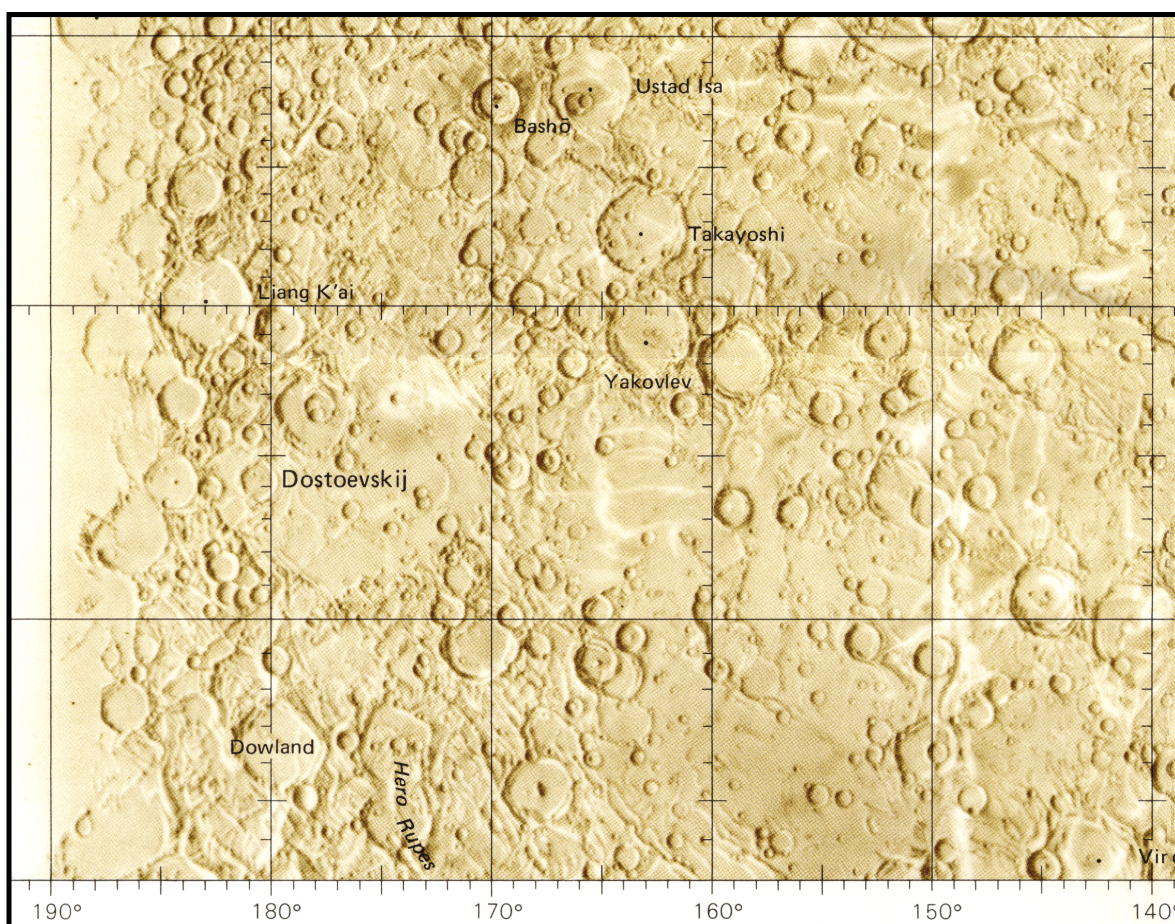
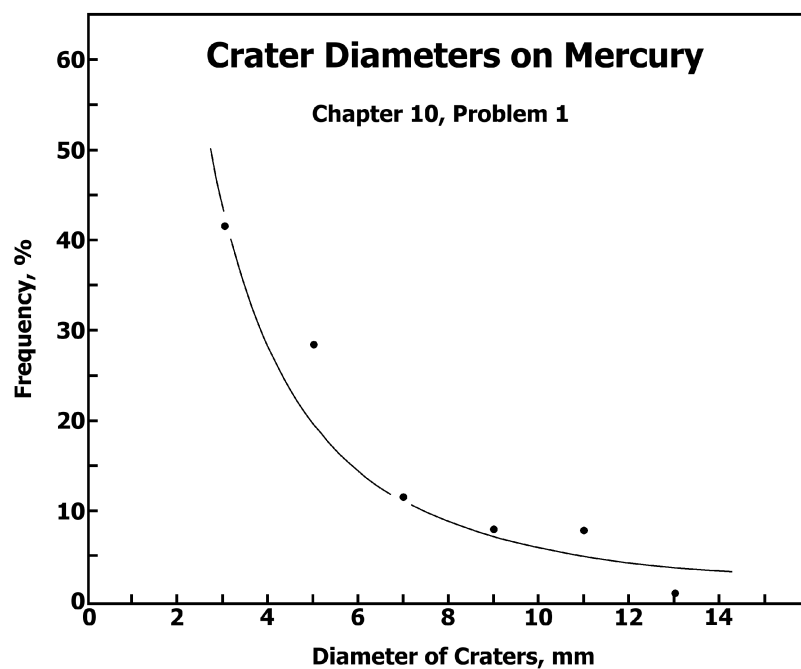


Fig. 10.7



2. The abundance of methane on Titan suggests that this compound may have existed in the early-formed atmospheres of the terrestrial planets, including Venus.
 3. In that case, one molecule of methane may have reacted with two molecules of water vapor to form one molecule of carbon dioxide and four molecules of hydrogen.
 4. The hydrogen would have escaped into interplanet space, leaving an atmosphere composed of carbon dioxide, molecular nitrogen, and the noble gases neon, argon, krypton, and xenon.
 5. This hypothesis explains how Venus may have lost most of the water it inherited from the planetesimals and comets that impacted on its surface.
 6. However, the exact path of this reaction and its rate in the primordial atmosphere of Venus remain to be investigated.
4. Explain why Venus does not have a magnetic field and consider the consequences for the environmental conditions on the surface of this planet.

Venus does not have a magnetic field because:

1. Its iron core is solid (not likely), or
2. it rotates too slowly (possible), or
3. it is currently undergoing a polarity reversal (also possible but unlikely), or
4. the absence of volcanic activity on Venus is allowing the temperature of the mantle to rise, which is neutralizing the thermal gradient within its outer (liquid) core. Therefore, the convection in the outer core, which induces the magnetic field, cannot occur under present conditions. (possible, but controversial).

The environmental consequences on the surface of Venus caused by the absence of a magnetic field include:

1. Cosmic rays and solar wind can enter the atmosphere without being diverted as on the Earth.
2. The rocks exposed at the surface may contain cosmogenic radionuclides formed by nuclear spallation reactions.
3. The rocks in the crust of Venus also do not contain remanent magnetization.
4. Navigation by means of a magnetic compass is not possible on the surface of Venus.
5. Auroral displays do not occur.
6. Energetic cosmic rays and ultraviolet radiation may break chemical bonds of compounds in the atmosphere thereby facilitating reactions that otherwise would not occur.

