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The sequential order in these notes is from equilibria to invariant tori, which means that the various types of bifurcations appear and re-appear in different chapters. Therefore the following overview on the main Hamiltonian bifurcations of co-dimension one might be helpful.

bifurcation	for equilibria	periodic orbits	quasi-periodic
centre-saddle	Examples 2.6, 2.19, 2.22 and 2.23	Theorem 3.1 Example 3.3	Corollary 4.2 Theorem 4.4 Examples 4.5 and 4.8
Hamiltonian flip period-doubling frequency halving	Theorem 2.17	Theorem 3.4	Theorem 4.18
Hamiltonian Hopf	Theorem 2.20 Theorem 2.25	Theorem 3.7 Example 3.8 Example 4.5	Theorem 4.27
Hamiltonian pitchfork	Example 2.6 Example 2.22	Theorem 3.6	Theorem 4.13 Example 4.15

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