Finally, section 1.5 contains formulation of the equations of coupled dynamic three-dimensional problems with physical non-linearities. Moreover, the finite difference methods, Runge-Kutta's method and the method of additional loads have been combined to form a numerical algorithm of solutions. Convergence of an approximate solution to the real one (the one searched for) has been analysed. The results of problems concerning thermal and mechanical impacts beyond the elasticity fields have been presented and the effects of the influence of reciprocal temperature and deformation fields' coupling on the analysed processes have also been investigated in this chapter.

1.1 Introduction

While designing and constructing electronic devices, industrial facilities, flying objects or technological instrumentation, the problems related to heat processes are particularly important. They appear due to the use of new materials, more complex loads affecting every single element of analysed objects, and also due to an increase of permissible heat loads in devices of smaller and smaller dimensions. As it is generally known, heat processes determine stability of functioning and durability of analysed objects. On the other hand though, numerous empirical measurements of heat processes are extremely complex and expensive. Therefore, exact computational analyses (numerical, as well as analytical) ought to be conducted in order to obtain constructions of optimum characteristics.

In fact, non-stationary temperature reactions in surrounding environment require more accurate calculations than classic modelling of thermomechanical phenomena. In 1845, Duhamel [188] was the first to formulate the theory of elasticity regarding thermal stresses. However it was not until 1956, that Biot [107] introduced a dissipation function into a thermal conduction equation to account for the heat caused by the material's deformation. Thus, the problem of thermoelasticity and the variational principle of coupled theory of thermoplasticity were first formulated. Since then there has been a great interest in that sort of problems.

Earlier works on the theory of thermoelasticity [188] presented a dominating view that a change of temperature within a time interval is small, and therefore it was possible to apply a simplified (quasistatic) method, that is to neglect inertial terms in equations of motion, without the risk of major errors. The next step, introduced by means of the theory of thermoelasticity to simplify the problem, was neglecting dilatation terms in heat conduction equations. Sometimes, when both of the above mentioned terms are neglected in differential equations [598], the solution of a static problem is found. It turns out though, that due to the significance of the problems such simplifications ought not to be made. Among such problems are: the problem of investigating stress waves in deformable bodies; the problems related to determining thermoelastic vibrations; the problems related to investigating stability of conservative elastic systems [119, 164, 267, 316, 356, 466]. In their works, Danilovskoya [160, 161, 162, 163, 164], Kartashova and Shefter [316] analysed the influence of inertial terms on bodies' behaviour considering the inertia forces. They also proved that neglecting a dilatation term does not ensure qualitatively satisfactory results due to inefficient examination of the coupling coefficient's influence on the phenomenon.

All the factors mentioned above caused a growth of interest in complete (i.e. not simplified) problems which fruited in numerous analytical works.

Works of Karlsoy and Eger [315], Lykov [451], Kovalenko [355] and Nowacki [512] contain analyses and generalisation of two, so far independent disciplines, i.e. the theory of elasticity and the theory of heat conduction, and also a definition of so called coupled problem. A full formulation of the principles of variational theories of thermoelasticity is to be found in works [107, 265]. Betti's theorem on reciprocity of virtual works is discussed in monograph [516], and a generalisation of Maizel's method may be found in work [453]. Formulation of flat and space problems of coupled quasistatic theory of thermoelasticity is described in the works of Podstrigach, Schvetz, and Nowacki [512, 516, 545, 546, 547, 548]. Nowacki's monograph [513] introduces equations of the coupled theory of thermoelasticity into wave equations and a method of solving linear and non-linear variants of the problems listed above. Many popular methods of solving the equations of Galerkin's [215] or Papkovich's [528] classic theories of elasticity are generalized in Podstrigach's or Nowacki's works and applied into the theory of coupled thermoelasticity. The method of solving problems of the coupled theory of thermoelasticity in case of a boundless space was proposed by Zorski [727], who used Green's function to solve a heat conduction equation and considered dilatation to be a heat source. Chadwick's work [145] takes up generalized problems of solving boundary problems of the coupled theory of thermoelasticity with the use of integral methods, whereas Souler and Brul use the small parameter method [632].

The problems related to accuracy of formulated boundary problems of the coupled theory of thermoelasticity were described first in book [119], which investigates an initial boundary problem for an isotropic body, later extended also onto an anisotropic body in Ionescu work [277].