Chapter 12: Mars, The Little Planet that Could

1. Calculate the escape speed for Mars in km/s.
(See Appendix 1 and Table 12.1).

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

$$M = 6.42 \times 10^{23} \text{ kg}; r = 3394 \text{ km}$$

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{3394 \times 10^3} \right)^{1/2} = (0.02523 \times 10^9)^{1/2}$$

$$v_e = 0.502 \times 10^4 \text{ m/s} = 5.02 \text{ km/s}$$

2. Calculate the mass in kilograms of a carbon dioxide molecule whose molecular weight is 44.009 atomic mass units.

(Remember that Avogadro's Number is 6.022×10^{23} molecules per gram-molecular weight)

1 gram-molecular weight of $\text{CO}_2 = 44.009 \text{ g}$

1 gram-molecular weight contains: 6.022×10^{23} molecules

Therefore, the mass of one molecule of CO_2 (M_{CO_2}) is:

$$M_{\text{CO}_2} = \frac{44.009}{6.022 \times 10^{23}} = 7.308 \times 10^{-23} \text{ g}$$

$$M_{\text{CO}_2} = \frac{7.308 \times 10^{-23}}{10^3} = 7.308 \times 10^{-26} \text{ kg}$$

3. Calculate the average speed of a molecule of carbon dioxide in the atmosphere of Mars assuming an atmospheric temperature of -83°C . Express the speed in units of km/s.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$k \text{ (Boltzmann constant)} = 1.38 \times 10^{-23} \text{ J/K}$$

$$T = 273.15 - 83 = 190.15 \text{ K}$$

$$m = 7.308 \times 10^{-26} \text{ kg}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 190.15}{7.308 \times 10^{-26}} \right)^{1/2} = (107.72 \times 10^3)^{1/2}$$

$$v_m = 328.0 \text{ m/s} = 0.328 \text{ km/s}$$

4. Interpret the results of calculations in problems 2 and 3 above (Consult Section 24 of Appendix 1)

$$v_e = 5.02 \text{ km/s}, v_m = 0.328 \text{ km/s}$$

Conclusion: $v_m \ll v_e$. Therefore, CO_2 cannot escape from Mars even if the molecular speed is multiplied by 6.

$6 v_m = 1.968 \text{ km/s}$ which is still less than the escape speed of 5.02 km/s .

5. Repeat the calculation of problem 3 for a temperature of -33°C and interpret the result.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$T = 240.15 \text{ K}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 240.15}{7.308 \times 10^{-26}} \right)^{1/2} = (136.04 \times 10^3)^{1/2}$$

$$v_m = 368.8 \text{ m/s} = 0.368 \text{ km/s}$$

6. Plot a graph of the variation of the average speed of CO_2 molecules as a function of temperature between $T = 180 \text{ K}$ and $T = 298 \text{ K}$. Let T be the x-axis and v be the y-axis. Write a descriptive caption for this diagram.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2} = \left(\frac{3 \times 1.38 \times 10^{-23} T}{7.308 \times 10^{-26}} \right)^{1/2}$$

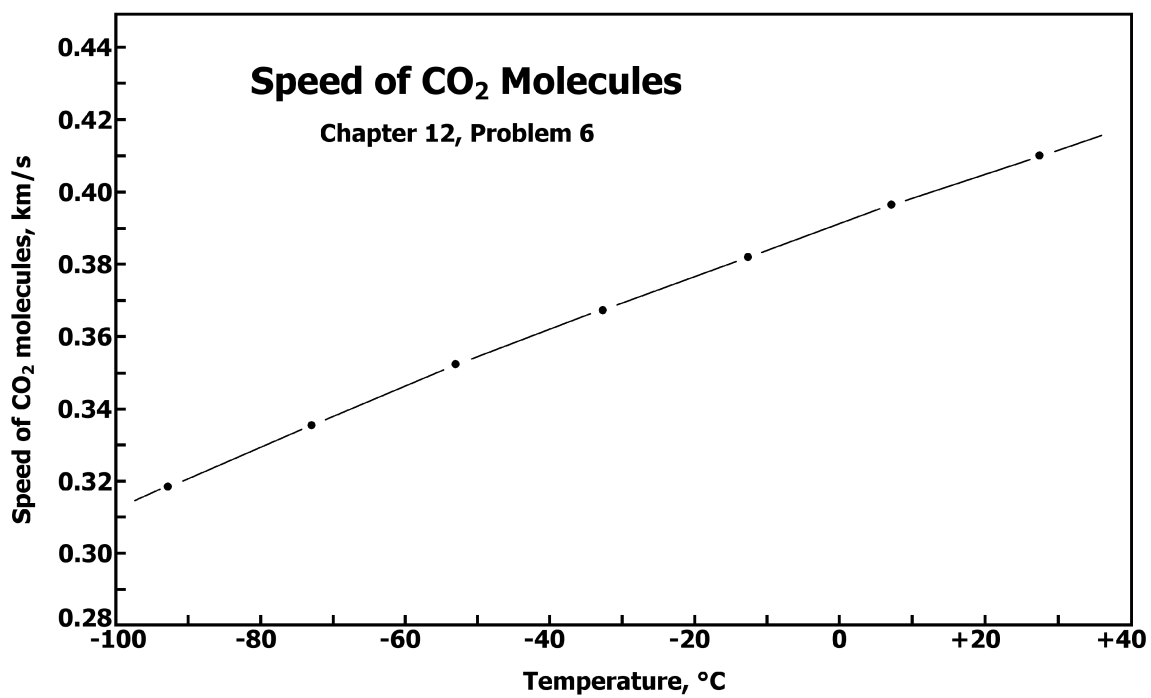
$$v_m = (0.5665 \times 10^3 T)^{1/2} = 23.80 T^{1/2}$$

$T = 180 \text{ K}$	$v_m = 0.319 \text{ km/s}$	-98°C
200 "	0.336 "	-73 "
220 "	0.353 "	-53 "
240 "	0.368 "	-33 "
260 "	0.383 "	-13 "
280 "	0.398 "	+7 "
300 "	0.412 "	+27 "

See graph

Descriptive Caption

Variation of the speed of carbon dioxide molecules in a gas with increasing temperature from



-100°C to + 40°C.

Note: The molecular speed also increases as the mass of the molecule under consideration decreases. In other words, low - mass molecules are more likely to escape from a planet or satellite than high-mass molecules, especially at elevated temperature.

Chapter 13: Asteroids: Shattered Worlds

1. Estimate the average distance between asteroids that reside between the orbits of Mars and Jupiter. Total area = $1.74 \times 10^{18} \text{ km}^2$. Assume that this area is populated by 2×10^6 individual asteroids and that the area surrounding each asteroid is circular in shape.

