2.8.1 Properties of Electron Bremsstrahlung

For the production of electron Bremsstrahlung two mechanisms of the electron–atom interaction are of importance:

- *Ordinary Bremsstrahlung*: Electron Bremsstrahlung as a result of photon emission of a charged particle decelerated in the field of a target electron or nucleus.
- *Polarizational Bremsstrahlung* (atomic Bremsstrahlung) as a result of the dynamical polarization of the atom by the incident particle (photon emission of the target electrons virtually excited by the projectile).

Because of its practical importance, in the following we will consider in detail ordinary Bremsstrahlung in comparison to polarization Bremsstrahlung. Ordinary Bremsstrahlung is much more important in most cases.

The production of electron Bremsstrahlung is characterized by a change of the momentum $\boldsymbol{p}$ of the impinging electron with simultaneous emission of electromagnetic radiation (Bremsstrahlung):

$$\mathrm{A} + e^-(\boldsymbol{p}_0) \rightarrow \mathrm{A} + e^-(\boldsymbol{p}_1) + \hbar\omega \ . \qquad (2.149)$$

In this way the inner state of the atom remains unchanged.

For electron Bremsstrahlung the spectrum is characteristic: quanta can be emitted up to a limiting energy of

$$E_{\max} = \frac{hc}{E_e}. \qquad (2.150)$$

The probability that a charged particle emits Bremsstrahlung is proportional to

$$\frac{q^2 Z^2 E}{m^2}$$

with q the charge of the particle (in units of the electron charge), E the particle energy and m the mass of the particle. This proportionality is the reason that especially light particles such as electrons can intensively emit Bremsstrahlung. On the other hand, in comparison to electrons, protons generate Bremsstrahlung with a probability six orders of magnitude lower.

For the case of the deceleration of an electron in the field of a nucleus the basic situation for the production of Bremsstrahlung is shown in Fig. 2.48.

2.8.2 Bremsstrahlung Cross-Sections

Fundamental calculations of the energy loss of electrons by Bremsstrahlung in the fields of atomic nuclei and atomic electrons were done by Bethe [59] and Heitler [231], among others. An overview on cross-sections for Bremsstrahlung production processes can be found by Koch and Motz [287]. Furthermore, a summary of the treatment of Bremsstrahlung photon emission in the framework of many-body perturbation theory was given by Amusia [13]. Basic formulas for Bremsstrahlung production and corresponding numerical material can also be found by Beyer et al. [62].

The energy loss $(-dE/dx)_{rad}$ of electrons during Bremsstrahlung emission can be explained in the framework of quantum electrodynamics using the Thomas–Fermi model

$$\left(-\frac{dE}{dx}\right) \approx 4\,\alpha\, r_e^2 N_0 E Z^2 \,\ln\frac{183}{Z^{1/3}} \ . \qquad (2.151)$$

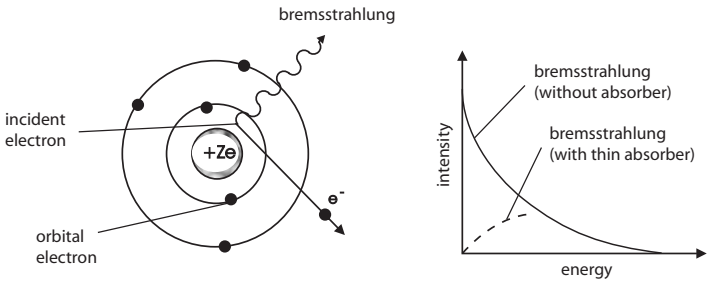

Fig. 2.48. Production of electron Bremsstrahlung by deceleration of electrons in the field of a nucleus. On the ride side the influence of a thin absorber on the undisturbed spectrum is shown.

Here the quantity $(-dE/dx)_{rad}$ describes the energy loss per electron and path length. Furthermore, it yields E, the electron energy, N_0, the atomic density of a medium with atomic number Z, $\alpha = e^2/\hbar c$, the fine-structure constant and $r_e = e^2/(4\pi\,\varepsilon_0\,mc^2) = \alpha^2\,a_0 = 2.818 \times 10^{-13}$ cm the classical electron radius.

2.8.3 Radiation Length and Radiation Density

The electron Bremsstrahlung loss increases with increasing electron energy. This will be transparent if we write (2.151) in the form

$$\left(-\frac{dE}{dx}\right) = \frac{1}{x_0}\,E\,. \tag{2.152}$$

Here the quantity x_0 describes the so-called *radiation length*, which characterizes the target thickness for which the particle energy is reduced to the e-th part of the impact energy:

$$\frac{1}{x_0} = 4\,\alpha\,r_e^2 N_0 Z^2\,\ln\frac{183}{Z^{173}}\,. \tag{2.153}$$

Furthermore, the product of the radiation length and the density ϱ of the medium is also applied for the characterization of the Bremsstrahlung and is known as the *radiation thickness* x_r:

$$x_r = \varrho\,x_0 = \frac{A}{4\,\alpha\,r_e^2 N_A Z^2\,\ln\dfrac{183}{Z^{1/3}}} \tag{2.154}$$

with A the atomic number of the target atoms and N_A the Avogadro constant.

2.8.4 Electron Bremsstrahlung from X-Ray Tubes

The photon spectrum from an X-ray tube contains contributions from the electron Bremsstrahlung of accelerated electrons impinging on the thick (in the most cases) anode and lines of X-ray transitions from the atoms of the anode material. Here we will tract in more detail the Bremsstrahlung part of the tube spectrum.

