9 Geophysics and Radio-Astronomy: VLBI – Very Long Base Interferometry

VLBI is an interferometry technique used in radio astronomy, in which two or more signals, coming from the same astronomical object, are received by antennas that are *very distant from each other*, recorded and then correlated in deferred time (Fig. 9.1). Due to the very long distance between the receivers and the fact that the resolution is proportional to that distance, a very high resolution can be obtained (see, for instance [26]).

In conventional interferometry techniques, the signals received by the antennas are directly transmitted via a physical link to the correlator, which produces the interference fringe in real-time; the antennas are physically connected to the correlator.

In VLBI, the received signals cannot be transmitted directly and in real-time to the correlator; the propagation time fluctuations in the physical links would completely cancel the correlation between them.

On the contrary, the signals are combined in differed time; they are converted to a lower standard frequency (IF) and recorded at each telescope on magnetic tape or hard disk, with a precise time base. The recorded

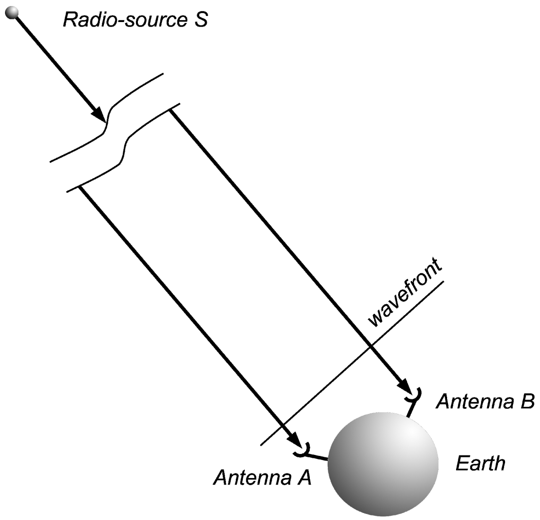

Fig. 9.1. The principle of VLBI

signals are then sent to a correlating centre, where they are synchronized, and due to the timing information, played together and combined just as if they were coming in real-time from the antennas. The correlated data can then, for instance, be turned into images using any appropriate software.

This is possible only if the phase noise of the local oscillators that down convert the signal frequency does not blur the interference fringe, and if the timestamps are accurate and stable during the duration of the experiment. In fact, only very stable atomic frequency standards can meet these requirements.

VLBI is most often performed at radio wavelengths and the following description is limited to radio signals; however, the technique has been extended to optics. The principle is very simple. Let

– **AB** be the baseline of an array of two antennas. It is the vector position of one antenna (B) with respect to the other (A).
– **s** be a unit vector in the direction of the source.

The time interval τ_{AB} (between the arrival of a wave front) to the antennas is

$$\tau_{\mathrm{AB}} = \frac{\mathbf{B} \bullet \mathbf{u}}{c} , \tag{9.1}$$

where c is the light velocity.

The measurement of τ_{AB} can provide one of the following types of information:

– the component of **s** along **AB** if this vector is known, or
– the component of **AB** along **s** if this vector is known.

Consequently, the applications of VLBI apply to the geodesic domain as well as the astronomic domain.

If the uncertainty on the measurement of τ_{AB} is 1 ps (1×10^{-12} s), (9.1) shows that

– if the position of the source is perfectly known, the uncertainty on the value of the baseline length is of the order of 1 mm, and
– if the baseline is perfectly known, the uncertainty on the position of the source is of the order of 1×10^{-9} rd ($\approx 10^{-3}$ arcsecond) for a baseline length of 1 000 km.

9.1 Principle of VLBI

The following description of astronomical interferometry is limited to 1D models but can easily be extended to the 2D model.

9.1.1 Interferometry

The application of interference methods to provide better resolution in astronomical measurements (in both optical and radio domains) is not a new concept (see, for instance [93, 108]).

The principle is the following: Consider two (optical or radio) receivers A and B, separated by a distance D and receiving the electromagnetic radiation emitted by a point source whose direction is at an angle α (see Fig. 9.2).