Area surrounding each asteroid (A)

$$A = \frac{1.74 \times 10^{18}}{2 \times 10^6} = 0.87 \times 10^{12} \text{ km}^2$$

Radius of the area (A)

$$A = r^2 \pi, r = \left(\frac{A}{\pi} \right)^{1/2}$$

$$r = \left(\frac{0.87 \times 10^{12}}{3.14} \right)^{1/2} = (0.2770 \times 10^{12})^{1/2} = 0.526 \times 10^6 \text{ km}$$

$$r = 526,000 \text{ km}$$

2. The orbit of the Apollo asteroid Geographos has an average radius $a = 1.24 \text{ AU}$ and an eccentricity $e = 0.34$. Calculate the distance of this asteroid from the Sun at perihelion (q) and at aphelion (Q).

$$e = \frac{f}{a}, f = e \times a$$

$$f = 0.34 \times 1.24 = 0.4216 \text{ AU}$$

$$q = a - f = 1.24 - 0.42 = 0.82 \text{ AU}$$

$$Q = a + f = 1.24 + 0.42 = 1.66 \text{ AU}$$

3. Calculate the book value of gold in the iron meteorite Lutetia (Science Brief 13.7.1) based on the following information: Diameter = 115 km, density = 7.90 g/cm^3 , concentration of gold = $0.6 \text{ } \mu\text{g/g}$ (ppm), price of gold = \$12,405 per kg (Kargel, 1994).

Volume of Lutetia (V):

$$V = \frac{4 \times \pi \times r^3}{3} = \frac{4 \times 3.14 \times (57.5)^3}{3} = 0.7959 \times 10^6 \text{ km}^3$$

$$V = 0.7959 \times 10^6 \times 10^{15} = 0.7959 \times 10^{21} \text{ cm}^3$$

Mass of Lutetia (M):

$$M = \text{Volume} \times \text{Density}$$

$$M = 0.7959 \times 10^{21} \times 7.90 = 6.2876 \times 10^{21} \text{ g}$$

Mass of gold in Lutetia (Au):

$$\text{Au} = \text{Mass} \times \text{Concentration}$$

$$\text{Au} = 6.2876 \times 10^{21} \times 0.6 = 3.77 \times 10^{21} \mu\text{g}$$

$$\text{Au} = \frac{3.77 \times 10^{21}}{10^6} = 3.77 \times 10^{15} \text{ g} = 3.77 \times 10^{12} \text{ kg}$$

Value of the gold (\$):

$$\text{\$} = 3.77 \times 10^{12} \times 12,405 = 4.67 \times 10^{16}$$

$$\text{Value of the gold} = \$4.67 \times 10^{16} \text{ US}$$

4. Write an essay about the topic: “How should we protect ourselves and our civilization from destruction by the impact of a 10-km asteroid?” (Assume that the impactor cannot be deflected).

Points to consider:

- a. If we are taken by surprise and have not prepared for such an event, the human race may perish as a result of the impact of a 10-km asteroid.
- b. The consequences of the impact on the survival of the human race depends on the location of the impact (e.g., polar regions, mid-latitude ocean, mid-latitude continent).
- c. Long-range preparations for a catastrophic impact of an asteroid upon the Earth should include subsurface shelters stocked with food and water to last one year.
- d. In addition, home owners could build private shelters where they may survive if they are not in the vicinity of the impact site.

- e. The preparations should include stockpiles of materials for reconstruction and information about post-impact conditions to be faced by survivors.
- f. In spite of such preparations, most humans would die as a direct consequence of the impact and its aftermath.
- g. The most effective survival strategy may be the establishment of self-sustaining colonies on Mars.

Chapter 14: Jupiter, Heavy-Weight Champion

1. Calculate the average speed of H_2 molecules in the atmosphere of Jupiter (consult Section 22 of Appendix 1), given that the mass of one H_2 molecule is 3.34×10^{-27} kg (Science Brief 14.7.4) and that the temperature of the upper atmosphere of Jupiter is 110 K (Freedman and Kaufmann, 2002) p. 195).

$$v_m \left(\frac{3kT}{m} \right)^{1/2}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 110}{3.34 \times 10^{-27}} \right)^{1/2} = (136.34 \times 10^4)^{1/2}$$

$$v_m = 11.67 \times 10^2 \text{ m/s} = 1.16 \text{ km/s}$$

2. Calculate the mass of one CO_2 molecule (M_{CO_2}) given that the atomic weights of carbon and oxygen are: C = 12.011 amu, O = 15.9994 amu. (Consult Science Brief 14.7.3).

$$\begin{aligned} \text{Molecular weight of } \text{CO}_2 &= 12.011 + 2 \times 15.9994 \\ &= 44.0098 \text{ amu} \end{aligned}$$

$$1 \text{ gram-molecular weight of } \text{CO}_2 = 44.0098 \text{ g}$$

$$1 \text{ gram-molecular weight contains } 6.022 \times 10^{23} \text{ molecules}$$

$$\begin{aligned} \text{Mass of one molecule of } \text{CO}_2 &= \frac{44.0098}{6.022 \times 10^{23}} \\ &= 7.308 \times 10^{-23} \text{ g} \\ &= 7.308 \times 10^{-26} \text{ kg} \end{aligned}$$

3. Demonstrate by calculation whether O_2 molecules can escape from the atmosphere of Venus. The mass of O_2 is 5.32×10^{-26} kg, the temperature is 743 K, the escape velocity from the surface of Venus is 10.4 km/s (Section 23, Appendix 1), and the Boltzmann constant is $1.38 \times$

10^{-23} J/K. (Consult Science Brief 14.7.4.)

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} - 26 \times 743}{5.32 \times 10^{-26}} \right)^{1/2} = (578.19 \times 10^3)^{1/2}$$

$$v_m = 760.3 \text{ m/s} = 0.760 \text{ km/s}$$

The speed of O_2 molecules in the atmosphere of Venus is not high enough to escape.

4. Calculate the Roche limit of Phobos and explain the significance of this result. (Use equation 14.2 of Science Brief 14.7.2).

Density of Phobos = 1.90 g/cm^3

Density of Mars = 3.94 g/cm^3

Radius of Mars = 3394 km

Radius of the orbit of Phobos = $9.38 \times 10^3 \text{ km}$

$$r_R = 2.5 \left(\frac{\rho_M}{\rho_m} \right)^{1/3} \times R$$

$$r_R = 2.5 \left(\frac{3.94}{1.90} \right)^{1/3} \times 3398 = 8495 (2.073)^{1/3}$$

$$r_R = 8495 \times 1.2752 = 10,832 \text{ km}$$

If the average radius of the orbit of Phobos is less than 10,832 km, the tidal force exerted by Mars could disrupt it.

Chapter 15: Galilean Satellites of Jupiter, Jewels of the Solar System.

1. Calculate the average distance of Europa from Jupiter using Newton's version of Kepler's third law (See Science Brief 15.6.1 and data in Appendix 1, Table 15.1 and other sources in this book).

Newton's version of Kepler's third law:

$$p^2 = \left(\frac{4\pi^2}{G(M+m)} \right) a^3$$

M = mass of Jupiter = $1.90 \times 10^{27} \text{ kg}$

m = mass of Europa = 4.79×10^{22} kg

p = period of revolution of Europa = 3.551 d

$$a = \left(\frac{p^2 G (M + m)}{4\pi^2} \right)^{1/3}$$

$$p^2 = (3.551 \times 24 \times 60 \times 60)^2 = 9.4130 \times 10^{10} \text{ s}^2$$

$$M + m = 1.90 \times 10^{27} + 4.79 \times 10^{22} = 1.90 \times 10^{27} \text{ kg}$$

$$\pi^2 = (3.14)^2 = 9.8596$$

$$a = \left(\frac{9.413 \times 10^{10} \times 6.67 \times 10^{-11} \times 1.90 \times 10^{27}}{4 \times 9.8596} \right)^{1/3}$$

$$a = (3.024 \times 10^{26})^{1/3} = 6.712 \times 10^8 \text{ m}$$

$$a = 6.712 \times 10^5 = 671.2 \times 10^3 \text{ km}$$

Note: The radius of the orbit of Europa in Table 15.1 is 671×10^3 km.