An X-ray tube working with an acceleration voltage U emits Bremsstrahlung quanta up to a threshold wavelength

$$\lambda_{\max} = \frac{hc}{eU} \qquad (\text{Duane-Hunt limit})\,. \tag{2.155}$$

After considering the values for the natural constants we get

$$\lambda_{\max}[\text{nm}] = 1.239 \times 10^9\,\frac{1}{U\,[\text{kV}]}\,. \tag{2.156}$$

For the emitted spectrum the intensity $I(\lambda)$ can be described by the Kramer rule [296]:

$$I(\lambda)\,d\lambda = \frac{KiZ}{\lambda^2}\left(\frac{\lambda}{\lambda_{\max}} - 1\right) \tag{2.157}$$

where K is the Kramer constant, i is the tube current and Z is the atomic number of the anode material. $I(\lambda)$ describes the intensity density and $I(\lambda)\,d\lambda$ the spectral intensity. The total radiation intensity emitted by an X-ray tube results as the integral of the spectral intensity integrated over all wavelengths of the spectrum. The units of the intensity density are $\text{s}^{-1}\text{nm}^{-1}$ or $\text{s}^{-1}\text{eV}^{-1}$ if the quantity is declared as $I(\lambda)$ or as $I(E)$.

For the X-ray intensity at a tube voltage U and C as a constant it follows that

$$I = CiZU^2\,. \tag{2.158}$$

The maximum wavelength for the intensity distribution of the Bremsstrahlung spectrum can be estimated from the critical wavelength $\lambda_{\max}$

$$\bar{\lambda} = \frac{3}{2}\,\lambda_{\max} \tag{2.159}$$

and depends on the tube voltage, the characteristics of the applied voltage and on the anode material (atomic number).

For the total absorption of an electron beam of current i and acceleration voltage U in a material of atomic number Z the Bremsstrahlung power P_X is

$$P_X[\text{W}] = 1.5 \times 10^{-9}\,Z\,i[\text{A}]\,U^2[\text{V}]\,. \tag{2.160}$$

In Fig. 2.49 the spectral distribution emitted by an X-ray tube is presented. For real tubes the long-wave part of the emitted radiation is attenuated by the tube window (as a rule beryllium or aluminum).

Ne Z=10

[He] $2s^2\ 2p^6$

Neon A = 20.1797(6) [260] ρ = 8.391(-4) g/cm^3 [547]

X-ray transitions

line	transition	E/eV		I/eV/$\hbar$		TPIV	Γ/eV	
K series								
K_{α_3}	KL_1	817.69 ± 0.56	[145]					
$K_{\alpha_{1,2}}$	$KL_{3,2}$	848.61±0.26	[50]	3.37(-3)	[488]		0.24	[74]
K_α	KL	848.32	[104]					
K_{α_2}	KL_2	838.10	[423]			4.643(-3)		
K_{α_2}	KL_2	849.09 ± 0.54	[145]					
K_{α_1}	KL_3	838.22	[423]			9.230(-3)		
K_{α_1}	KL_3	849.17 ± 0.54	[145]					
K_β	KM	857.89 ± 0.44	[50]					
	KM	857.44	[104]					

level characteristics

level	E_B/eV		ω_{nlj}		Γ/eV		AE/eV	
K	867.0	[503]	1.52(-2)	[39]	0.24	[301]	866.90	[51]
	866.9±0.3	[51]	1.80(-2)	[301]	0.243	[423]	870.73 ± 0.16	[145]
	870.1±0.2	[350]	1.82(-2)	[347]	0.24	[94]		
			1.64(-2)	[557]				
			2.04(-2)	[293]				
			1.39(-2)	[423]				
			1.80(-2)	[310]				
L_1	45.0	[503]			< 0.1	[301]	45.0	[51]
	48.5	[350]					53.04 ± 0.40	[145]
L_2	18.0	[503]			0.01	[94]	18.3	[51]
	18.3±0.4	[51]					21.63 ± 0.39	[145]
	21.7	[493]						
L_3	18.0	[503]			0.01	[94]	18.3	[51]
	18.3±0.4	[51]					21.55 ± 0.38	[145]
	21.6	[493]						
IP	21.5646	[222]						

Pr Z=59

[Xe] $4f^3\ 6s^2$

Praseodymium

A = 140.90765(2) [222]

ϱ = 6.772 g/cm^3 [547]
ϱ = 6.71 g/cm^3 [483]