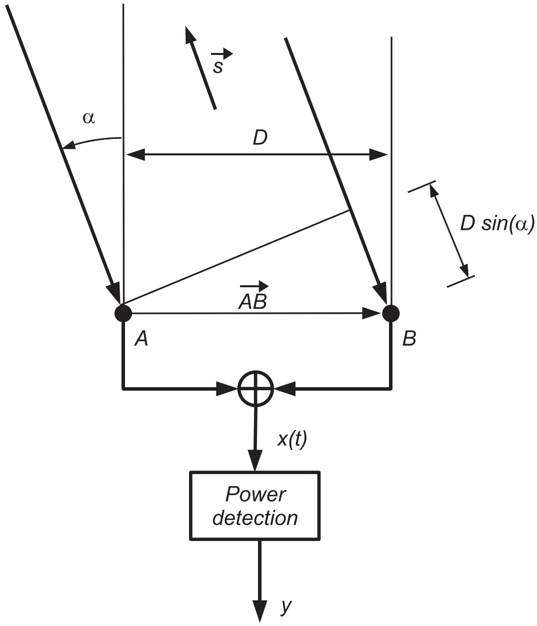

Fig. 9.2. The principle of interferometric measurements

Monochromatic Plane Waves

Consider in a first step that the incoming wave is plane and perfectly monochromatic, the frequency is ν, the wavelength is λ, the wave number is $k = \frac{2\pi}{\lambda}$ and the amplitude is X. The source is very far from Earth and its direction is indicated by the unit vector $\mathbf{s}$. The direction of the wave propagation is given by the unit vector $\mathbf{e} = -\mathbf{s}$. The equation of the wave is as follows:

$$x(t, \mathbf{r}) = X \exp\left[2\pi\nu\left(t - \frac{\mathbf{r}\cdot\mathbf{e}}{c}\right)\right] . \tag{9.2}$$

The vector $\mathbf{r} = \mathbf{OM}$ corresponds to a point M in the vicinity of the Earth. The origin O of $\mathbf{r}$ is the barycenter of the geoid, for instance. The two receivers are located at points A and B, respectively.

Supposing that the wave front is not perturbed by the atmosphere, the antennas receive the signals $x_{\mathrm{A}}(t)$ and $x_{\mathrm{B}}(t)$,

$$x_{\mathrm{A}}(t) = X \exp\left[2j\pi\nu\left(t - \frac{\mathbf{r}_{\mathrm{A}} \cdot \mathbf{e}}{c}\right)\right] \tag{9.3}$$

and

$$\begin{aligned} x_{\mathrm{B}}(t) &= X \exp\left[2j\pi\nu\left(t - \frac{\mathbf{r}_{\mathrm{B}} \cdot \mathbf{e}}{c}\right)\right] && (9.4)\\ &= x_{\mathrm{A}} \exp\left[-2j\pi\nu\left(\frac{\mathbf{AB} \cdot \mathbf{e}}{c}\right)\right] && (9.5)\\ &= x_{\mathrm{A}} \exp\left[j\left(\mathbf{AB} \cdot \mathbf{k}\right)\right] . && (9.6) \end{aligned}$$

The vector $\mathbf{k}$ is

$$\begin{aligned} \mathbf{k} &= \frac{2\pi}{\lambda}\mathbf{e} && (9.7)\\ &= k\mathbf{e} . && (9.8) \end{aligned}$$

$x_{\mathrm{A}}(t)$ and $x_{\mathrm{B}}(t)$ can also be expressed as functions of the baseline length D and the direction of the source α,

$$\begin{aligned} x_{\mathrm{B}}(t) &= x_{\mathrm{A}} \exp\left(-2j\pi\frac{D\sin(\alpha)}{\lambda}\right) && (9.9)\\ &= x_{\mathrm{A}} \exp\left(-jkD\sin(\alpha)\right) . && (9.10) \end{aligned}$$