2. Use the orbital parameters of Ganymede to calculate the mass of Jupiter. Use data sources available in this book.

$$p^2 = \left(\frac{4\pi^2}{G (M + m)} \right) a^3$$

$$p = 7.155 \text{ d} = 7.155 \times 24 \times 60 \times 60 = 6.1819 \times 10^5 \text{ s}$$

$$a = 1071 \times 10^3 \text{ km} = 1071 \times 10^6 \text{ m}$$

$$m = 1.48 \times 10^{23} \text{ kg}$$

$$m \ll M \text{ (mass of Jupiter)}$$

$$a^3 = \frac{p^2 GM}{4\pi^2}$$

$$M = \frac{a^3 4\pi^2}{p^2 G}$$

$$a^3 = (1071 \times 10^6)^3 = 1.228 \times 10^{27}$$

$$p^2 = (6.1819 \times 10^5)^2 = 38.2158 \times 10^{10}$$

$$M = \frac{1.228 \times 10^{27} \times 4 \times (3.14)^2}{38.215 \times 10^{10} \times 6.67 \times 10^{-11}} = 0.1900 \times 10^{28} \text{ kg}$$

$$M = 1.90 \times 10^{27} \text{ kg}$$

3. Calculate the escape speed of Ganymede (See Appendix 1 and Table 15.1)

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

$$M = 1.48 \times 10^{23} \text{ kg}$$

$$r = 2640 \text{ km}$$

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 1.48 \times 10^{23}}{2640 \times 10^3} \right)^{1/2} = (0.007478 \times 10^9)^{1/2}$$

$$v_e = 2.734 \times 10^3 \text{ m/s} = \frac{2.734 \times 10^3}{10^3} \text{ km/s} = 2.73 \text{ km/s}$$

4. Calculate the average speed of N_2 on the surface of Ganymede (molecular weight = 28.013 atomic mass units, Avogadro's Number = 6.022×10^{23} molecules/mole, surface temperature $T = 107 \text{ K}$).

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{Mass of one } N_2 \text{ molecule } m = \frac{28.013}{6.022 \times 10^{23}}$$

$$m = 4.65 \times 10^{-23} \text{ g} = 4.65 \times 10^{-26} \text{ kg}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 107}{4.65 \times 10^{-26}} \right)^{1/2} = (95.26 \times 10^3)^{1/2}$$

$$v_m = 308.5 \text{ m/s} = 0.308 \text{ km/s}$$

5. Determine whether Ganymede can retain N_2 molecules.

$$v_m = 0.302 \text{ km/s}$$

$$v_e = 2.73 \text{ km/s}$$

$$6 v_m = 0.302 \times 6 = 1.812$$

$v_e > 6 v_m$. Therefore, N_2 molecules cannot escape from Ganymede.

6. Suggest one or several plausible explanations why Ganymede does not have an atmosphere (containing N_2) even though Titan (Chapter 13) does.

Hypothetical explanations for the absence of a N₂-bearing atmosphere on Ganymede:

1. The temperature in the space where Ganymede formed was low enough to cause N₂ gas to condense. Therefore, solid N₂ may be present in the crust of Ganymede but it does not sublime because the surface temperature is low enough to prevent it.
2. Given that Ganymede can retain N₂, it follows that it can also retain CO₂. Ganymede does not have a CO₂-bearing atmosphere because CO₂ gas condenses at the low surface temperature.

Chapter 16: The Beauty of Rings

1. Calculate what fraction expressed in percent of the volume of Rhea is composed of rocky material. Density of Rhea = 1.33 g/cm³, density of water ice = 0.919 g/cm³, density of rocky material = 3.0 g/cm³.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}, \text{Mass} = \text{Volume} \times \text{Density}$$

Mass of Rhea = Mass of the core + mass of the mantle

$$M_R = M_C + M_m \quad (1)$$

$$V_R d_R = V_C d_C + V_m d_m \quad (2)$$

$$d_R = \left(\frac{V_C}{V_R} \right) d_C + \left(\frac{V_m}{V_R} \right) d_m \quad (3)$$

$$\left(\frac{V_C}{V_R} \right) + \left(\frac{V_m}{V_R} \right) = 1 \quad (4)$$

$$\left(\frac{V_m}{V_R} \right) = 1 - \left(\frac{V_C}{V_R} \right) \quad (5)$$

Substituting equation 5 into equation 3:

$$d_R = \left(\frac{V_C}{V_R} \right) d_C + \left(1 - \frac{V_C}{V_R} \right) d_m \quad (6)$$

Solving for $\frac{V_c}{V_R}$

$$d_R = \left(\frac{V_c}{V_R}\right)d_c + d_m - \left(\frac{V_c}{V_R}\right)d_m$$

$$d_R = \left(\frac{V_c}{V_R}\right)(d_c - d_m) + d_m$$

$$\left(\frac{V_c}{V_R}\right) = \frac{d_R - d_m}{d_c - d_m} \quad (7)$$

Substituting values:

$$\left(\frac{V_c}{V_R}\right) = \frac{1.33 - 0.919}{3.0 - 0.919} = \frac{0.411}{2.081} = 0.197$$

$$\left(\frac{V_c}{V_R}\right) = 19.7\%$$

2. Calculate the Roche limit for a comet composed of water ice that is approaching Saturn (density of the comet = 1.10 g/cm³, density of Saturn = 0.690 g/cm³). Consult Section 14 of Appendix 1 and use the equation for a body of zero strength.

Density of the comet: $\rho_c = 1.10 \text{ g/cm}^3$

Density of Saturn: $\rho_s = 0.690 \text{ g/cm}^3$

$$r_R = 2.44 \left(\frac{\rho_s}{\rho_c}\right)^{1/3} R$$

R = radius of Saturn = 60,330 km

$$r_R = 2.44 \left(\frac{0.690}{1.10}\right)^{1/3} \times 60,300$$

$$\frac{0.690}{1.10} = 0.62721$$

$$\frac{1}{3} \log 0.6272 = -\frac{0.20259}{3} = -0.06753$$

$$\left(\frac{0.690}{1.10}\right)^{1/3} = 0.8559$$

$$r_R = 2.44 \times 0.8559 \times 60,330 = 125,993 \text{ km}$$

$$r_R = 125,993 \text{ km}$$

3. Verify that Mimas is outside of the Roche limit for a body consisting of particles that are touching. Density of Mimas = 1.20 g/cm^3 ; density of Saturn = 0.690 g/cm^3 .