X-ray transitions

line	transition	E/eV		I/eV/$\hbar$		TPIV	Γ/eV	
K series								
K_{α_3}	KL_1	35157.3 ± 1.4	[145]				19.74	[423]
$K_{\alpha 2}$	KL_2	35550.59 ± 0.30	[50]	4.15	[488]	2.630(-1)	20.10	[301]
		35550.00	[104]				19.08	[423]
$K_{\alpha 1}$	KL_3	36026.71 ± 0.31	[50]	7.57	[488]	4.802(-1)	19.80	[301]
		36027.00	[104]				18.87	[423]
	KM_1	40484.3 ± 1.4	[145]				17.14	[423]
$K_{\beta 3}$	KM_2	40653.27 ± 0.99	[50]	0.742	[488]	4.706(-2)	21.79	[423]
		40654.00	[104]					
$K_{\beta 1}$	KM_3	40748.67 ± 0.79	[50]	1.436	[488]	9.112(-2)	22.73	[423]
		40749.00	[104]					
$K_{\beta_5^{II}}$	KM_4	41039.2 ± 1.5	[145]			5.856(-4)	16.65	[423]
$K_{\beta_5^{I}}$	KM_5	41060.5 ± 1.5	[145]			7.851(-4)	16.53	[423]
	KN_1	41688.1 ± 4.3	[145]				24.03	[423]
$K_{\beta 2}$	$KN_{2,3}$	41774.4 ± 4.2	[50]					
$K_{\beta_2^{II}}$	KN_2	41738.40	[104]	0.1583	[488]	1.006(-2)	27.62	[423]
		41754 ± 16	[145]					
$K_{\beta_2^{I}}$	KN_3	41775.40	[104]	0.308	[488]	1.956(-2)	20.13	[423]
		41774.3 ± 1.4	[145]					
$K_{\beta_4^{II}}$	KN_4	41872.4 ± 4.5	[145]			1.245(-4)	16.56	[423]
$K_{\beta_4^{I}}$	KN_5	41876.35 ± 0.97	[145]			1.670(-4)	16.31	[423]
L series								
	L_1M_1	5326.9 ± 1.7	[145]				5.32	[423]
$L_{\beta 4}$	L_1M_2	5498.1 ± 1.4	[50]	7.97(-2)	[488]	1.853(-2)	9.97	[423]
		5497.90	[104]					
$L_{\beta 3}$	L_1M_3	5591.8 ± 1.1	[50]	0.1199	[488]	2.787(-2)	10.91	[423]
		5593.60	[104]					
$L_{\beta 10}$	L_1M_4	5884.0 ± 1.7	[50]	1.43(-3)	[488]	3.328(-4)	4.83	[423]

line	transition	E/eV		I/eV/$\hbar$		TPIV	Γ/eV	
		5885.20	[104]					
$L_{\beta 9}$	L_1M_5	5902.8 ± 1.7 5904.10	[50] [104]	2.14(-3)	[488]	4.976(-4)	4.71	[423]
	L_1N_1	6530.7 ± 4.6	[145]				12.21	[423]
$L_{\gamma 2}$	L_1N_2	6598.0 ± 2.1 6600.40	[50] [104]	1.837(-2)	[488]	4.269(-3)	15.80	[423]
$L_{\gamma 3}$	L_1N_3	6615.9 ± 2.1 6618.80	[50] [104]	2.861(-2)	[488]	6.649(-3)	8.31	[423]
	L_1N_4	6715.1 ± 4.8	[145]			4.432(-5)	4.74	[423]
$L_{\gamma_{11}}$	L_1N_5	6719.0 ± 1.3	[145]			6.789(-5)	4.49	[423]
	$L_1N_{4,5}$	6718.70	[104]	2.92(-4)	[488]			
	$L_1N_{6,7}$	6830.40	[501]					
$L_{\gamma 4}$	$L_1O_{2,3}$	6815.00 ± 1.50 6811.90	[50] [104]	4.09(-3)	[488]			
	L_1O_2					6.221(-4)		
	L_1O_3					9.503(-4)		
L_η	L_2M_1	4935.6 ± 8.7 4935.00	[50] [104]	1.002(-2)	[488]	4.563(-3)	16.81	[423]
	L_2M_2	5104.0 ± 1.5	[145]				9.31	[423]
$L_{\beta_{17}}$	L_2M_3	5200.4 ± 1.4	[145]			7.667(-5)	10.25	[423]
$L_{\beta 1}$	L_2M_4	5488.9 ± 1.1 5489.60	[50] [104]	0.365	[488]	1.042(-1)	4.17	[423]
	L_2M_5	5510.0 ± 1.1	[145]				4.05	[423]
$L_{\gamma 5}$	L_2N_1	6136.2 ± 1.8 6135.70	[50] [104]	2.34(-3)	[488]	7.698(-4)	11.55	[423]
	L_2N_2	6204 ± 15	[145]				15.14	[423]
	L_2N_3	6223.8 ± 1.0	[145]			1.317(-5)	7.65	[423]
$L_{\gamma 1}$	L_2N_4	6322.1 ± 1.4 6323.50	[50] [104]	6.13(-2)	[488]	1.747(-2)	4.08	[423]
	L_2N_5	6325.84 ± 0.61	[145]				3.83	[423]
L_ν	L_2N_6	6436.70 6431.16 ± 0.81	[104] [145]	1.06(-5)	[488]	3.030(-6)		
	L_2N_7	6436.70	[104]					
$L_{\gamma 8}$	L_2O_1	6403.00 ± 1.30 6403.30	[50] [104]	4.05(-4)	[488]	1.164(-4)	3.87	[423]
L_l	L_3M_1	4453.23 ± 0.95 4453.60	[50] [104]	1.328(-2)	[488]	8.390(-3)	16.59	[423]