Numerous dynamical problems of mathematical physics apply various integral transformations, including Laplace's transformation [294], the solution of which is related to the use of Fourier's series. In their work, Kupradze and others [398] propose their theory of multidimensional singular integral equations that makes it possible to investigate the static and dynamic problems of stabilised continuous systems' vibrations. Hybrid problems, investigated by Magnaradze [452], Kupradze and Burchuadze [397] may be solved with generalized integrals that correspond to differential equations with the use of harmonic and analytical functions.

Defermos' work [175] contains many theorems concerning basic problems of the theory of thermoelasticity, including their proofs. Work [101] investigates the so-called second and third boundary and initial boundary problems of the coupled theory of thermoelasticity with the use of the method of potential and Laplace's transformation. Work [397] analyses four basic three-dimensional boundary problems of the theory of thermoelasticity in case of harmonic vibrations of a homogeneous isotropic medium with the following conditions set in its boundaries:

1) displacement and distribution of temperature; 2) thermal stress and thermal flux; 3) displacement and thermal flux; 4) thermal stress and distribution of temperature. In addition, the authors formulate and prove many theorems concerning the existence and uniqueness of the above mentioned problems. The solutions to all of the four types of boundary conditions, presented in the form of generalized Fourier's series, are to be found in Burchuladze's work [135]. Fundamental results referring to the initial boundary problems of the theory of thermoelasticity have been obtained in the work of Kachnashviliev [294]. Nevertheless, fundamental solutions are still being perceived as classic. The conditions of smoothness appear to be too difficult to achieve for solutions of a wave equation describing impact processes. Due to the fact that such solutions do not have derivatives of the first order, they need to be examined from a generalized perspective. Integral relations contain information about solutions and emphasise physical phenomena because information on solution's smoothness is partially lost in differential equations.

The generalized mathematical theory on differential equations of the coupled theory of thermoelasticity described by means of both hyperbolic and parabolic equation has been formulated relatively recently. The works of Ladyzhenskaya [405] and Ilyisn [276] that were published in early fifties, contain numerous vital results referring to the theory of boundary problems for one hyperbolic or parabolic equation of a general type. In order to prove the existence and uniqueness of a generalized equation, it is necessary to make an entirely new a priori estimation that would take into account the right parts of equations in the form of the weakest norm and thus would accurately emphasise the physical aspect of the problem.

Qualitatively most adequate examinations of general solutions seem to be the ones that apply the finite difference method. The method definitely stands out among many other approximate methods. Owing to continuing research of Samarskiy, Gulin, Nikolaev [591, 593, 594, 595], a large number of problems concerning stability of difference schemes for all types of one-dimensional equations in mathematical physics have been solved. This also started the research on difference schemes in the theory of elasticity. Let us list only a few examples of important results obtained with the use of the theory of difference schemes. Work [419] describes an a priori estimation of a solution in spaces $W_2^{2,2}$, $W_2^{2,1}$ made by means of energy inequalities for dynamic problems of the theory of thermoelasticity using Dirichlet's homogeneous boundary conditions. The authors have also constructed and examined a non-overt difference scheme and proved its convergence. In his work [483], Moskalkov presents a method of constructing difference schemes for the coupled theory of thermoelasticity boundary problems that is also useful for the equations of variable or discontinuous coefficients. Work [541] proposes a variational-difference formulation of the difference scheme of the coupled theory of thermoelasticity problems. Work [341] proves convergence of the difference solution towards the solution of a general hybrid problem for a hyperbolic equation with variable coefficients. It also shows how to improve the accuracy of presently applied difference schemes. In works [419, 694], the relation between the smoothness of a solution to the coupled theory of thermoelasticity one-dimensional dynamic problems and the smoothness of input data is examined. Smoothness is examined with the use of terminology applied for Hilbert and Sobolev's spaces. Two difference scheme families have been constructed and their stability and convergence have been studied. Works [419, 693] extend the investigated problems by taking into account two-dimensionality or many so-called layer problems. It is worth noticing that at present, many finite differential problems modelling the flat problem of the dynamic theory of elasticity and the theory of thermoelasticity have already been solved. A large number of schemes described by displacements of high accuracy, stability and short computation time have also been presented [79, 96, 97, 345, 484, 591, 592, 664]. Among the less thoroughly examined problems are the ones that refer to the differential method of solving initial-boundary problems of the three-dimensional theory of elasticity and the theory of thermoelasticity. A review work by Suslova [643] contains a broad bibliography of works on research focused on solving boundary problems of the three-dimensional theory of elasticity. It also lists several works concerning the theory of thermoelasticity [142, 293, 643]. In works [198, 199] Ermolenko describes constructing the solution of a hybrid problem for a cubicoid by cutting the finite space out and he proves stability and convergence of the cubic difference process by applying the transformation of Lamé's equations. He compares the result obtained in this way to the accurate one. In works [339, 340] Konovalov describes stability conditions for difference schemes for two-dimensional dynamic and static hybrid problems.