Density of Mimas: $d_M = 1.20 \text{ g/cm}^3$ (ρ_M)

Density of Saturn: $d_S = 0.690 \text{ g/cm}^3$ (ρ_S)

Roche limit, touching bodies:

$$r_R = 2.54 \left(\frac{\rho_S}{\rho_M} \right)^{1/3} \times R$$

Radius of Saturn: $R = 60,330 \text{ km}$

$$\frac{\rho_S}{\rho_M} = \left(\frac{0.690}{1.20} \right)^{1/3} = (0.575)^{1/3} = 0.83155$$

$$r_R = 2.54 \times 0.83155 \times 60,330 = 127,425 \text{ km}$$

The average radius of the orbit of Mimas $a = 185,540 \text{ km}$. Therefore, Mimas is outside of the

Roche limit because $a > r_R$

4. Calculate the average speed of water molecules on the surface of Enceladus and the escape velocity. Interpret the results by concluding whether water molecules can escape from the gravitational field of Enceladus. (Molecular weight of water = 18.0152 amu ; mass of Enceladus = $8.4 \times 10^{19} \text{ kg}$; radius of Enceladus = 250 km ; $T = 100 \text{ K}$.)

Escape speed:
$$v_e = \left(\frac{2GM}{R} \right)^{1/2}$$

Average speed of a molecules:
$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

Escape speed of Enceladus:

$$v_e = \left(\frac{2GM}{r} \right)^{1/2} = \left(\frac{2 \times 6.67 \times 10^{-11} \times 8.4 \times 10^{19}}{250 \times 10^3} \right)^{1/2}$$

$$V_e = (0.4482 \times 10^5)^{1/2} = 211.68 \text{ m/s}$$

$$V_e = 0.2117 \text{ km/s}$$

Mass of one molecule of water:

$$m = \frac{18.0152}{6.022 \times 10^{23}} = 2.991 \times 10^{-23} \text{ g}$$

$$M = 2.991 \times 10^{-26} \text{ kg}$$

Average speed of water molecules:

$$v_m = \left(\frac{3kT}{m} \right)^{1/2} = \left(\frac{3 \times 1.38 \times 10^{-23} \times 100}{2.991 \times 10^{-26}} \right)^{1/2}$$

$$v_m = (1.384 \times 10^5)^{1/2} = 371.9 \text{ m/s}$$

$$v_m = 0.372 \text{ km/s}$$

Conclusion: $v_m (\text{H}_2\text{O}) > v_e$. Therefore, molecules of water can escape from Enceladus.

Chapter 17: Titan, An Ancient World in Deep Freeze

1. Calculate the mass of Saturn based on the orbit of Titan by means of Newton's form of Kepler's third law (Appendix 1, Section 11). Mass of Titan (M) = $1.35 \times 10^{23} \text{ kg}$; average radius of the orbit of Titan (a) = $1222 \times 10^3 \text{ km}$; period of revolution of Titan (p) = 15.94 d; $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

Let $m \ll M$, $M + m \approx M$

$$p^2 = \frac{4\pi^2}{GM} a^3$$

$$M = \frac{4\pi^2 a^3}{Gp^2}$$

$$p^2 = (15.94 \times 24 \times 60 \times 60)^2 = 1.8967 \times 10^{12}$$

$$\pi^2 = (3.14)^2 = 9.8596$$

$$a^3 = (1222 \times 10^6)^3 = 1.824 \times 10^{27}$$

$$M = \frac{4 \times 9.8596 \times 1.824 \times 10^{27}}{6.67 \times 10^{-11} \times 1.8967 \times 10^{12}} = 5.686 \times 10^{26} \text{ kg}$$

$$M = 5.69 \times 10^{26} \text{ kg}$$

2. Calculate the average distance of a spacecraft in a “stationary” orbit around Titan using Newton’s form of Kepler’s third law. All data needed for this calculation are provided in Problem 1 above. Express the result in terms of 10^3 km.

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

$$a^3 = \frac{G(M+m)p^2}{4\pi^2}$$

Let $m \ll M$

$$a = \left(\frac{G M p^2}{4\pi^2} \right)^{1/3} = \left(\frac{6.67 \times 10^{-11} \times 1.35 \times 10^{23} \times 1.8967 \times 10^{12}}{4 \times 9.8596} \right)^{1/3}$$

Note: Titan has 1:1 spin - orbit coupling (i.e., its period or revolution is equal to its period of rotation).

$$a = (0.4330 \times 10^{24})^{1/3} = 0.7565 \times 10^8 \text{ m}$$

$$a = 0.7565 \times 10^5 \text{ km} = 75.65 \times 10^3 \text{ km}$$

3. Plot two points A and B in Figure 17.4 and determine the states of aggregation (or phases) of water and methane that coexist in each of the two environments.

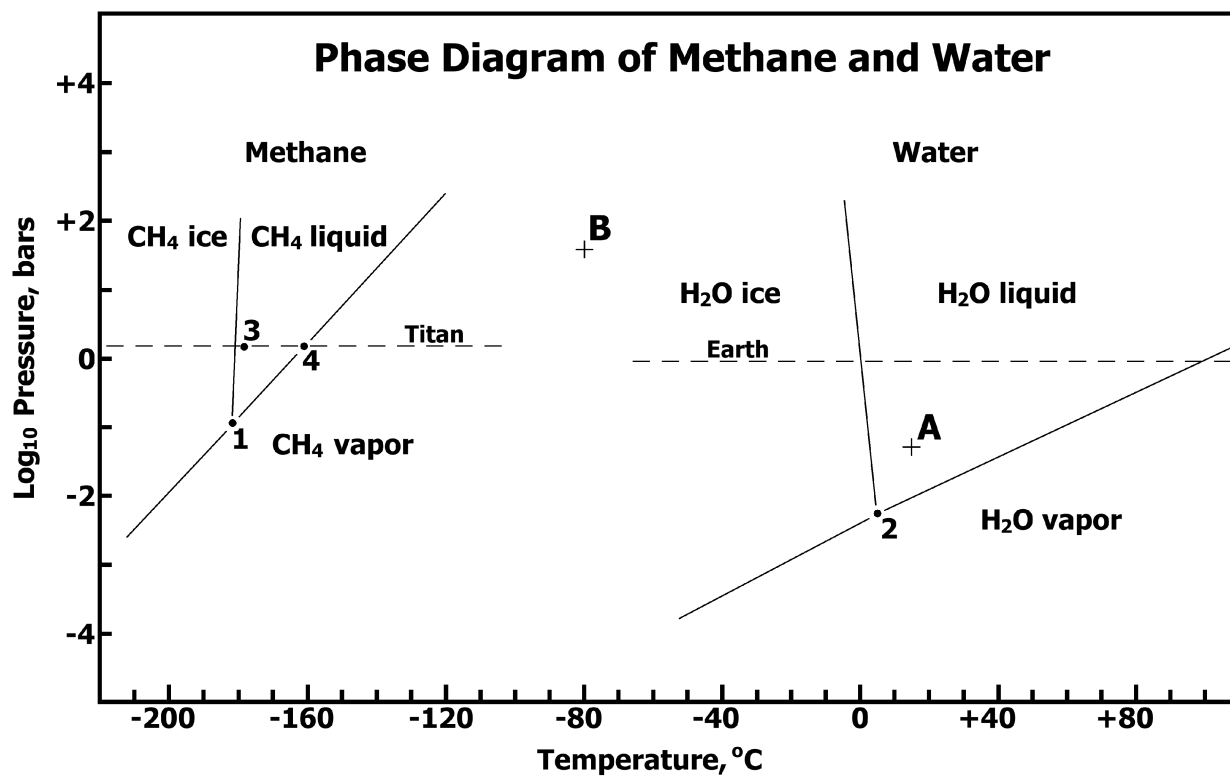
Point A: $P = 3 \times 10^{-2}$ bars, $T = 15^\circ\text{C}$