Using the small angle approximation, which is of course not necessary (the source position angle α being supposed small),

$$x_{\mathrm{B}}(t) = x_{\mathrm{A}} \exp\left(-jkD\alpha\right) . \tag{9.11}$$

The two signals $x_{\mathrm{A}}(t)$ and $x_{\mathrm{B}}(t)$ are added to give $x(t)$,

$$\begin{aligned} x(t) &= x_{\mathrm{A}}(t) + x_{\mathrm{B}}(t) && (9.12)\\ &= x_{\mathrm{A}} \times \left[1 + \exp\left(-j\mathbf{AB} \cdot \mathbf{k}\right)\right] && (9.13)\\ &= x_{\mathrm{A}} \times \left[1 + \exp\left(-jkD\alpha\right)\right] && (9.14)\\ &= 2x_{\mathrm{A}} \exp\left(-j\frac{kD\alpha}{2}\right) \times \cos\left(\frac{kD\alpha}{2}\right) && (9.15)\\ &= 2x_{\mathrm{A}} \exp\left(-j\frac{\mathbf{AB} \cdot \mathbf{k}}{2}\right) \times \cos\left(\frac{\mathbf{AB} \cdot \mathbf{k}}{2}\right) . && (9.16) \end{aligned}$$

The output of the square law detector is, consequently,

$$\begin{aligned} y(\alpha) &= 4X^2 \cos^2\left(\frac{kD\alpha}{2}\right) && (9.17)\\ &= 4X^2 \cos^2\left(\frac{\mathbf{AB} \cdot \mathbf{k}}{2}\right) && (9.18)\\ &= 2X^2[1 + \cos(\mathbf{AB} \cdot \mathbf{k})] . && (9.19) \end{aligned}$$

This is the classical interference pattern for monochromatic radiation. The central fringe is obtained for $\alpha = 0$ ($\tau_{AB} = 0$).

For $\alpha \neq 0$, the value of τ_{AB} can be measured by introducing in one of the arms of the interferometer a delay $\tau_{AB'} = \pm\tau_{AB}$, which compensates τ_{AB}.

Due to the rotation of the Earth, the value of α (or $\mathbf{AB}$) varies continuously and a series of configurations can consequently be studied.

Quasi-Monochromatic Plane Wave

In fact, the radiation emitted by the source is never perfectly monochromatic. Phase and amplitude fluctuations occur,

$$x(t, \mathbf{r}) = X(t) \exp\left[2j\pi\nu\left(t - \frac{\mathbf{r}\cdot\mathbf{e}}{c}\right)\right] \tag{9.20}$$

with

$$X(t) = X_0[1 + a(t)] \exp[j\phi(t)] \ , \tag{9.21}$$

where $a(t)$ represents the relative amplitude fluctuations and $\phi(t)$ the phase fluctuations. These fluctuations are small,

$$|a(t)| \ll 1 \ , \tag{9.22}$$

$$\left|\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\right| \ll 2\pi\nu \ . \tag{9.23}$$

The signals received by the two antennas are

$$\begin{aligned} x_A(t) &= X\left(t - \frac{\mathbf{r}_A\cdot\mathbf{e}}{c}\right) \exp\left[2j\pi\nu\left(t - \frac{\mathbf{r}_A\cdot\mathbf{e}}{c}\right)\right] && (9.24)\\ &= X\left(t - \tau_A\right) \exp\left[2j\pi\nu\left(t - \tau_A\right)\right] && (9.25)\\ &= X\left(t_A\right) \exp\left(2j\pi\nu t_A\right) && (9.26)\end{aligned}$$

and

$$\begin{aligned} x_B(t) &= X\left(t - \frac{\mathbf{r}_B\cdot\mathbf{e}}{c}\right) \exp\left[2j\pi\nu\left(t - \frac{\mathbf{r}_B\cdot\mathbf{e}}{c}\right)\right] && (9.27)\\ &= X\left(t_A - \tau_{AB}\right) \exp\left[2j\pi\nu\left(t_A - \tau_{AB}\right)\right] \ . && (9.28)\end{aligned}$$

In these expressions,

$$\tau_A = \frac{\mathbf{r}_A\cdot\mathbf{e}}{c} \ , \ t_A = t - \tau_A \ , \ \tau_B = \frac{\mathbf{r}_B\cdot\mathbf{e}}{c} \ , \ \tau_{AB} = \tau_B - \tau_A = \frac{\mathbf{AB}\cdot\mathbf{e}}{c} \ . \tag{9.29}$$