The development of computational methods using computers and special algorithms has led to a sudden progress in the discussed field of science. A major contribution in the development of computational methods in the research on the dynamics of continuous media has been brought by the works of Godunov [224], Kukudzanov [393], Neuman [500], Rachmatulin [561, 562], Richtmyer [572], Wilkins [703] and Janenko [287]. Numerous examples of computations regarding the mechanics of a continuous medium are included in monographs [225, 287, 394, 573]. The problem of the coupled theory of thermoelasticity still remains a live issue due to its potential application and the numerical methods allow drawing a great deal of conclusions of a general nature. The examples of these may be the research and solutions of coupled thermoelasticity problems with the use of numerical methods for a number of particular issues: in work [546], Galerkin's method is applied for solving a coupled problem in a finitely dimensional space with the use of a three-dimensional model; in work [616], the same method is applied to solve a two-dimensional problem; in work [430], a half-space finite difference method is applied for a one-dimensional problem, and in works [220, 721] – for a three-dimensional problem.

In work [266], Huang and Shich compare solutions of free vibration problems regarding thermal processes in plates and spherical shells by applying dynamic and quasistatic theories. Non-stationary thermoelasticity problems for an infinite two-layered and initially heated plate consisting of various materials and thermally processed through interaction with fluids within Newton's laws, have been examined in work [646]. Work [649] analyses stress-strain states of thick two-layered spheres with regard to axially symmetrical heat sources (the problem has been solved with the use of the quasistatic theory). Work [648] investigates a system of coupled thermoelasticity differential equations with the use of a cylindrical coordinate system. Fourier's method has been used to examine stress-strain states in a long circular cylinder with inserted rigid rings in work [504]. The finite difference method has been used to solve the problem of thermoelasticity for a rectangular orthotropic plate with regard to the dependence of its certain characteristics on temperature in work [641]. Work [663] investigates a non-stationary coupled thermoelasticity problem for an infinitely long, thick plate. The plate's surfaces have been subjected to intensive heating and the coupling between the temperature field and the deformation has been analysed. The distribution of the temperature field in time has also been examined, as well as concentration of the stresses depending on the size of the stress field and the material's thermodynamic properties. Dynamic loss of stability of thin plates has been analysed with the use of finite difference method in work [191], taking into account the effect of reciprocal coupling of the temperature field and the deformation field. Work [324] presents a solution to the coupled thermoelasticity problem for a thin rectangular shell affected by a three-dimensional temperature field. It also mathematically proves the convergence of the obtained approximate solution.

All of the above mentioned works point out the differences which appear in solutions if the coupling of the deformation (strain) fields and the temperature fields are not taken into account. An increase of the coupling coefficient leads to an increase of interactions, which consequently leads to damping of the produced thermoelastic waves. Works by Karnauchov [312] and Pobedria [541] are focused on the problem of coupling in the theory of thermoelasticity. The influence of coupling on the stress-strain state of elastic and elastoplastic constructions has been investigated in work [359]. Several works of Day [169, 170, 171, 172, 173, 174] are also worth attention since the author investigates the conditions of legitimacy of applying approximations of unbounded theory of thermoelasticity and also the conditions of applying the properties of the solutions of heat conductivity equations to the solutions of a onedimensional dynamic coupled thermoelasticity problem's equations.

