4. Dynamics of Fibre Formation Processes

4.1 Task

It is well-known that textile fibres are produced with essentially three basic technologies:

a) The separating, refining, strengthening and winding up of spinnable liquid mass streams. All organic and inorganic chemical fibres are to be subsumed to this group independently, if they are produced in a melt, dry or wet spinning process.
b) The separating, parallel join and twist of fibres. All fibre yarns are to be classified into this group independently, if the single fibres come from natural (animal, vegetable) or chemical sources. The latter case is mostly extending in front of a chemical fibre spinning process as seen in case a) with following cut process.
c) Cut of plain sheets of organic polymers into thin, tape like stripes (slit film yarn).

Each fibre formation process aims at the manufacturing of yarns with equal properties along to the yarn length axis. This means in conformity with the given definitions, that all product variables, which estimate the textile processing and wear properties of the yarn, should be as constant as possible. The case of effect yarn manufacture with consciously determined periodic or stochastic disturbed yarn structures along its length axis is an exception that should be mentioned. However, it will not be subject of the following considerations.

The processing of textile yarns and their wear behaviour is characterised by the product variable mass (fineness) along the yarn length axis or deformation resistance (elastic modulus) along the fibre length axis. These product variables oscillate around their averages caused by oscillations of raw materials and process variables. Therefore these product variables characterise the yarn unevenness, in which the fineness characterises the so-called outer yarn unevenness and the elastic modulus characterises the so-called inner yarn unevenness.

4.2 Melt Spinning of Polymers

4.2.1 Variable Fibre Fineness

The yarn finenesses and the yarn orientation are the most important among the different variables which describe the yarn quality. Therefore, the development of a mathematical model for these two yarn variables should be demonstrated here. At first, we start the investigations with the variable fibre fineness.

Cause-Effect-Scheme

The cause-effect relations of the process and the product variables for the target quantity yarn fineness should be demonstrated in the following. The recommended first step of the modelling process (registration and order of relevant process and product variables; see Sect. 2.5.1) can be carried out best through the elaboration of a cause-effect-scheme. The technological scheme of a melt spinning process is shown in Fig. 4.1.

It is to be remarked that this scheme is strongly simplified. It only contains the absolutely necessary tools and variables for our considerations. For instance thread guides, the oiling system and in some cases existing godets before the winder are not drawn. It is assumed that the heating system for the spinning die is an electrical resistance heating equipment. This is usual for laboratory equipment. Typical for the polymer melt spinning process is, that the thermoplastic melt (produced normally by means of an extruder) is fed to the single spinning positions along a melt distribution system by means of an exactly feeding volume conveyor tool for each (gear pump, spinning pump). After passing the spinning die (the tool, which distributes the melt stream into the number of filaments in the yarn) the single thin melt filaments are rapidly deformed, cooled and strengthened. At this complicated rheological and structural formation, shift processes take place in the filaments, which are caused directly or indirectly by the take-up velocity, created by the winder. For the target quantity or effect variable "fineness of the spun yarn" the process and product cause variables that are probably interesting at such a spinning position are shown in Fig. 4.1 as well.
Figure 4.2 shows the cause-effect-scheme for the target quantity fineness Tt_{s} (designed on this basis).

The cause-effect-arrows go from the cause to the effect. The box of the target quantity fineness is thickly framed, boxes of quantities at the process periphery are shaded (from these arrows only lead off). The fineness Tt_{s} is only caused under static conditions from the take-down (spinning) velocity at the output of the fibre formation distance v_{s} and from the throughput through the spinneret Q_{s}, which feeds the fibre formation distance at its input. The basic equation uses the suitable dimensions