line	transition	E/eV		I/eV/$\hbar$		TPIV	Γ/eV	
L_t	L_3M_2	4628.2 ± 1.4	[145]			5.080(-5)	9.10	[423]
L_s	L_3M_3	4724.6 ± 1.3	[145]			4.697(-5)	10.04	[423]
$L_{\alpha 2}$	L_3M_4	5013.64 ± 0.90 5013.00	[50] [104]	3.42(-2)	[488]	1.004(-2)	3.96	[423]
$L_{\alpha 1}$	L_3M_5	5033.79 ± 0.60 5033.60	[50] [104]	0.302	[488]	8.893(-2)	3.84	[423]
$L_{\beta 6}$	L_3N_1	5659.7 ± 1.5 5660.20	[50] [104]	3.04(-3)	[488]	9.185(-4)	11.33	[423]
	L_3N_2	5724.20 5728 ± 15	[501] [145]			6.199(-6)	14.92	[423]
	L_2N_3	5748.06 ± 0.97	[145]			5.978(-6)		
$L_{\beta 15}$	L_3N_4	5851.30 5846.1 ± 4.1	[104] [145]	5.54(-3)	[488]	1.628(-3)	3.87	[423]
L_{β_2}	L_3N_5	5850.08 ± 0.54	[145]			1.455(-2)	3.62	[423]
$L_{\beta 15,2}$	$L_3N_{4,5}$	5849.9 ± 1.6	[50]					
L_u	$L_3N_{6,7}$	5721.10	[104]	8.91(-6)	[488]			
L'_u	L_3N_6	5955.40 ± 0.75	[145]			4.573(-7)		
L_u	L_3N_7					2.621(-6)		
$L_{\beta 7}$	L_3O_1	5927.00 ± 1.10 5926.40	[50] [104]	5.27(-4)	[488]	1.588(-4)	3.66	[423]
	$L_3O_{2,3}$	5940.80	[501]					
	L_3O_2					1.168(-6)		
	L_3O_3					1.037(-6)		
M series								
$M_{\gamma 2.1}$	$M_3N_{4,5}$	1127.30 ± 0.90 1127.30 1129.20	[50] [104] [263]	5.80(-3)	[335]			
$M_{\gamma 2}$	M_3N_4					2.051(-4)	7.73	[423]
$M_{\gamma 1}$	M_3N_5					1.679(-3)	7.48	[423]
$M_{\xi 2}$	M_4N_2	715.34	[423]			3.233(-3)	12.71	[423]
M_δ	M_4N_3	736.42	[423]			4.784(-4)	5.22	[423]
M_β	M_4N_6	950.00 ± 1.00 951.50	[50] [263]	1.37(-3)	[335]	3.579(-4)		
M_η	M_4O_2	928.72	[423]			5.254(-4)		
$M_{\xi 1}$	M_5N_3	714.00 ± 1.60 716.70	[50] [263]			3.580(-3)	5.10	[423]
$M_{\alpha_{2,1}}$	$M_5N_{6,7}$	929.20 ± 0.35	[50]	1.37(-3)	[335]			

line	transition	E/eV		I/eV/$\hbar$	TPIV	Γ/eV
		930.80	[263]			
M_{α_2}	M_5N_6				1.878(-5)	
M_{α_1}	M_5N_7				3.667(-4)	
	N series					
	$N_{4,5}N_{6,7}$	109.50 ± 1.00	[50]		3.667(-4)	
	$N_{4,5}O_{2,3}$	80.80 ± 0.30	[43]		3.667(-4)	

level characteristics

level	E_B/eV		ω_{nlj}		Γ/eV		AE/eV	
K	41991.00	[503]	0.917	[301]	10.20	[301]	41990.6	[51]
	41990.60 ± 0.50	[493]			15.78	[423]	42002 ± 11	[50]
			0.915	[38]	16.2	[94]	41994.11 ± 0.75	[145]
			0.9140	[39]				
			0.9180	[423]				
			0.917	[310]				
			0.923	[33]				
			0.916	[242]				
L_1	6835.00	[503]	6.10(-2)	[301]	4.34	[301]	6834.8	[51]
	6834.80 ± 0.50	[493]	6.04(-2)	[423]	3.96	[423]	6834.4 ± 2.8	[50]
			6.50(-2)	[436]	2.7	[94]	6836.8 ± 1.1	[145]
L_2	6441.00	[503]	0.117	[301]	3.89	[301]	6440.4	[51]
	6440.40 ± 0.50	[493]	0.1272	[423]	3.30	[423]	6439.0 ± 2.5	[50]
			0.128	[436]	3.48	[94]	6443.60 ± 0.39	
L_3	5965.00	[503]	0.118	[301]	3.60	[301]	5964.3	[51]
	5964.30 ± 0.40	[493]	0.1248	[423]	3.09	[423]	5963.3 ± 2.1	[50]
			0.126	[436]	3.27	[94]	5967.84 ± 0.33	[145]
M_1	1511.00	[503]	1.01(-3)		11.8	[94]	1511.0	[51]
	1511.00 ± 0.80	[493]	9.7(-4)	[423]	13.51	[423]		
M_2	1338.00	[503]	9.42(-3)	[512]	4.5	[94]	1337.4	[51]
	1337.40 ± 0.70	[493]	2.49(-3)	[423]	6.01	[423]		
M_3	1243.00	[503]	1.05(-3)	[512]	6.7	[94]	1242.2	[51]