Point B: $P = 20$ bars, $T = -80^\circ\text{C}$

Point A: water is liquid, methane is vapor

Point B: water is ice, methane is vapor

4. Investigate a possible resonance between Titan and Rhea whose periods of revolution are: Titan = 15.94d; Rhea = 4.518d. Express the resonance in terms of whole numbers and explain in words how it works.



$$\frac{\text{Titan}}{\text{Rhea}} = \frac{15.94}{4.518} = \frac{3.528}{1}$$

Rhea makes 3.528 revolutions for each revolution of Titan. Resonance is 3.528:1

Multiplying by 2 yields:

$$\frac{3.528 \times 2}{1 \times 2} = \frac{7.056}{2}$$

$$2 \times 15.94 = 7.056 \times 4.518$$

$$31.88 = 31.87$$

When Rhea and Titan are in conjunction as seen from Saturn and travel in their orbits for 31.88 days, they return into conjunction.

- Investigate the effect of the orbit of Titan on its day/night cycle and on the seasonality of its climate. Present your results in the form of a term paper including neatly drawn diagrams.

Points to consider:

- Titan has 1:1 spin - orbit coupling. Therefore one day/night cycle is equal to the period of its rotation /revolution.
- The illumination of Titan at night is enhanced by the light that is reflected by Saturn.
- The axial obliquity of Titan is close to 0°. However, Titan does have seasons. Why?
- Day and night have the same length.

Chapter 18: Uranus, What Happened Here?

- Calculate the radius of a hypothetical satellite of Uranus that has a mass of 1.00×10^{19} kg (i.e., equal to the mass of its ring particles) assuming that these particles are composed to water ice having a density of 1.00 g/cm^3 .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}; \text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

Volume (V) of the hypothetical satellite:

$$\text{A mass of } 1.00 \times 10^{19} \times 10^3 \text{ g of ice has a volume of } 1 \times 10^{22} \text{ cm}^3 = \frac{1 \times 10^{22}}{10^{15}} \text{ km}^3$$

$$V = 1 \times 10^7 \text{ km}^3$$

$$V = \frac{4}{3} \pi r^3$$

$$r = \left(\frac{3V}{4\pi} \right)^{1/3} = \left(\frac{3 \times 10^7}{4 \times 3.14} \right)^{1/3} = (0.2388 \times 10^7)^{1/3}$$

$$r = 133.6 \text{ km or } 134 \text{ km}$$

2. Estimate the radius of the orbit of such a hypothetical satellite of Uranus by calculating the Roche limit for such an object using equation 14.1 in Appendix 1.

$$r_R = 2.5 \left(\frac{\rho_M}{\rho_m} \right)^{1/3} \times R$$

$$\rho_M = 1.29 \text{ g/cm}^3; R = 25,559 \text{ km}$$

$$\rho_m = 1.00 \text{ g/cm}^3$$

$$r_R = 2.5 (1.29)^{1/3} 25,559$$

$$1.29^{1/3} = 1.0885$$

$$r_R = 2.5 \times 1.0885 \times 25,559 = 69,552 \text{ km}$$

The radius of the orbit of the hypothetical satellite of Uranus must be greater than 69,552 km.

3. Calculate the perigee (q) and the apogee (Q) distances from the center of Uranus for the “exotic” satellite Caliban (See Table 18.4).

$$a = 7,168.9 \times 10^3 \text{ km}, e = 0.2$$

$$e = \frac{f}{a}, f = e \times a = 0.2 \times 7,168.9 \times 10^3$$

$$= 1433.78 \times 10^3 \text{ km}$$

$$q = a - f = 7,168.9 \times 10^3 - 1433.78 \times 10^3 = 5735.12 \times 10^3 \text{ km}$$

$$Q = a + f = (7,168.9 + 1433.78) \times 10^3 = 8.602.68 \times 10^3 \text{ km}$$

$$q = 5,735 \times 10^3 \text{ km}$$

$$Q = 8,603 \times 10^3 \text{ km}$$

4. Investigate whether any two of the known satellites of Uranus in Table 18.4 are “in resonance” with each other.

Satellites	Period of revolution, d
Miranda	1.414
Ariel	2.520
Umbriel	4.144
Titania	8.706
Oberon	13.463

$$\frac{\text{Oberon}}{\text{Titania}} = \frac{13.463}{8.706} = \frac{1.546}{1} = \frac{3.09}{2}$$

Oberon: 2 revolution, Titana 3.0 revolutions

$$\frac{\text{Oberon}}{\text{Umbriel}} = \frac{13.463}{4.144} = \frac{3.2}{1} = \frac{6.497}{2} = \frac{12.99}{4}$$

Umbriel: Oberon: 4 revolutions, Umbriel:13 revolutions

$$\frac{\text{Titania}}{\text{Umbriel}} = \frac{8.706}{4.144} = \frac{2.10}{1} = \frac{4.2}{2} \text{ (no resonance)}$$

Many additional resonances remain to be discovered.

5. Calculate the Roche limit of the satellite Ophelia assuming that its density is 1.0 g/cm^3 (i.e., water ice) and compare that limit to the radius of its orbit. Use equation A1.3 of Appendix 1. Derive a conclusion from this comparison.

$$r_R = 2.5 \left(\frac{\rho_M}{\rho_m} \right)^{1/3} \times R$$

$$\rho_M = 1.290 \text{ g/cm}^3$$

$$\rho_m = 1.0 \text{ g/cm}^3$$

$$R = 25,559 \text{ km}$$

$$r_R = 2.5 \left(\frac{1.290}{1.00} \right)^{1/3} \times 25,559 = 2.5 \times 1.0885 \times 25,559$$

$$r_R = 69,552 \text{ km}$$

$$\text{Radius of the orbit of Ophelia} = 53.8 \times 10^3 \text{ km}$$

The orbit of Ophelia is located inside the Roche limit as calculated. Therefore, Ophelia would be unstable if its density actually is 1.00 g/cm^3 (i.e., it is composed of water ice).

If it is a rocky object with a bulk density of about 3.2 g/cm^3 , its Roche limit would be $r_R = 47,200 \text{ km}$. In that case, Ophelia's orbit lies outside the Roche limit and the satellite would be stable.

Given that Ophelia does appear to be stable, its Roche limit is less than $53,800 \text{ km}$, which indicates that it is not composed of ice.

Note: This calculation can be extended to determine the density of Ophelia that yields a Roche limit of $53,800 \text{ km}$.

The resulting density may indicate that Ophelia has a rocky core covered by an ice mantle.

The radius of the hypothetical core and the thickness of the ice mantle can be calculated based on assumed densities.

Chapter 19: Neptune, More Surprises

1. Calculate the period of revolution (p) of Triton in its orbit around Neptune from Newton's version of Kepler's third law (See Tables 18.1 and 19.2 and refer to Section 11 of Appendix 1.