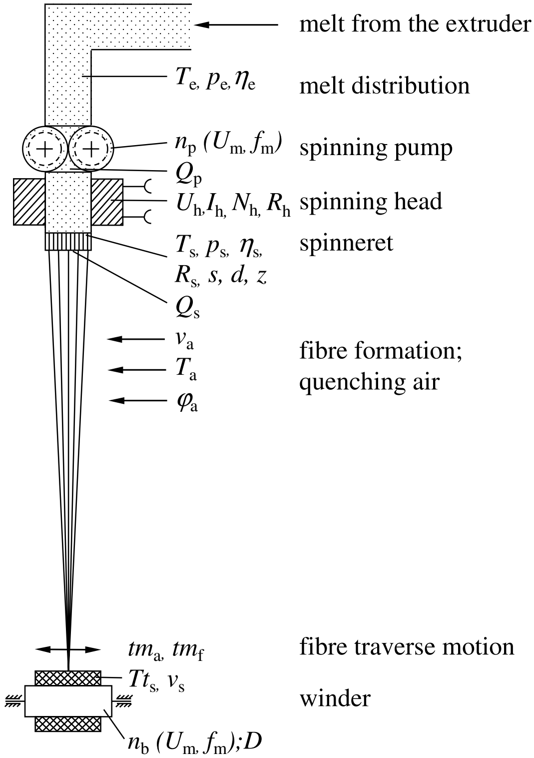

Fig. 4.1. Technological scheme (simplified) of a polymer melt spinning process

$$Tt_s[\text{tex}] = \frac{Q_s[\text{g/min}]}{v_s[\text{km/min}]} \tag{4.1}$$

It should be added, that changes of the quenching air velocity Δv_a below the die, and the necessary traverse motion at the bobbin of the winder, which is effected by the amplitude tm_a and the frequency tm_f of the thread guide for the traverse motion, also effect changes of the yarn fineness. However, they do not effect changes to the mean fineness. They only change the value of the fineness differentially. These 3 boxes are dotted frames. More detailed explanations to the latter are in Sects. 4.1.1.3 and 5.1.5.4.

The main cause variables for the fineness Tt_s, namely the throughput Q_s and the take-down (spinning) velocity v_s, can be traced back now regarding their cause process and product variables. The take-down velocity v_s is caused by the speed of the bobbin n_b, which is caused by the mains supply voltage U_m and the mains supply frequency f_m if an asynchronous drive motor is

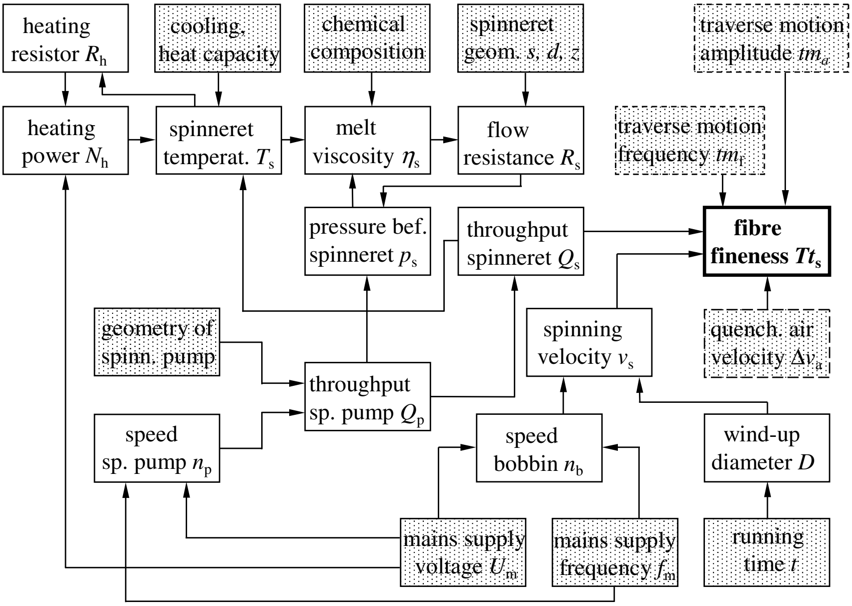

Fig. 4.2. Cause-effect-scheme for the target quantity fineness Tt_s of a polymer melt spinning process

used, and the wind-up diameter D. The latter of course increases with the running time t.

The throughput Q_s is only caused by the throughput of the spinning pump Q_p, which depends upon itself because of its geometrical design and its speed n_p. Back stream leakages of the spinning pump, which would mean $Q_s < Q_p$, are not regarded here. If the spinning pump would be driven by an asynchronous motor from the same mains supply U_m and f_m to Tt_s would be effectively doubled along the cause-effect-chains:

$$U_m,\ f_m \to n_b \to v_s \to Tt_s \text{ and}$$
$$U_m,\ f_m \to n_p \to Q_p \to Q_s \to Tt_s$$