The sum of the two signals $x_{\mathrm{A}}(t)$ and $x_{\mathrm{B}}(t)$ is

$$x_{\mathrm{A}}(t) + x_{\mathrm{B}}(t) = X(t_{\mathrm{A}}) \exp[2j\pi\nu(t_{\mathrm{A}})] + X(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \exp[2j\pi\nu(t_{\mathrm{A}} - \tau_{\mathrm{AB}})] \quad (9.30)$$

$$= [X(t_{\mathrm{A}}) + X(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \exp(-2j\pi\nu\tau_{\mathrm{AB}})] \times \exp(2j\pi\nu t_{\mathrm{A}}) \; . \quad (9.31)$$

The output of the square law detector is

$$|x_{\mathrm{A}}(t) + x_{\mathrm{B}}(t)|^2 = |X(t_{\mathrm{A}})|^2 + |X(t_{\mathrm{A}} - \tau_{\mathrm{AB}})|^2 \quad (9.32)$$

$$+ X(t_{\mathrm{A}}) X^*(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \exp(2j\pi\nu\tau_{\mathrm{AB}}) \quad (9.33)$$

$$+ X^*(t_{\mathrm{A}}) X(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \exp(-2j\pi\nu\tau_{\mathrm{AB}}) \; . \quad (9.34)$$

This result is integrated over a time Δt, chosen much longer than the period of the signals but much shorter than the characteristic time of variation of the direction α of the source due to the Earth's rotation. The output $y(\tau_{\mathrm{AB}})$ of the interferometer is, consequently,

$$y(\tau_{\mathrm{AB}}) = \left\langle |X(t_{\mathrm{A}})|^2 \right\rangle + \left\langle |X(t_{\mathrm{A}} - \tau_{\mathrm{AB}})|^2 \right\rangle + \langle X(t_{\mathrm{A}}) X^*(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \rangle \exp(2j\pi\nu\tau_{\mathrm{AB}}) + \langle X^*(t_{\mathrm{A}}) X(t_{\mathrm{A}} - \tau_{\mathrm{AB}}) \rangle \exp(-2j\pi\nu\tau_{\mathrm{AB}}) \; . \quad (9.35)$$

The mean value of the amplitude is constant,

$$\left\langle |X(t_{\mathrm{A}})|^2 \right\rangle = \left\langle |X(t_{\mathrm{A}} - \tau_{\mathrm{AB}})|^2 \right\rangle = X_0^2 \; . \quad (9.36)$$

The mean values of the products

$$X^*(t_{\mathrm{A}}) X(t_{\mathrm{A}} - \tau_{\mathrm{AB}})$$

and

$$X(t_{\mathrm{A}}) X^*(t_{\mathrm{A}} - \tau_{\mathrm{AB}})$$

are related to the autocorrelation function $\gamma_X(t)$ of $X(t)$,

$$\gamma_X(t) = \langle X^*(\tau) X(t + \tau) \rangle \; . \quad (9.37)$$

Consequently,

$$y(\tau_{\mathrm{AB}}) = 2X_0^2 + \gamma_X(\tau_{\mathrm{AB}}) \exp(2j\pi\nu\tau_{\mathrm{AB}}) + \gamma_X(-\tau_{\mathrm{AB}}) \exp(-2j\pi\nu\tau_{\mathrm{AB}}) \; . \quad (9.38)$$

The following property of the autocorrelation function results from its definition (the autocorrelation is a Hermitian operator):

$$\gamma_X(-t) = \gamma_X^*(t) \; . \quad (9.39)$$

Consequently,

$$y(\tau_{\mathrm{AB}}) = 2X_0^2 + 2\Re\left\{\gamma_X(\tau_{\mathrm{AB}})\exp\left(2j\pi\nu\tau_{\mathrm{AB}}\right)\right\} , \tag{9.40}$$

where $\Re(z)$ is the real part of the complex number z.

Notice that

$$\gamma_X(\tau_{\mathrm{AB}})\exp\left(2j\pi\nu\tau_{\mathrm{AB}}\right) = \gamma_x(\tau_{\mathrm{AB}}) , \tag{9.41}$$

where $\gamma_x(t)$ is the autocorrelation function of $x(t, \mathbf{r})$.

