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Abbreviations

Arithmetics

a, \dots, z	variables, $a, \dots, z \in \mathbb{R}$
$\underline{a}, \dots, \underline{z}$	vectors
$\underline{A}, \dots, \underline{Z}$	matrices
\sum	addition operator of real-valued variables
\prod	multiplication operator of real-valued variables
\oplus	$l_\alpha r_\alpha$ -addition operator of fuzzy variables
\ln	natural logarithm

Analysis

$(\dots; \dots)^T$	elements of a column matrix
$(\dots; \dots)$	open interval
$[\dots; \dots]$	closed interval
$ \dots $	absolute value
\lim	limes, limit
∞	infinity
d	differentiation
∂	partial differentiation
\int	integration
\rightarrow	mapping

Set Theory

$\mathbf{A}, \dots, \mathbf{Z}$	fundamental sets
A, \dots, Z	sets
\mathbb{N}	natural numbers
\mathbb{R}	real numbers
\mathbb{R}^n	n-dimensional Euclidean space
$\{\dots; \dots\}$	set of ..., elements of a set
\in	element of
\subseteq	subset of
\cap	intersection
\cup	union
\emptyset	empty set

Logic

\wedge	conjunction, logical <i>and</i>
\vee	alternative, logical <i>or</i>
$ $	for which the following holds
\forall	universal quantifier, for all
\exists	existential quantifier, there exists

Fuzzy Set Theory

\sim	fuzziness
$\tilde{a}, \dots, \tilde{z}$	fuzzy variables
x_l	peak point of \tilde{x}
$S_{\tilde{x}}$	support of \tilde{x}
$\mu_{\tilde{x}}(x)$	membership function of \tilde{x}
α_i	i th α -level
X_{α_i}	α -level set of \tilde{x} for $\alpha = \alpha_i$
n	number of discrete α -levels
$\Delta x_{\alpha_i l}$	l_α -increment of \tilde{x} for $\alpha = \alpha_i$
$\Delta x_{\alpha_i r}$	r_α -increment of \tilde{x} for $\alpha = \alpha_i$
Δx_i	i th element of the $l_\alpha r_\alpha$ -increment representation of \tilde{x}
$L_{\alpha_i, \alpha_{i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leq \alpha < \alpha_{i+1}$ (left)
$R_{\alpha_i, \alpha_{i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leq \alpha < \alpha_{i+1}$ (right)
$\hat{\tilde{z}}$	best possible approximation of \tilde{z}

$d_F(\tilde{a}; \tilde{b})$	distance between two fuzzy variables \tilde{a} and \tilde{b}
$d_H(A; B)$	Hausdorff distance between two sets A and B
$d_E(a; b)$	Euclidean distance between two real-valued variables a and b

sup	supremum operator
inf	infimum operator
max	maximum operator
min	minimum operator

Fuzzy Probabilistics

A, \dots, Z	random variables
$\tilde{A}, \dots, \tilde{Z}$	fuzzy random variables
ω	random elementary event
Ω	space of the random elementary events
$P(\square)$	probability of occurrence of \square
$\tilde{P}(\square)$	fuzzy probability of occurrence of \square
$F_X(x)$	probability distribution function of X
$f_X(x)$	probability density function of X
$\tilde{F}_{\tilde{X}}(x)$	fuzzy probability distribution function form I of \tilde{X}
$F_{\tilde{X}}(\tilde{x})$	fuzzy probability distribution function form II of \tilde{X}
${}_{lr}F_{\tilde{X}}(\tilde{x})$	fuzzy probability distribution function form II of \tilde{X} using $l_\alpha r_\alpha$ -discretization
${}_{lr}f_{\tilde{X}}(\tilde{x})$	fuzzy probability density function form II of \tilde{X} using $l_\alpha r_\alpha$ -discretization

Artificial Neural Networks for Fuzzy Variables

I	input layer
H_i	i th hidden layer
O	output layer
n_I	number of neurons in the input layer
n_{H_i}	number of neurons in the i th hidden layer
n_O	number of neurons in the output layer
\tilde{o}_i^\square	fuzzy output value of the i th neuron of the layer \square
$\underline{W}_{ij}^\square$	weighting matrix of the j th neuron of the layer \square referring to the i th neuron of the preceding layer
\tilde{net}_i^\square	fuzzy netto input of the i th neuron of the layer \square
$\tilde{f}_A(\cdot)$	fuzzy activation function