Mass of Neptune $M_N = 1.02 \times 10^{26} \text{ kg}$
 Triton, $a = 354.8 \times 10^3 \text{ km}$

$$p^2 = \left(\frac{4\pi^2}{G(M + m)} a^3 \right)$$

$$p^2 = \frac{4 \times 3.14 \times 3.14 \times (354.8 \times 10^6)^3}{6.67 \times 10^{-11} \times 1.02 \times 10^{26}} = \frac{4 \times 9.8596 \times 44.66 \times 10^{24}}{6.8034 \times 10^{15}}$$

$$p^2 = 258.88 \times 10^9 \text{ s}$$

$$p = 508,744.8 \text{ s} = \frac{508,744.8}{60 \times 60 \times 24} \text{ d}$$

$$p = 5.88 \text{ d}$$

2. Calculate the escape speed from the surface of Triton.

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

Radius of Triton $r = 1350 \text{ km}$

Mass of Triton $M = 2.14 \times 10^{22} \text{ kg}$

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 2.14 \times 10^{22}}{1350 \times 10^3} \right)^{1/2}$$

$$v_e = (0.02114 \times 10^8)^{1/2} = 1453.9 \text{ m/s}$$

$$v_e = 1.453 \text{ km/s}$$

3. Calculate the average speed of molecules of N_2 in space adjacent to the surface of Triton. $T = -240^\circ\text{C}$, molecular weight of $N_2 = 28.013 \text{ amu}$. Consult Science Brief 14.7.3 and use equation A1.19 of Appendix 1.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$\text{Mass of } N_2 = \frac{28.013}{6.022 \times 10^{23}} = 4.6517 \times 10^{-23} \text{ g}$$

$$M = 4.65 \times 10^{-26} \text{ kg}$$

Temperature $T = -240 + 273 = +33 \text{ K}$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 33}{4.65 \times 10^{-26}} \right)^{1/2} = (29.380 \times 10^3)^{1/2}$$

$$v_m = 171.39 \text{ m/s} = 0.171 \text{ km/s}$$

$$6 v_m = 1.028 \text{ km/s}$$

4. $6 v_m < v_e$. Therefore, N_2 does not escape from Triton.
5. Calculate the speed of H_2 molecules at $T = 33 \text{ K}$ on the surface of Triton. The molecular weight of H_2 is 2.0159 amu .

$$\begin{aligned} \text{Mass of } H_2 (m) &= \frac{2.0159}{6.022 \times 10^{23}} = 0.3347 \times 10^{-23} \text{ g} \\ &= 0.3347 \times 10^{-26} \text{ kg} \end{aligned}$$

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 33}{0.3347 \times 10^{-26}} \right)^{1/2} = (408.18 \times 10^3)^{1/2}$$

$$v_m = 638.8 \text{ m/s} = 0.638 \text{ km/s}$$

6. The molecular speed is expressed by the equation.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

The speed increases with increasing temperature and decreases with increasing mass of the molecule.

Chapter 20: Pluto and Charon, The Odd Couple

1. Calculate the distance between the center of gravity of the Earth-Moon system and the center of the Earth.

$$\text{Mass of the Earth } (m_E) = 5.98 \times 10^{24} \text{ kg}$$

$$\text{Mass of the Moon } (m_M) = 7.35 \times 10^{22} \text{ kg}$$

$$\text{Radius of the orbit of the moon } r_M = 384.4 \times 10^3 \text{ km}$$

Let x be the distance between the center of the Earth and the center of gravity of the Earth-Moon system. Taking moments around the center of gravity:

$$5.98 \times 10^{24} (x) = 7.35 \times 10^{22} (384.4 \times 10^3 - x)$$

Solving for x :

$$5.98 \times 10^{24} (x) = 2825.3 \times 10^{25} - 7.35 \times 10^{22} (x)$$

$$598 \times 10^{22} (x) + 7.35 \times 10^{22} (x) = 2825.3 \times 10^{25}$$

$$605.35 \times 10^{22} (x) = 2825.3 \times 10^{25}$$

$$x = \frac{2825.3 \times 10^{25}}{605.35 \times 10^{22}}$$

$$x = 4.667 \times 10^3 \text{ km}$$

2. Calculate the escape speed from the surface of Charon.

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}$$

Mass of Charon (M) = 1.77×10^{21} kg

Radius of Charon (r) = 595 km

$$v_e = \left(\frac{2 \times 6.67 \times 10^{-11} \times 1.77 \times 10^{21}}{595 \times 10^3} \right)^{1/2}$$

$$v_e = (0.03968 \times 10^7)^{1/2} = 629.9 \text{ m/s}$$

$$v_e = 0.630 \text{ km/s}$$

3. Calculate the synodic period of Pluto.

For a superior planet:

$$\frac{1}{P} = \frac{1}{E} - \frac{1}{S}$$

Period of revolution of Pluto (P) = 247.7 y

Period of revolution of Earth (E) = 1.0 y

$$\frac{1}{247.7} = \frac{1}{1} - \frac{1}{S}$$

$$0.004037 = 1 - \frac{1}{S}$$

$$\frac{1}{S} = 1 - 0.004037 = 0.99496$$

$$S = \frac{1}{0.99496} = 1.00405 \text{ y}$$

$$S = 1.00405 \times 365.25 = 366.73 \text{ d}$$

4. Calculate the average speed of molecules of CH_4 (molecular weight = 16.043 amu) on the surface of Charon ($T = 37\text{K}$) and determine whether Charon can retain this compound.

$$v_m = \left(\frac{3kT}{m} \right)^{1/2}$$

$$\text{Mass of a methane molecule} = \frac{16.043}{6.022 \times 10^{23}}$$

$$= 2.664 \times 10^{-26} \text{ kg}$$

$$v_m = \left(\frac{3 \times 1.38 \times 10^{-23} \times 37}{2.664 \times 10^{-26}} \right)^{1/2} = (57.5 \times 10^3)^{1/2}$$

$$v_m = 239.7 \text{ m/s} = 0.239 \text{ km/s}$$

$$6 \times v_m = 1.438 \text{ km/s}$$

$$v_e = 0.630 \text{ km/s}$$

Methane can escape from Charon because $6 \times v_m = 1.438 \text{ km/s}$ is greater than the escape velocity ($v_e = 0.630 \text{ km/s}$).

Chapter 21: Ice Worlds at the Outer Limits

1. Write a term paper on the theme: “Questions to be answered about the origin and evolution of KBOs by the New Horizons spacecraft”

Points to consider:

1. KBOs that were expelled by gravitational interactions (i.e, tides) may have experienced cryovolcanic activity and structural deformation (e.g., rifting).
 2. KBOs that were expelled from the space between Saturn and Neptune may have different chemical compositions than indigenous KBOs that formed outside of the orbit of Neptune.
 3. All KBOs have surface deposits containing organic compounds, amorphous carbon, and refractory compounds similar to the crusts of short-period comets.
 4. Some KBOs may have a rocky core that formed by internal differentiation.
 5. It may be possible in the future to land on a KBO and to cause it to collide with Mars thereby increasing the atmospheric pressure and changing the climate on that planet.
2. Calculate the radius of Eris using equation 21.1, where R = radius in km, r = heliocentric distance (97.6 AU), Δ = distance from the Earth (96.6 AU), p = albedo expressed as a decimal fraction (0.86), and V = visual magnitude (18.8).

$$R^2 = \frac{4.53 \times 10^5 r^2 \Delta^2}{10^{0.4V} \times p}$$

$$0.4 \times 18.8 = 7.52$$

$$10^{7.52} = 33.113 \times 10^6$$

$$R^2 = \frac{453 \times 10^5 \times (97.6)^2 (96.6)^2}{33.113 \times 10^6 \times 0.86}$$

$$R^2 = 14.140 \times 10^6 \times 10^{-1}$$

$$R = (14.140 \times 10^5)^{1/2} = 1189.0 \text{ km}$$

Note: The radius of Pluto is 1150 km (i.e., Eris is larger than Pluto).