Finally,

$$y(\tau_{\mathrm{AB}}) = 2X_0^2 + 2\Re\left\{\gamma_x(\tau_{\mathrm{AB}})\right\} . \tag{9.42}$$

The conclusions are the following:

1. The useful information is contained in the periodic part of $y(\tau_{\mathrm{AB}})$, which is the autocorrelation function $\gamma_x(\tau_{\mathrm{AB}})$ of the plane wave emitted by the point source.
2. The time delay τ_{AB} that connects the baseline and the source position appears in the value of $\gamma_x(\tau_{\mathrm{AB}})$.
3. The ratio of the periodic part of $y(\tau_{\mathrm{AB}})$ to its constant one is called the complex fringe visibility. It is proportional to the autocorrelation function of the plane wave.
4. The autocorrelation function $\gamma_x(t)$ is maximal for $t = 0$. This means that the fringe visibility is reduced when the delay τ_{AB} increases, this is due to the limited coherence time of the radiation, related to its linewidth.
   The autocorrelation function is linked to the spectral density of the line (the Wiener–Khinchin theorem). If $C(t)$ is the autocorrelation function of a time function $f(t)$ whose Fourier transform is $F(\nu)$, then $C(t)$ is the Fourier transform of the absolute square of $F(\nu)$, which is the spectral density of $f(t)$.
   Consequently, the order of magnitude of the coherence time of the incoming wave is given by the inverse of its linewidth. This will ultimately limit the resolution of the observation. For instance, a linewidth of 1 kHz gives an upper limit of only 1.6×10^{-4} s for τ_{AB}, corresponding to

   $$|\mathbf{AB} \bullet \mathbf{s}| \approx 5\,\mathrm{km}$$

   and, for a baseline length of 5 000 km, to $\alpha = 1 \times 10^{-3}$ rd.
   In fact, since the signals are correlated in deferred time, it is possible to shift one record until the time difference is canceled and the correlation function is maximal. The shift gives the value of the time difference τ_{AB}.
5. The main part of the processing of the signals received by any array of radio antennas is consequently the calculation of their correlation.
   Although the signals received by different antennas come from the same source, the quantity computed is called the cross-correlation, taking into

account the fact that each signal may have been modified in a different and non-correlated way (atmospheric perturbations, additive noise of the receiver, etc.). This calculation is made by a specialized data processing system called the *correlator*.

Extended Source

If the radiation source is extended around the incoming direction $\mathbf{e_0}$, but is incoherent (different points of the source radiate independently), there is no interference between the contribution of the different points and their power contributions are simply added.

The following model is limited to a one-dimensional source. It is easy to extend the model to the real case of two-dimensional sources.

The position of each point of the source is characterized by the unit vectors $\mathbf{e}$ (from the source) or $\mathbf{s} = -\mathbf{e}$ (toward the source) or by the angle α between the perpendicular to the baseline and $\mathbf{s}$.

The sky brightness $B(\alpha)$ of the point in the direction α is proportional to the square of the mean amplitude $X_0(\alpha)$ of the incoming radiation from that direction and the total brightness is $B_\mathrm{t} = \int_\alpha B(\alpha)\mathrm{d}\alpha$.

The output y_c of the correlator is the sum of the elementary cross-correlation functions corresponding to all the points of the source,

$$\begin{aligned} y_\mathrm{c} &= \int_\alpha \gamma_x(\alpha, \tau_\alpha)\mathrm{d}\alpha \\ &= \int_\alpha \gamma_X(\alpha, \tau_\alpha)\exp\left(j2\pi\nu\tau_\alpha\right)\mathrm{d}\alpha\ . \end{aligned} \tag{9.43}$$