level	E_B/eV		ω_{nlj}		Γ/eV		AE/eV	
	1242.20 ± 0.60	[493]	2.68(-3)	[423]	6.95	[423]		
M_4	951.00	[503]	2.7(-3)	[512]	0.75	[94]		
	951.10 ± 0.60	[493]	4.67(-3)	[423]	0.87	[423]		
M_5	931.00	[503]	4.56(-3)	[423]	0.75	[94]	928.8	[51]
	931.00 ± 0.60	[493]			0.75	[423]		
N_1	305.00	[503]	1.2(-4)	[423]	3.7	[94]		
	304.50 ± 0.90	[493]			8.25	[423]		
N_2	237.00	[503]	2.2(-4)	[423]	5.08	[94]		
	236.30 ± 1.50	[493]			11.84	[423]		
N_3	218.00	[503]	7.0(-5)	[423]	1.75	[94]		
	217.60 ± 1.10	[493]			4.35	[423]		
N_4	114.00	[503]	1.0(-5)	[423]	0.78	[94]		
	113.20 ± 0.70	[51]						
	117.90 ± 0.70	[493]			0.78	[423]		
N_5	114.00	[503]	1.0(-5)	[423]				
	110.10 ± 0.70	[493]			0.53	[423]		
N_6	2.00	[503]			0.53	[94]		
	2.00 ± 0.60	[493]						
O_1	38.00	[503]			0.57	[423]		
	37.40 ± 1.00	[493]						
O_2	23.00	[503]						
	22.30 ± 0.70	[51]						
	24.60 ± 0.70	[493]						
O_3	23.00	[503]						
	21.20 ± 0.70	[493]						
IP	5.464	[222]						

6 X-Ray Transition Energies: Ordered by Energy/Wavelength

In the following we give ordered by the wavelength and transition energy emission lines and absorption edges for all elements up to americium (Z=95). To restrict the volume of the table we give for each quantity only one selected value what in a first step is sufficient to indentify lines or edges in a spectrum of interest.

Tabulated are

λ – wavelength of the tabulated transition or absorption edge;
E – energy of the tabulated transition or absorption edge;
Z – atomic number.

λ [pm]	E [keV]	Z	element	transition	notation	reference
10.0299	123.6152	95	Am	KN_2	$K_{\beta_{21}}$	[51]
10.2719	120.7032	94	Pu	KN_3	$K_{\beta_{22}}$	[51]
10.2939	120.4459	94	Pu	KN_2	$K_{\beta_{21}}$	[51]
10.3012	120.3600	95	Am	KM_3	K_{β_1}	[51]
10.3913	119.3168	95	Am	KM_2	K_{β_3}	[51]
10.5438	117.5912	93	Np	KN_3	$K_{\beta_{21}}$	[51]
10.5654	117.3503	93	Np	KN_2	$K_{\beta_{22}}$	[51]
10.5734	117.2614	94	Pu	KM_3	K_{β_1}	[51]
10.6629	116.2768	94	Pu	KM_2	K_{β_3}	[51]
10.7235	115.606	92	U	abs. edge	K	[50]
10.8190	114.6	92	U	KN_3	$K_{\beta_{21}}$	[50]
10.8379	114.4	92	U	KN_2	$K_{\beta_{22}}$	[50]
10.8527	114.2433	93	Np	KM_3	K_{β_1}	[51]
10.9420	113.3118	93	Np	KM_2	K_{β_3}	[51]
11.0110	112.601	91	Pa	abs. edge	K	[51]
11.1078	111.62	91	Pa	KN_3	$K_{\beta_{21}}$	[50]
11.1297	111.4	91	Pa	KN_2	$K_{\beta_{22}}$	[50]
11.1397	111.3	92	U	KM_3	K_{β_1}	[50]
11.2295	110.41	92	U	KM_2	K_{β_3}	[50]
11.3078	109.646	90	Th	abs. edge	K	[50]
11.3228	109.5	90	Th	$KO_{2,3}$		[50]
11.4044	108.717	90	Th	KN_3	$K_{\beta_{21}}$	[50]
11.4260	108.511	90	Th	KN_2	$K_{\beta_{22}}$	[50]

λ [pm]	E [keV]	Z	element	transition	notation	reference
11.4349	108.427	91	Pa	KM_3	K_{β_1}	[50]
11.5228	107.6	91	Pa	KM_2	K_{β_3}	[50]
11.6140	106.755	89	Ac	abs. edge	K	[51]
11.6393	106.5229	95	Am	KL_3	K_{α_1}	[51]
11.7122	105.86	89	Ac	KN_3	$K_{\beta_{21}}$	[50]
11.7310	105.69	90	Th	KM_3	K_{β_1}	[50]
11.7332	105.67	89	Ac	KN_2	$K_{\beta_{22}}$	[50]
11.8272	104.831	90	Th	KM_2	K_{β_3}	[50]
11.9306	103.922	88	Ra	abs. edge	K	[51]
11.9491	103.7612	94	Pu	KL_3	K_{α_1}	[51]
12.0292	103.07	88	Ra	KN_3	$K_{\beta_{21}}$	[50]
12.0503	102.89	88	Ra	KN_2	$K_{\beta_{22}}$	[50]
12.0550	102.85	89	Ac	KM_3	K_{β_1}	[50]
12.1435	102.1	89	Ac	KM_2	K_{β_3}	[50]
12.1455	102.083	95	Am	KL_2	K_{α_2}	[51]
12.2591	101.137	87	Fr	abs. edge	K	[51]
12.2675	101.068	93	Np	KL_2	K_{α_2}	[51]
12.2891	100.89	88	Ra	KN_2	$K_{\beta_{22}}$	[50]
12.3577	100.33	87	Fr	KN_3	$K_{\beta_{21}}$	[50]
12.3787	100.16	87	Fr	KN_2	$K_{\beta_{22}}$	[50]
12.3824	100.13	88	Ra	KM_3	K_{β_1}	[50]
12.4543	99.5518	94	Pu	KL_2	K_{α_2}	[51]
12.4696	99.43	88	Ra	KM_2	K_{β_3}	[50]
12.5951	98.439	92	U	KL_3	K_{α_1}	[50]
12.5996	98.404	86	Ra	abs. edge	K	[51]
12.6982	97.646	86	Ra	KN_3	$K_{\beta_{21}}$	[50]
12.7203	97.47	86	Ra	KN_2	$K_{\beta_{22}}$	[50]
12.7203	97.47	87	Fr	KM_3	K_{β_1}	[50]
12.7718	97.0775	93	Np	KL_2	K_{α_2}	[51]
12.8071	96.81	87	Fr	KM_2	K_{β_3}	[50]
12.9329	95.868	91	Pa	KL_3	K_{α_1}	[50]
12.9516	95.7299	85	At	abs. edge	K	[51]
13.0524	94.99	85	At	KN_3	$K_{\beta_{21}}$	[50]
13.0690	94.97	86	Ra	KM_3	K_{β_1}	[50]
13.0731	94.84	85	At	KN_2	$K_{\beta_{22}}$	[50]
13.0973	94.665	92	U	KL_2	K_{α_2}	[50]
13.1563	94.24	86	Ra	KM_2	K_{β_3}	[50]
13.2818	93.35	90	Th	KL_3	K_{α_1}	[50]
13.3167	93.105	84	Po	abs. edge	K	[51]
13.4183	92.4	84	Po	KN_3	$K_{\beta_{21}}$	[50]
13.4328	92.3	85	At	KM_3	K_{β_1}	[50]
13.4347	92.287	91	Pa	KL_2	K_{α_2}	[50]