Research on thermal processes with regard to finite velocity of heat transfer is another direction in the development of the theory of thermoelasticity, since an entire class of physical processes (highly intensive thermal processes, laser rays) should be presented from the perspective of generalized Fourier's law [451]. Works [323, 429, 495, 496, 558, 627] have been dedicated to the research on dynamic processes in solid bodies with regard to the heat transfer finite speed. In the works of Engelbrecht and Ivanov [285], an analysis of one- and two-dimensional models of wave processes have been made. In Kolyano and Shter's work [337], a variational principle of reciprocal coupling of thermoelasticity for non-homogeneous media has been investigated using a cantilever beam as an example. Coupling of the deformation field and the temperature field significantly affects the solution's character, especially in the problems of spreading impact fields in thermoelastic bodies. Therefore, the research on the dynamic coupling effects occurring in thermoelastic bodies subjected to simultaneous thermal, impulse, impact and mechanic treatment is one of the most important issues these days. Danilovskaya [163, 164] was the first to examine the dynamic effect in the "impact" problem along a half-space. The research was consequently carried on by Mura [489]. If the temperature on the surface of a body changes at a limited speed instead of sudden leaps, then the problem may be solved with a small parameter method [494]. In Pobrushin's work [544], an analysis of some one-dimensional initial-boundary problems with thermal and mechanical impacts along the symmetry axis of an infinite rod has been made. The dynamic coupled thermoelasticity problem for a half-infinite plate at a simultaneous increase of temperature on its edge and with the use of Laplace's integral transformation including the small parameter method has been solved in Sidlar's work [617]. Dynamic behaviour of thin cylindrical shells subjected to impetuous thermal treatment has been investigated in work [632]. A coupled system of differential equations is derived with the use of Bubnov-Galerkin method and variational theorems, and also a simple-supported infinite cylindrical shell is investigated. Work [359] investigates dynamic thermoelastic processes during heat impacts in such construction elements as plates or spherical and cylindrical shells. The research has been conducted with the use of dynamic coupled thermoelastic equations and dynamic non-coupled equations of thermoelastoplasticity, and with the method of reduction to a series of non-coupled quasistatic problems, which in turn have been solved with Krylov-Bogolubov method. In Kuvyrkin's work [402], a heat impact in the surface layer of a body limited by a curvilinear surface has been investigated. Shatalov's work [608] shows that a decrease of equations' couplings leads to a decrease of strain in the front of a thermoelastic wave. A method of expansion into power series in regard to a small parameter being the thermomechanical coupling has been applied in that case. Gayvas' work [221] presents an analytical solution to a thermoelasticity problem for a plate with discontinuity caused by heat impact. The behaviours of plates subjected to steady mechanical load and rapid thermal transients on their both surfaces have been investigated in work [231]. Few of the solved problems that are related to impacts belong to the class of problems with aperiodic excitations. In this respect the theory of thermoelasticity seems to be a little underdeveloped and it faces some significant mathematical problems. Due to simultaneous mechanical and thermal impacts in constructions some small plastic deformations are ignored. The first work focused on investigation of elastoplastic stress states was published by Iliushin [272], and later by Rogoshinskov, who took non-uniformity of heating into account. Many works analyse also particular problems. Ionov's works [278, 498] based on the theory of small elastoplastic deformations are among them. Work [148] describes a stress-strain state of an infinitely long cylindrical shell subjected to heating.

In a series of works by Piskun [538, 539], cylindrical shells subjected to non-uniform heating and internal pressure have been examined. Work [307] contains some computations of thermoplastic deformations based on the variational-difference method, and work [109] describes a stress-strain state of rotational shells in conditions of axially symmetrical heating. Monographs [609, 610] present a theory and computational methods concerning many problems of thermoplasticity at variable loads including also the history of loading (the objects of study included cylinders, disks and low lift rotational shells). Work [126] applies Iliushin's theory of plasticity to deal with heating an isotropic sphere with heat impacts of various shape and length (the problem was solved as a non-coupled one). Analytical description of thermoelastoplastic deformations is published in work [583]. In work [242], Birger's method is applied to solve non-linear elasticity problems. Many interesting conclusions concerning dependence of physical and material parameters on temperature and work regime related to cooling shells and plates have been drawn in work [417]. Work [399] formulates a functional in order to find a variational solution to a plasticity theory problem at changing temperature for an elastoplastic material. Work [261] investigates the influence of the temperature load history, and work [150] analyses unique and continuous dependence on initial conditions in dynamic problems of non-linear thermoelasticity. A theory and a method of solving problems of thin-walled constructions heated by stationary and non-stationary heat sources are described in work [336], in which the dependence of physical and mechanical characteristics on temperature has been taken into account. A combination of the method applied for the theory of thermoelasticity with Vlasov's variational method has been used to solve a three-dimensional problem of non-linear thermoelasticity in work [357]. It needs to be emphasised that coupling of the temperature and deformation fields (also in a quasistatic case) for problems of non-elastic material characteristics is taken into account only in selected works [180, 217, 259, 350, 584].

A recent Polish publication edited by Woźniak [708] contains a synthetic and abundant presentation of the level of modern knowledge of the theory of elastic plates and shells with specific reference to the contribution of Polish scientists in its development. In contrast to that approach this monograph puts more light to the contribution of scientists from the former eastern bloc into the development of the theory of plates in the temperature field. It is worth emphasising that names of the two first authors of this book are connected with a series of monographs on the theory of plates and shells published in Polish [37, 38, 39, 48, 50, 51, 53]. The latest theoretical achievements in non-classic analyses of the thermoelastic shell theory problems are described in monograph [39].