The integral is to be taken over the radio source. Every point of the source corresponds to a value of α,

$$\alpha = \alpha_0 + \delta\alpha \tag{9.44}$$

with

$$\delta\alpha \ll 1\ , \tag{9.45}$$

where α_0 corresponds to an arbitrary reference point of the source, τ_α is the value of τ_AB for the value α of the angle between the perpendicular to the baseline, and $\mathbf{s}$

$$\begin{aligned} \tau_\alpha &= -\frac{D\sin\alpha}{c} \\ &= -\frac{D\sin\alpha_0}{c} - \frac{D\cos\alpha_0}{c}\delta\alpha\ , \end{aligned} \tag{9.46}$$

$$2\pi\nu\tau_\alpha = -2\pi\frac{D}{\lambda}\sin\alpha_0 - 2\pi\frac{D}{\lambda}\cos\alpha_0\delta\alpha\ , \tag{9.47}$$

where λ is the wavelength of the radiation. $\gamma_X(\delta\alpha, \tau_{\delta\alpha})$ is the correlation function of the sky brightness in the direction $\alpha = \alpha_0 + \delta\alpha$. Consequently,

$$y_c = \int_{\delta\alpha} \gamma_X(\delta\alpha, \tau_{\delta\alpha}) \exp\left(-j2\pi\frac{D\sin\alpha_0}{\lambda}\right) \times \exp\left(-j2\pi\frac{D\cos\alpha_0}{\lambda}\delta\alpha\right) \mathrm{d}(\delta\alpha) \;. \tag{9.48}$$

This expression shows that the output of the correlator is the Fourier transform of the function of $\delta\alpha$,

$$\gamma_{X,\alpha_0}(\delta\alpha, \tau_{\delta\alpha}) = \gamma_X(\delta\alpha, \tau_{\delta\alpha}) \exp\left(-j2\pi\frac{D\sin\alpha_0}{\lambda}\right) \;. \tag{9.49}$$

This function is closely related to the correlation function of the sky brightness [117],

$$y_c\left(\frac{D\cos\alpha_0}{\lambda}\right) = \mathcal{F}_\alpha[\gamma_{X,\alpha_0}(\alpha, \tau_\alpha)]\left(\frac{D\cos\alpha_0}{\lambda}\right) \;. \tag{9.50}$$

The conclusions are the following:

1. The correlator gives the Fourier transform of the cross-correlation function of the amplitude $X(\alpha, t_{\mathrm{AB}})$ of the signal emitted by the source. This correlation function is calculated for the delay t_{AB} between the two receivers.
2. The sky brightness can be calculated from this result if this Fourier transform is known for different sampled values of its parameter $\frac{D\cos\alpha_0}{\lambda}$, i.e. for different values of D, the distance between the two receivers involved in the calculation of the Fourier transform and/or different values of α_0. In the first case, an array of receivers is used, in the second case, the motion of the vector **AB** due to the rotation of the Earth is used.

Examples

In the following simple examples

1. $\alpha_0 \ll 1$: $\cos\alpha_0 = 1$ and $\sin\alpha_0 = 0$.
   Consequently, (9.48) simplifies to

$$y_c = \int_{\delta\alpha} \gamma_X(\delta\alpha, \tau_{\delta\alpha}) \times \exp\left(-j2\pi\frac{D}{\lambda}\delta\alpha\right) \mathrm{d}(\delta\alpha) \tag{9.51}$$

   and the output of the correlator is the Fourier transform of the cross-correlation function $\gamma_{\mathrm{X}}(\delta\alpha, \tau_{\delta\alpha})$.
2. The linewidth of the radiation emitted by the source is supposed to be narrow enough so that the cross-correlation function $\gamma_{\mathrm{X}}(\delta\alpha, \tau_{\delta\alpha})$ is (a monochromatic wave)

$$\gamma_{\mathrm{X}}(\delta\alpha, \tau_{\delta\alpha}) = X_0^2(\delta\alpha) \;. \tag{9.52}$$

The Rectangular Sky Brightness Function

The object is centered at $-\alpha_0/2 \ll 1$ and its width is $2\Delta\alpha$,

$$X_0^2(\delta\alpha) = \begin{cases} 0 & \begin{cases} \delta\alpha < -\alpha_0/2 - \Delta\alpha \\ \delta\alpha > -\alpha_0/2 + \Delta\alpha \end{cases}, \\ X_0^2 & -\alpha_0/2 - \Delta\alpha \le \delta\alpha \le -\alpha_0/2 + \Delta\alpha \; . \end{cases} \tag{9.53}$$