λ [pm]	E [keV]	Z	element	transition	notation	reference
101.2041	12.251	77	Ir	L_2N_2		[50]
101.2041	12.2510	81	Tl	L_3N_4	$L_{\beta_{15}}$	[50]
101.2637	12.2438	93	Np	L_3M_2	L_t	[51]
101.4677	12.2264	80	Hg	L_3O_3		[50]
101.5165	l2.2133	81	Tl	L_2M_4	L_{β_1}	[50]
101.5614	12.2079	80	Hg	L_3O_2		[50]
101.6588	12.1962	88	Ra	L_3M_4	L_{α_2}	[50]
101.6772	12.1940	80	Hg	L_3N_7	L_u	[50]
101.7723	12.1826	80	Hg	L_3N_6	L_u	[50]
101.9405	12.1625	80	Hg	L_3O_1	L_{β_7}	[50]
102.0672	12.1474	79	Au	L_1M_5	L_{β_9}	[50]
102.1042	12.143	81	Pb	L_3N_1	L_{β_6}	[50]
102.1783	12.1342	77	Ir	L_2N_1	L_{γ_5}	[50]
102.2389	12.127	81	Pb	L_2M_3	$L_{\beta_{17}}$	[50]
102.2643	12.124	94	Pu	L_3M_1	L_l	[50]
102.4705	12.0996	74	W	abs. edge	L_1	[51]
102.5069	12.0953	79	0s	L_2N_4	L_{γ_1}	[50]
102.5094	12.095	74	W	$L_1O_{4,5}$		[50]
102.6163	12.0824	75	Re	L_1N_3	L_{γ_3}	[50]
102.7779	12.0634	74	W	L_1O_3	L_{γ_4}	[50]
102.7924	12.0617	79	Au	L_1M_4	$L_{\beta_{10}}$	[50]
102.8666	12.0530	74	W	L_1O_2	$L_{\gamma_{4'}}$	[50]
102.8666	12.053	81	Tl	L_3N_3		[50]
103.0522	12.0313	87	Fr	L_3M_5	L_{α_1}	[50]
103.1748	12.017	74	W	L_1O_1		[50]
103.2349	12.010	82	Pb	L_1M_1		[50]
103.2366	12.0098	75	Re	L_1N_2	L_{γ_2}	[50]
103.3614	11.9953	80	Hg	L_1M_3	L_{β_3}	[50]
103.4762	11.982	92	U	L_3M_2	L_t	[50]
103.4934	11.98	83	Bi	L_2M_2		[50]
103.7186	11.954	75	Re	abs. edge	L_2	[51]
103.7012	11.956	75	Re	L_2O_4	L_{γ_6}	[50]
103.8793	11.9355	79	Au	$L_3P_{2,3}$		[50]
103.8854	11.9348	79	Au	L_3P_1		[50]
103.9220	11.9306	81	Tl	L_1M_2	L_{β_4}	[50]
103.9708	11.925	75	Re	L_2O_3		[50]
103.9769	11.9243	76	0s	L_2N_3		[50]
103.9777	11.9242	35	Br	KL_3	K_{α_1}	[50]
103.9786	11.9241	80	Hg	L_3N_5	L_{β_2}	[50]
104.0040	11.9212	79	Au	abs. edge	L_3	[50]
104.0406	11.917	75	Re	L_2N_6	L_ν	[50]
104.0467	11.9163	79	Au	$L_3O_{4,5}$	L_{β_5}	[50]