The pressure before the spinning die p_s does not appear as a cause variable in regard to Q_s. It only depends, corresponding to the HAGEN-POISEUILLE-law for laminar flows in the tube, see [279], on the flow resistance inside the capillary holes of the die R_s and from Q_p. The flow resistance R_s depends on its part from the geometry of the capillary holes (length s, diameter d, number z) and from the melt viscosity η_s. The dependence of the melt viscosity η_s on the spinneret temperature T_s and on the chemical polymer composition is comprehensible by reason of simple basic physical laws. The dependence of T_s on the cooling conditions at the spinning die and on the heating power N_h

(itself depending on the mains supply voltage U_m and on the OHM'S heating resistance R_h) can be concluded with the same reasons.
In the next step it is necessary to set up the DEq. for each cause-effect relation or, if impossible, to investigate the dynamic signal transfer properties of the partial transfer systems. The signal transfer and signal interlacing character can be better represented by means of the so-called *functional block diagram*. In automatic control this is an often used scheme, which develops formally from the cause-effect-scheme by means of technological and *prior* physical knowledge. This only contains the change or oscillating parts of the process and product variables as their signals are connected together by cause-effect relations. The dynamic transfer properties are represented by the blocks, *"black boxes"*, which are unknown at the beginning of the analysis. This procedure will be demonstrated more fully in Sect. 4.3 with the example of "glass fibre spinning". In the following, the set up of the dynamic model will be demonstrated, which on the one hand describes the cause-effect relations between the cause variables throughput spinneret Q_s and the spinning velocity v_s and the fibre fineness Tt_s which is effected by these variables, on the other. The final goal of this procedure is to prepare technological statements about the disturbance transfer properties of the fibre formation distance.

Specified Differential Equation of a Fibre Formation Distance (simplified)

Figure 4.3 shows the fibre formation distance of the melt spinning process, only one monofilament fibre, which is reduced to the most necessary of elements and variables.

A melt stream is pressed through the capillary hole, cross section q_i, of the spinneret with the velocity v_i (input velocity or injection velocity into the fibre formation distance). At the length l between the spinneret and the take-up rolls with the output velocity v_s it is drawn, solidified and transported. The ready fibre with the cross section q_s esp. the fineness Tt_s appears at the take-up rolls. The relationship between the fibre fineness and the cross section is given by the density ϱ in the following manner:

$$Tt_\mathrm{s}[\mathrm{tex}] = \varrho[\mathrm{g/cm}^3] \cdot q_\mathrm{s}[\mu\mathrm{m}^2] \cdot 10^{-3} \tag{4.2}$$

The following relationship exists between the mass discharge per time (or throughput) Q_s and the input variables of the fibre formation distance:

$$Q_\mathrm{s} = \varrho \cdot q_\mathrm{i} \cdot v_\mathrm{i} \tag{4.3}$$

In Eq. 4.3 it is not distinguished between ϱ_fibre and ϱ_melt, because a constant factor exists between these two densities and an influence on the time and frequency oscillation relations will not be given. We split according to the

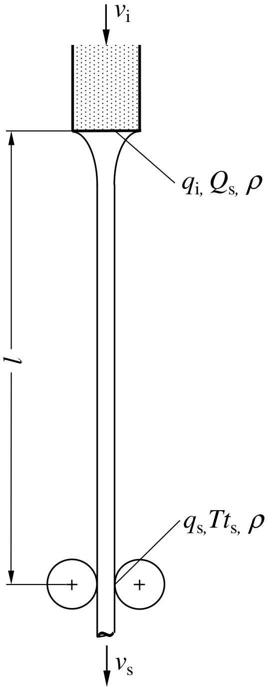

Fig. 4.3. Fibre formation distance (simplified) of the polymer melt spinning

agreement the variables, if necessary, into the mean value and the fluctuating part, consequently for instance:

$$v_i = v_{im} \pm \Delta v_i,$$
$$q_s = q_{sm} \pm \Delta q_s \text{ and so on}$$

The question is: Which Tt_s-fluctuations appear and if fluctuations of the variables v_i, q_i, ϱ, v_s or l appear? The question can only be answered on the basis of a dynamic fibre formation model. Ingeniously the starting point is the dynamic continuum equation (2.18).
Applied to the present fibre formation distance are:

$$\text{mass inflow/time} = q_i \cdot v_i \cdot \varrho$$
$$\text{mass discharge/time} = q_s \cdot v_s \cdot \varrho = Tt_s \cdot v_s$$
$$\text{change of stored mass} = \dot{q}_s \cdot l \cdot \varrho = \dot{T}t_s \cdot l$$