Numerous aspects of non-linear dynamics of shells and plates, including bifurcations, chaos and solitons, have been analysed in other works of the two first authors of this monograph [41, 45, 46, 47, 49, 52, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 389, 390], which also seem to be worth recommendation for readers who wish to broaden their knowledge in the field of shells and plates.

At this point, several conclusions need to be drawn. (i) All of the above mentioned works investigate classic initially-boundary problems, while a typical (combined) boundary conditions are the most important in the theory of elasticity and thermoelasticity. There is a noticeable lack of solutions of that type in both linear and non-linear problems. (ii) There is no evidence for stability of difference schemes of the coupled theory of thermoelasticity in three-dimensional formulation for a cubicoid. (iii) Complexity of a physically non-linear system of differential equations limits the number of examples of solutions to thermoelastoplastic problems to only a few.

The authors of this chapter focused their attention on solving the following problems: 1) construct a system of differential equations of the coupled dynamic theory of thermoelasticity taking into account a three-dimensional model and singularities of all kinds; 2) apply the variational-difference method for solving the coupled thermoelasticity theory problems; 3) prove stability of the difference approximation for the examined class of problems; 4) solve a typical problems of the theory of elasticity and the theory of thermoelasticity; 5) formulate a method and solve physically non-linear, initially-boundary problems for a three-dimensional plate in the dynamic coupled approach, and examine the influence of temperature and deformation fields' coupling.

The following notation is used:

$x_i,\ i = 1, ..., 3$ - coordinate of a point in space;
$W(x)$ - examined field;
t - time;
$Q(x,t)$ - $\{x \in \Omega(x),\ \tau \in (\tau_0, \tau_1)\}$;
h_α - step in a mesh: $h_\alpha = \dfrac{l_\alpha}{N_\alpha}$;
n - normal unit vector directed outside the field: $n_{i,j+m} = \cos(n_{i,j+m}, x_i)$;
$U(u_1, u_2, u_3)$ - displacement vector;
$T = T_0 + \theta$ - absolute temperature;
T_0 - absolute temperature in a stress-free state;
θ - temperature increase;
α_τ - linear coefficient of thermal expansion;
λ_q - heat conduction coefficient;
λ - heat emission coefficient;
c - thermal capacity;
e_{ij} - strain tensor coefficient;
σ_{ij} - stress tensor coefficient;
e - volumetric strains: $e = \sum_{i=1}^{3} e_{ii}$;
λ, μ - Lamé's coefficients: $\lambda = \dfrac{E\nu}{(1+\nu)(1-2\nu)}, \mu = \dfrac{E}{2(1+\nu)}$;
E - Young's modulus;
ρ - material's density;
ν - Poisson's ratio;
P^4 - heat sources' unit power;
$\partial\Omega_i$ - plate's wall;
$P(P^1, P^2, P^3)$ - volume (mass) force;
$f(f_1, f_2, f_3)$ - surface force;

l_α - plate's dimension along x_α axis;

N_α - set of points of division towards x_α axis;

$\omega(\omega_1 \times \omega_2 \times \omega_3)$ - mesh surface: $\omega = \{x(x_1, x_2, x_3), x_\alpha \in \omega_\alpha, \alpha = 1, ..., 3\}$, $\bar{\omega}_\alpha = \{x_\alpha^{i_\alpha}, i_\alpha = 0, 1, ..., N_{\alpha-1}, N_\alpha\}$, $\omega_\tau = \omega_1 \times \omega_2 \times \omega_3 \times \omega_4 = \omega \times \omega_4 = \{x(x_1, x_2, x_3, x_4), x_\alpha \in \omega_\alpha, \alpha = 1, ..., 4\}$;

$S(S^1, S^2, S^3)$ - entropy vector;

s - entropy flux;

$L_2(\Omega)$ - Banach functional space of the following properties:

$$\|u\|_{2,\Omega} = \left(\int_\Omega |u|^2\, dx\right)^{\frac{1}{2}} \sim \|u\|_{2,\bar{\omega}} = (u, u)_{\bar{\omega}}^{\frac{1}{2}}, \quad \|u_x\|_{2,\Omega} = \left(\int_\Omega u_x^2 dx\right)^{\frac{1}{2}};$$