λ [pm]	E [keV]	Z	element	transition	notation	reference
104.1542	11.9040	80	Hg	L_3N_4	$L_{\beta_{15}}$	[50]
104.1980	11.899	75	Re	L_1N_1		[50]
104.2330	11.8950	87	Fr	L_3M_4	L_{α_2}	[50]
104.2J69	11.890	93	Np	L_3M_1	L_l	[50]
104.3857	11.8776	35	Br	KL_2	K_{α_2}	[50]
104.4015	11.8758	75	Re	L_2O_1	L_{γ_8}	[50]
104.4966	11.865	33	As	abs. edge	K	[51]
104.4965	11.865	79	Au	$L_3O_{2,3}$		[50]
104.5036	11.8642	33	As	$KN_{2,3}$	K	[50]
104.5759	11.856	74	W	L_1N_5	$L_{\gamma_{11}}$	[50]
104.6818	11.844	74	W	L_1N_4		[50]
104.7552	11.8357	79	Au	$L_3N_{6,7}$	L_u	[50]
104.8713	11.8226	80	Hg	L_2M_4	L_{β_1}	[50]
104.8766	11.822	33	As	$KM_{4,5}$	K_{β_5}	[50]
104.9762	11.8118	81	Tl	L_3N_1	L_{β_6}	[50]
104.9779	11.8106	79	Au	L_3O_1	L_{β_7}	[50]
105.4502	11.7577	78	Pt	L_1M_5	L_{β_9}	[50]
105.6119	11.7397	81	Tl	L_2M_3	$L_{\beta_{17}}$	[50]
105.6965	11.7303	76	0s	L_2N_1	L_{γ_5}	[50]
105.7262	11.7270	86	Rn	L_3M_5	L_{α_1}	[50]
105.7334	11.7262	33	As	KM_3	K_{β_1}	[50]
105.7867	11.7203	33	As	KM_2	K_{β_3}	[50]
105.8526	11.713	80	Hg	L_3N_3		[50]
105.8598	11.7122	83	Bi	L_2M_1	L_η	[50]
106.1026	11.6854	75	Re	L_2N_4	L_{γ_1}	[50]
106.1335	11.682	73	Ta	abs. edge	L_1	[51]
106.1862	11.6762	78	Pt	L_1M_4	$L_{\beta_{10}}$	[50]
106.1953	11.6752	73	Ta	$L_1O_{4,5}$		[50]
106.2035	11.6743	74	W	L_1N_3	L_{γ_3}	[50]
106.3611	11.6570	73	Ta	L_1N_6, 7		[50]
106.4433	11.648	81	Tl	L_1M_1		[50]
106.4433	11.648	82	Pb	L_2M_2		[50]
106.4698	11.6451	73	Ta	L_1O_3	L_{γ_4}	[50]
106.4982	11.642	80	Hg	L_3N_2		[50]
106.5476	11.6366	73	Ta	L_1O_2	$L_{\gamma_{4'}}$	[50]
106.7154	11.6183	92	U	L_3M_1	L_l	[50]
106.7751	11.6118	73	Ta	L_1O_1		[50]
106.7889	11.6103	79	Au	L_1M_3	L_{β_3}	[50]
106.8101	11.6080	74	W	L_1N_2	L_{γ_2}	[50]
106.9031	11.5979	86	Rn	L_3M_4	L_{α_2}	[50]
107.0249	11.5847	79	Au	L_3N_5	L_{β_2}	[50]
107.1915	11.5667	79	Au	L_3N_4	$L_{\beta_{15}}$	[50]

11 Fit Parameters for the Calculation of Mass Attenuation Coefficients

In the following fit parameters for the approximation of mass attenuation coefficients μ for the elements up to plutonium (Z=94) are given for the energy region 1 keV $\leq E \leq$ 150 keV.

As basis mass attenuation coefficients determined by [547] derived from a wide varity of literature values are used in the approximation. The fit is provided with a χ^2-approximation for the function

$$\ln \mu = x_0 + x_1 \ln E + x_2 (\ln E)^2 + x_3 (\ln E)^3$$

As criteria for the evaluation of the used parameter sets the approximation quality is given by a mean relative error quotient (*RFQ*)

$$RFQ = \frac{1}{m} \sum_{i=1}^{m} \frac{|\mu_{th}(i) - \mu_{ex}(i)|}{\Delta\mu_{ex}(i)}$$

Here μ_{th} describes the approximation results, μ_{ex} the original values and and $\Delta\mu_{ex}$ the error of the original values. For instance $RFQ = 0.1$ means that the deviations of the fit values do not exceed 10% of the experimental errors, i.e. they are an order of magnitude smaller as the the experimental errors.

In the column "region" intervals between the corresponding absorption edges are indicated. A characterizes the start value, E the end value and Z are intermediate values for reaching a higher approximation quality.

Z		region	energy interval [keV]			x_0	x_1	x_2	x_3	RFQ
1	H	(A-Z)	1.000	-	3.000	1.9782	-3.1810	0.2663	0.4646	0.0001
1	H	(Z-Z)	3.000	-	8.000	2.3064	-4.7411	2.3680	-0.4032	0.0098
1	H	(Z-Z)	8.000	-	40.000	-1.1768	0.3192	-0.1256	0.9120	0.0540
1	H	(Z-E)	40.000	-	150.000	-1.5268	0.3818	-0.0725	0.0009	0.0124
2	He	(A-Z)	1.000	-	4.000	4.1342	-0.1842	-0.1900	0.2179	0.0252
2	He	(Z-Z)	4.000	-	10.000	6.2952	-7.0916	2.0715	-0.1909	0.0233
2	He	(Z-Z)	10.000	-	40.000	2.1555	-2.9261	0.7622	-0.0689	0.0455
2	He	(Z-E)	40.000	-	150.000	-1.9594	0.2798	-0.0624	0.0009	0.0412
3	Li	(A-Z)	1.000	-	4.000	5.4603	-3.1208	-0.1262	0.0759	0.0115
3	Li	(Z-Z)	4.000	-	10.000	4.9815	-1.9013	-1.1537	0.3625	0.0257
3	Li	(Z-Z)	10.000	-	40.000	10.6173	-10.1751	2.8249	-0.2666	0.0331
3	Li	(Z-E)	40.000	-	150.000	-2.6383	0.6895	-0.1623	0.0087	0.0466
4	Be	(A-Z)	1.000	-	5.000	6.4310	-2.9919	-0.1353	0.0474	0.0319