3. Calculate the magnitude of the force of gravity exerted by the Sun on an object in the Oort cloud that has a diameter of 100 km and a density of 1.00 g/cm^3 at a heliocentric distance of 100,000 AU.

$$\text{Solar mass} = 1.99 \times 10^{30} \text{ kg}$$

Mass of the ice object in the Oort cloud:

$$V = \frac{4}{3} \pi r^3 = \frac{4 \times 3.14 \times (50 \times 1000 \times 100)^3}{3}$$

$$V = 4.186 \times (50 \times 10^5)^3 = 523,250 \times 10^{15} \text{ cm}^3$$

$$V = 5.232 \times 10^{20} \text{ cm}^3$$

$$M = 5.232 \times 10^{20} \text{ g} = 5.232 \times 10^{17} \text{ kg}$$

$$F = \frac{GM \times m}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.232 \times 10^{17}}{(100,000 \times 149.6 \times 10^6 \times 10^3)^2}$$

$$F = \frac{69.445 \times 10^{36}}{(149.6 \times 10^{14})^2} = \frac{69.445 \times 10^{36}}{22380 \times 10^{28}}$$

$$F = \frac{69.445 \times 10^{36}}{2.238 \times 10^{32}} = 31.029 \times 10^4 \text{ N}$$

4. Calculate the period of revolution of an object in the Oort cloud at an average distance of 100,000 AU from the Sun traveling on a circular path.

$$\text{Kepler's third law: } p^2 = a^3$$

p = period of revolution in years

a = average orbital radius in AU

$$p^2 = 100,000^3 = 10^{15}$$

$$p = (10^{15})^{1/2} = 31.62 \times 10^6 \text{ y}$$

5. Calculate the average orbital speed of an object on a circular orbit in the Oort cloud at a distance of 100,000 AU

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2r\pi}{p}$$

$$v = \frac{2 \times 100,000 \times 149.66 \times 10^6 \times 3.14}{31.62 \times 10^6 \times 365.25 \times 24 \times 60 \times 60}$$

$$v = \frac{939.48 \times 10^{11}}{9.9785 \times 10^{14}} = 94 \times 10^{-3} \text{ km / s}$$

$$v = 0.094 \text{ km/s}$$

Chapter 22: Comets, Coming Inside from the Cold

1. Calculate the area of the Edgeworth-Kuiper belt located between the orbit of Neptune (30 AU) and 50 AU. Express the result as a fraction of the total area of the solar system extending to 50 AU.

$$\text{Area of a circle (A)} = r^2 \pi$$

$$r = 50 \text{ AU } A_{50} = (50)^2 \pi = 7850 \text{ (AU)}^2$$

$$r = 30 \text{ AU } A_{30} = (30)^2 \pi = 2826 \text{ (AU)}^2$$

$$A_{50} - A_{30} = (7850 - 2826) \text{ (AU)}^2 = 5024 \text{ (AU)}^2$$

$$\frac{A_{50} - A_{30}}{A_{50}} = \frac{5024 \times 100}{7850} = 64\%$$

2. The orbital parameters of the short-period comet Crommelin (Science Brief 22.9.3) are: $p = 27.89 \text{ y}$ and $q = 0.743 \text{ AU}$. Use these data to calculate values of a , Q , and e .

$$\text{Kepler's third law: } p^2 = a^3$$

$$a = (p^2)^{1/3}, p^2 = (27.89)^2 = 777.8521$$

$$a = (777.8521)^{1/3} = 9.196 \text{ y}$$

$$f = a - q = 9.196 - 0.743 = 8.453 \text{ AU}$$

$$Q = a + f = 17.649 \text{ AU}$$

$$e = \frac{f}{a} = \frac{8.453}{9.196} = 0.919$$

3. The orbital parameters of the long-period comet Hale-Bopp after its last pass through the solar system include $Q = 358 \text{ AU}$ and $q = 0.914 \text{ AU}$. Calculate the corresponding values of p and e of its new orbit.

$$a = \frac{q + Q}{2} = \frac{0.914 + 358}{2} = 179.457 \text{ AU}$$

$$p^2 = a^3 = (179.457)^3 = 5.7793 \times 10^6$$

$$p = (5.7793 \times 10^6)^{1/2} = 2.404 \times 10^3 \text{ y}$$

$$e = \frac{f}{a}, f = a - q = 179.457 - 0.914 = 178.543 \text{ AU}$$

$$e = \frac{178.543}{179.457} = 0.995$$

$$p = 2.404 \times 10^3 \text{ y}$$

$$e = 0.995$$

4. Explain why Kepler's third law applies to the orbits of comets but not to the orbits of satellites.

Kepler's laws apply to objects that revolve around the Sun. Its complete form is:

$$p^2 = k a^3$$

where k is a proportionality constant.

When this relation is applied to the Earth for which $p = 1.0 \text{ y}$ and $a = 1.0 \text{ AU}$, $k = 1.0$. Therefore, Kepler's third law takes the form:

$$p^2 = a^3$$

but p must be expressed in years and a must be in AU and the object must revolve around the Sun rather than around a planet.

5. Does Newton's version of Kepler's third law apply to comets? Make a calculation for the comet Encke and find out.

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

All variables must be expressed in SI units because that is required if $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

Let $a = 2.21 \text{ AU (comet Encke)}$
 $a = 2.21 \times 149.6 \times 10^6 \times 10^3 \text{ m}$
 $a = 330.616 \times 10^9 \text{ m}$

The mass of the Sun (M) = $1.989 \times 10^{30} \text{ kg}$

The mass of Encke (m) = $\ll M$

$$p^2 = \frac{4 \times 3.14 \times 3.14 \times (330.616 \times 10^9)^3}{6.67 \times 10^{-11} \times 1.989 \times 10^{30}}$$

$$p = \left(\frac{4 \times 9.8596 \times 3.618 \times 10^{34}}{6.67 \times 10^{-11} \times 1.989 \times 10^{30}} \right)^{1/2}$$

$$p = (10.7429 \times 10^{15})^{1/2} = 1.03647 \times 10^8 \text{ s}$$

$$p = \frac{1.03647 \times 10^8}{60 \times 60 \times 24 \times 365.25} = 3.28 \text{ y}$$

The period of revolution of comet Encke is 3.30 y. Evidently, Newton's form of Kepler's third law does apply to objects that revolve around the Sun including comets.

Introduction to Planetary Science

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Faure, G.; Mensing, T.M.

2007, XX, 526 p., Hardcover

ISBN: 978-1-4020-5233-0