The assumed simplification at the formulation of the stored mass is, that the deformation range of the melt stream until it reaches its solidification point is not considered. At this point the diameter or the fineness of the spun fibre is reached. However, this range of the whole fibre formation distance is relatively small ($\sim$ 0.5...0.8 m), and the related mistakes do not prevent qualitatively correct results. An exact and quantitatively correct consideration of this range makes a correct mathematical solution impossible. In Sect. 3. the very complex processes which take place in the fibre formation distance are explained in more detail.

Using the expressions above the complete DEq. of the fibre formation distance can now be written as:

$$q_{\mathrm{i}} \cdot v_{\mathrm{i}} \cdot \varrho - Tt_{\mathrm{s}} \cdot v_{\mathrm{s}} - \dot{T}t_{\mathrm{s}} \cdot l = 0 \tag{4.4}$$

By marking the whole left side of the DEq. 4.4 with the letter $\varPhi$ and introducing the LAPLACE-operator $p = \dfrac{\mathrm{d}}{\mathrm{d}t}$, the DEq. is converted into the transformed quantic:

$$\varPhi = q_{\mathrm{i}} \cdot v_{\mathrm{i}} \cdot \varrho - Tt_{\mathrm{s}} \cdot v_{\mathrm{s}} - p \cdot Tt_{\mathrm{s}} \cdot l = 0 \tag{4.5}$$

Equation 4.5 represents a nonlinear DEq. first order, because all variables in the single terms, which can fluctuate, are multiplicatively connected together. Equation 4.5 can be linearised by means of the partial differentiation as follows:

$$\frac{\partial \varPhi}{\partial q_{\mathrm{s}}} \cdot \Delta q_{\mathrm{s}} + \frac{\partial \varPhi}{\partial v_{\mathrm{s}}} \cdot \Delta v_{\mathrm{s}} + \frac{\partial \varPhi}{\partial q_{\mathrm{i}}} \cdot \Delta q_{\mathrm{i}} + \frac{\partial \varPhi}{\partial v_{\mathrm{i}}} \cdot \Delta v_{\mathrm{i}} + \frac{\partial \varPhi}{\partial l} \cdot \Delta l + \frac{\partial \varPhi}{\partial \varrho} \cdot \Delta \varrho = 0 \tag{4.6}$$

The instruction of Eq. 4.6 means, that the whole DEq. 4.5 is to be derived partially with respect to each single variable of change. The mean value is to be set by the single derivation step for these variables which are not to be derived. The following linearised complete DEq. is achieved as the mathematical dynamic model of the fibre formation distance after the partial derivation and order of the single terms:

$$\begin{aligned} &(v_{\mathrm{sm}} + p \cdot l_{\mathrm{m}}) \cdot \Delta Tt_{\mathrm{s}} + Tt_{\mathrm{sm}} \cdot \Delta v_{\mathrm{s}} - v_{\mathrm{im}} \cdot \varrho_{\mathrm{m}} \cdot \Delta q_{\mathrm{s}} \\ &\quad - q_{\mathrm{im}} \cdot \varrho_{\mathrm{m}} \cdot \Delta v_{\mathrm{i}} + p \cdot Tt_{\mathrm{sm}} \Delta l - q_{\mathrm{im}} \cdot v_{\mathrm{im}} \cdot \Delta \varrho = 0 \end{aligned} \tag{4.7}$$

Equation 4.7 is a multilateral applicable dynamic model equation for fibre formation and fibre transport processes (see also Sect. 5.1). The performed linearisation (each term of the Eq. 4.7 contains only variables of change in time) is connected to the following consequences regarding the analysis:
Equation 4.5 represents primarily a nonlinear relationship. This is imaginable as a spatial multidimensional *curved* sheet and it is approached by a *plane* tangential sheet in the technological operating point. It is determined by the mean values of the single variables. The linearised relation is better validated the nearer the analytical investigation remains at this technological operation point. That means that the linearised Eq. 4.7 is valid more exact the smaller the investigated change quantities are in relation to their mean values. In practice it should be kept for any change variable x:

$$\Delta x \leqq 0.1 \cdot x_{\mathrm{m}} \tag{4.8}$$

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