$f_A(\cdot)$	deterministic activation function
$\tilde{f}_O(\cdot)$	fuzzy output function
$f_O(\cdot)$	deterministic output function
$\tilde{\theta}_i^o$	fuzzy bias of the i th neuron of the layer \square
\tilde{x}_i	i th fuzzy input vector
\tilde{y}_i	i th fuzzy control vector

Analysis of Time Series comprised of Fuzzy Data

τ	time coordinate
\mathbf{T}	set of equidistant points in time
$(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$	fuzzy time series
\tilde{x}_τ	fuzzy variable at point in time τ
$\tilde{t}(\tau)$	fuzzy trend function
\tilde{t}_τ	functional value of $\tilde{t}(\tau)$ at point in time τ
$\tilde{z}(\tau)$	fuzzy cycle function
\tilde{z}_τ	functional value of $\tilde{z}(\tau)$ at point in time τ
$(\tilde{R}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random residual process
\tilde{r}_τ	realization of $(\tilde{R}_\tau)_{\tau \in \mathbf{T}}$ at point in time τ , fuzzy residual component
$t_j^*(\tau)$	trend function of the j th $l_\alpha r_\alpha$ -increment
λ_j	frequency of the cyclic variation of the j th $l_\alpha r_\alpha$ -increment
$z_j^*(\tau)$	cycle function of the j th $l_\alpha r_\alpha$ -increment
\tilde{x}	fuzzy mean value of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\sigma}_{\tilde{x}_\tau}^2$	$l_\alpha r_\alpha$ -variance of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\sigma}_{\tilde{x}_\tau}$	$l_\alpha r_\alpha$ -standard deviation of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\hat{K}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$ -covariance function of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\hat{R}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$ -correlation function of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
L	linear filter for fuzzy time series
c_i	i th filter coefficient
D^p	fuzzy difference filter of the order p
L_e	extended linear filter for fuzzy time series
\underline{C}_i	i th coefficient matrix
D_e^p	extended fuzzy difference filter of the order p
$(\tilde{X}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random process
\tilde{X}_τ	fuzzy random variable at point in time τ
$E[\tilde{X}_\tau], \tilde{m}_{\tilde{X}_\tau}$	fuzzy expected value of \tilde{X}_τ
$l_r Var[\tilde{X}_\tau], l_r \underline{\sigma}_{\tilde{X}_\tau}^2$	$l_\alpha r_\alpha$ -variance of \tilde{X}_τ

$l_r \underline{\sigma}_{\tilde{X}_\tau}$	$l_\alpha r_\alpha$ -standard deviation of of \tilde{X}_τ
$l_r \underline{K}_{\tilde{X}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$ -covariance function of $(\tilde{X}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{R}_{\tilde{X}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$ -correlation function of $(\tilde{X}_\tau)_{\tau \in \mathbf{T}}$
AR	autoregressive
MA	moving average
ARMA	autoregressive moving average
$(\tilde{\mathcal{E}}_\tau)_{\tau \in \mathbf{T}}$	fuzzy white-noise prozess
ε_τ	realization of $(\tilde{\mathcal{E}}_\tau)_{\tau \in \mathbf{T}}$ at point in time τ
p	order of fuzzy AR processes
q	order of fuzzy MA processes
\underline{A}_i	i th coefficient matrix of fuzzy AR processes
\underline{B}_i	i th coefficient matrix of fuzzy MA processes
\underline{C}_i	i th coefficient matrix of inverted fuzzy MA processes
$\underline{\Theta}_i$	i th parameter matrix of the WILSON-algorithm for fuzzy MA processes
$\underline{\Delta}_i$	i th correction matrix of the WILSON-algorithm for fuzzy MA processes

Forecasting of Time Series comprised of Fuzzy Data

N	number of observed fuzzy variables of a fuzzy time series
h	forecasting step size
$(\vec{X}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random forecasting process
\vec{X}_τ	conditional fuzzy random variable at point in time τ
\vec{x}_{N+h}	realization of \vec{X}_τ at point in time $\tau = N + h$
\hat{x}_{N+h}	optimum forecast at point in time $\tau = N + h$
\tilde{x}_{N+h}^κ	fuzzy forecast interval with confidence level κ
\tilde{x}_{N+h}^*	conditional fuzzy forecast interval
$P_{X_{N+h}}^*$	conditional probability for $\hat{X}_{N+h} \subseteq \tilde{x}_{N+h}^*$



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