$W_2^1(\Omega)$ - space of elements $L_2(\Omega)$ with generalized derivatives of the first order due to Ω and of the following properties:

$$(u, \upsilon)_{2,\Omega}^{(1)} = \int_\Omega (u\upsilon + u_x\upsilon_x)dx \sim (u, \upsilon)_{2,\bar{\omega}}^{(1)} = \sum_{i=0}^{N} \upsilon(x)u(x)h,$$

$$\|u\|_{2,\Omega}^{(1)} = \left((u, u)_{2,\Omega}^{(1)}\right)^{\frac{1}{2}} \sim \left((u, u)_{2,\bar{\omega}}^{(1)}\right)^{\frac{1}{2}};$$

$W_2^{1,0}(\Omega)$ - Hilbert's space composed of elements $u(x, \tau)$ belonging to space $L_2(Q_\tau)$, which have generalized derivatives of the first order due to Q_τ of the following properties:

$$(u, \upsilon)_{2,Q_\tau}^{(1,0)} = \int_{Q_\tau} (u\upsilon + u_x\upsilon_x)dxd\tau,$$

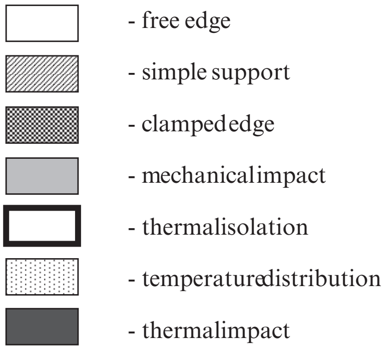
$$\|u\|_{2,Q_\tau}^{(1,0)} = \left((u, u)_{2,Q_\tau}^{(1,0)}\right)^{\frac{1}{2}};$$

$$\beta = 3(\lambda + \frac{2}{3}\mu)\alpha_\tau, \quad \hbar_\alpha = \begin{cases} h_\alpha, x_\alpha \in \omega_\alpha \\ h_{\frac{\alpha}{2}}, x_\alpha \in 0, l_\alpha \end{cases}, \quad h_4 = \frac{\tau_1 - \tau_0}{M},$$

$$\upsilon_x = \frac{\upsilon_{i+1} - \upsilon_i}{h}, \quad \upsilon_{\bar{x}} = \frac{\upsilon_i - \upsilon_{i-1}}{h}, \quad \upsilon_{0x} = \frac{\upsilon_{i+1} - \upsilon_{i-1}}{2h},$$

$$\upsilon_{\bar{x}x} = \frac{\upsilon_{i+1} - 2\upsilon_i + \upsilon_{i+1}}{h^2}, \quad \upsilon_{xy} = \frac{\upsilon_{i+1,j+1} - \upsilon_{i,j+1} - \upsilon_{i+1,j} + \upsilon_{ij}}{h_i h_j}.$$

The following markings are applied:



1.2 Coupled 3D Thermoelasticity Problem for a Cubicoid

This chapter presents a variational method-based derivation of a system of coupled thermoelasticity differential equations for a three-dimensional plate, taking into account material's non-homogeneity. The system includes equations within the plate's field, at its edges, ribs in its corners and at simple contact points of numerous boundary conditions, which allows solving a substantial number of problems. A difference system is derived with the use of the variational-difference method by approximating the initial differential system with accuracy of such small values as $O(h^2)$. The obtained difference scheme's stability theorem has been proven.

1.2.1 Variational equations

We shall consider interaction between an elastic non-homogeneous body Ω and a medium that surrounds it in conditions in which thermal and mechanical processes are taken into account. Let us assume that at time instant $\tau = \tau_0$ the body does not remain in the state of stress, i.e. the thermodynamic quantities that characterise the body such as absolute temperature $T = T_0$, strain and stress tensor components and displacement vector components are equal to zero. Mechanical interaction makes displacement fields appear in the body. In every general case they accompany the change of the temperature field. Heating the body also causes perturbations in the investigated fields. Heat conductivity involves producing entropy, and strains cause a decrease of it, which in result leads to producing heat. Although thermoelastic damping is usually weak and for a short time interval it may be neglected (the non-coupled thermoelasticity theory), the relatively long-lasting processes require taking energy dissipation into account (the combined theory of thermoelasticity).

Dissipation energy can be described by the following relation [63]:

$$D = \frac{1}{2} \iiint\limits_{\Omega} \frac{T_0}{\lambda_q} \frac{\partial S}{\partial \tau}^2 d\tau, \tag{1.1}$$

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