Z		region	energy interval [keV]			x_0	x_1	x_2	x_3	RFQ
4	Be	(Z-Z)	5.000	-	15.000	6.2244	-2.0962	-0.9718	0.2721	0.0525
4	Be	(Z-Z)	15.000	-	50.000	14.0212	-11.8160	2.9831	-0.2557	0.0630
4	Be	(Z-E)	50.000	-	150.000	-2.6112	0.7384	-0.1809	0.0105	0.0182
5	B	(A-Z)	1.000	-	4.000	7.1423	-2.9402	-0.1013	0.0216	0.0115
5	B	(Z-Z)	4.000	-	10.000	6.5962	-1.8275	-0.8584	0.1942	0.0191
5	B	(Z-Z)	10.000	-	40.000	15.6224	-11.4874	2.4828	-0.1742	0.0707
5	B	(Z-E)	40.000	-	150.000	-1.3145	0.2090	-0.1264	0.0108	0.0926
6	C	(A-Z)	1.000	-	4.000	7.7102	-2.8547	-0.0778	0.0003	0.0068
6	C	(Z-Z)	4.000	-	10.000	9.0028	-5.1288	1.2123	-0.2324	0.0540
6	C	(Z-Z)	10.000	-	40.000	10.0877	-4.8492	0.1742	0.0769	0.1210
6	C	(Z-E)	40.000	-	150.000	4.6797	-3.5526	0.6729	-0.0459	0.0833
7	N	(A-Z)	1.000	-	4.000	8.1495	-2.7883	-0.1042	0.0107	0.0215
7	N	(Z-Z)	4.000	-	10.000	6.4505	0.1488	-1.7796	0.3270	0.0889
7	N	(Z-Z)	10.000	-	40.000	8.6328	-2.3069	-0.7958	0.1840	0.0701
7	N	(Z-E)	40.000	-	150.000	9.8129	-6.6764	1.3102	-0.0894	0.0996
8	O	(A-Z)	1.000	-	5.000	8.4664	-2.6619	-0.1544	0.0220	0.0028
8	O	(Z-Z)	5.000	-	15.000	8.1187	-2.0347	-0.5296	0.0948	0.0068
8	O	(Z-Z)	15.000	-	50.000	9.9595	-2.9911	-0.5859	0.1532	0.0999
8	O	(Z-E)	50.000	-	150.000	6.0915	-3.8800	0.6363	-0.0367	0.0854
9	F	(A-Z)	1.000	-	5.000	8.7139	-2.6522	-0.1305	0.0137	0.0069
9	F	(Z-Z)	5.000	-	15.000	8.4276	-2.1423	-0.4321	0.0730	0.0089
9	F	(Z-Z)	15.000	-	50.000	10.3174	-2.9096	-0.6130	0.1494	0.0781
9	F	(Z-E)	50.000	-	150.000	19.7311	-12.6064	2.4979	-0.1695	0.0343
10	Ne	(A-Z)	1.000	-	5.000	8.9794	-2.5658	-0.1526	0.0176	0.0126
10	Ne	(Z-Z)	5.000	-	15.000	8.9435	-2.4401	-0.2552	0.0413	0.0126
10	Ne	(Z-Z)	15.000	-	50.000	8.6594	-1.0223	-1.1564	0.1952	0.0710
10	Ne	(Z-E)	50.000	-	150.000	19.8683	-12.0722	2.2690	-0.1457	0.0983
11	Na	(A-K)	1.000	-	1.072	6.492	-2.5589	0.2069	-1.9866	0.0002
11	Na	(K-Z)	1.072	-	5.000	9.1937	-2.5599	-0.1406	0.0144	0.0043
11	Na	(Z-Z)	5.000	-	20.000	8.7473	-1.8442	-0.5181	0.0797	0.0182
11	Na	(Z-Z)	20.000	-	60.000	11.5477	-3.2117	-0.5236	0.1297	0.0121
11	Na	(Z-E)	60.000	-	150.000	2.3765	-0.0597	-0.4497	0.0572	0.0819
12	Mg	(A-K)	1.000	-	1.305	6.8628	-2.8442	-0.2259	0.5522	0.0038
12	Mg	(K-Z)	1.305	-	5.000	9.3765	-2.4337	-0.2069	0.0307	0.0079
12	Mg	(Z-Z)	5.000	-	20.000	8.9194	-1.7667	-0.5141	0.0736	0.0108
12	Mg	(Z-Z)	20.000	-	60.000	12.0705	-3.4060	-0.4459	0.1164	0.0781
12	Mg	(Z-E)	60.000	-	150.000	4.9596	-1.1838	-0.3026	0.0525	0.0917
13	Al	(A-K)	1.000	-	1.560	7.0813	-2.7215	-0.0598	0.0746	0.0053
13	Al	(K-Z)	1.560	-	6.000	9.4955	-2.3690	-0.1814	0.0149	0.0177
13	Al	(Z-Z)	6.000	-	20.000	10.1688	-3.1658	0.1124	-0.0174	0.0629
13	Al	(Z-Z)	20.000	-	60.000	9.3874	-0.9191	-1.1208	0.1732	0.0885
13	Al	(Z-E)	60.000	-	150.000	9.8438	-3.7963	0.1608	0.0251	0.0921

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