The Fourier transform $\mathcal{F}_\alpha[\gamma_X(\alpha,\tau_\alpha)]\left(\frac{D}{\lambda}\right)$ is

$$Y\left(\frac{D}{\lambda}\right) = 2X_0^2\delta\alpha \exp\left(-j2\pi\frac{D}{\lambda} \times \frac{-\alpha_0}{2}\right) \operatorname{sinc}\left(2\pi\frac{D}{\lambda}\delta\alpha\right) \; . \tag{9.54}$$

The complex fringe visibility $\Gamma\left(D/\lambda\right)$ is

$$\Gamma\left(\frac{D}{\lambda}\right) = \exp\left(-j2\pi\frac{D}{\lambda} \times \frac{-\alpha_0}{2}\right) \operatorname{sinc}\left(2\pi\frac{D}{\lambda}\delta\alpha\right) \; . \tag{9.55}$$

The modulus of the complex visibility is consequently maximal for small values of the ratio

$$\frac{\delta\alpha}{\lambda/D} \; ,$$

i.e. for objects whose angular diameter is of the order of or smaller than λ/D; VLBI is used to observe very compact sources.

A Pair of Rectangular Sky Brightness Functions

As a second example, consider a pair of rectangular sky brightness functions centered at $\pm\alpha_0/2$ and having width $2\Delta\alpha$,

$$X_0^2(\delta\alpha) = \begin{cases} 0 & \begin{cases} \delta\alpha < -\alpha_0/2 - \Delta\alpha \\ -\alpha_0/2 + \Delta\alpha < \delta\alpha < +\alpha_0/2 - \Delta\alpha \\ \delta\alpha > +\alpha_0/2 + \Delta\alpha \end{cases}, \\ X_0^2 & \begin{cases} -\alpha_0/2 - \Delta\alpha \le \delta\alpha \le -\alpha_0/2 + \Delta\alpha \\ +\alpha_0/2 - \Delta\alpha \le \delta\alpha \le +\alpha_0/2 + \Delta\alpha \; . \end{cases} \end{cases} \tag{9.56}$$

In this case, the Fourier transform $Y_2\left(\frac{D}{\lambda}\right)$ of the sky brightness function is

$$Y_2\left(\frac{D}{\lambda}\right) = 2X_0^2\delta\alpha \operatorname{sinc}\left(2\pi\frac{D}{\lambda}\delta\alpha\right) \times \left[\exp\left(-j2\pi\frac{D}{\lambda} \times \frac{-\alpha_0}{2}\right) + \exp\left(-j2\pi\frac{D}{\lambda} \times \frac{+\alpha_0}{2}\right)\right] \tag{9.57}$$

$$= 4X_0^2\delta\alpha \operatorname{sinc}\left(2\pi\frac{D}{\lambda}\delta\alpha\right) \cos\left(2\pi\frac{D}{\lambda} \times \frac{\alpha_0}{2}\right) \; . \tag{9.58}$$

The complex fringe visibility $\Gamma\left(\frac{D}{\lambda}\right)$ is

$$\Gamma\left(\frac{D}{\lambda}\right) = \operatorname{sinc}\left(2\pi\frac{D}{\lambda}\delta\alpha\right)\cos\left(2\pi\frac{D}{\lambda}\times\frac{\alpha_0}{2}\right) . \tag{9.59}$$

The conclusions are the following:

1. As in the previous example, the modulus of the complex visibility is maximal for small values of the ratio

$$\frac{\delta\alpha}{\lambda/D} .$$

2. The modulus of the complex visibility is maximal for small values of the quantity $(2\pi D\alpha_0)/(2\lambda)$, i.e. for

$$\frac{\lambda}{D} < \pi\alpha_0 . \tag{9.60}$$

   The resolution of the interferometer is consequently given by the ratio of the distance between the two receivers to the wavelength of the radiation.

9.1.2 Processing of the Signals

The previous discussions show that processing the signal received by the antennas allows one to

– produce an image of an astronomical object (aperture synthesis);
– precisely determine the relative position of the antennas if the emitting object is distant and stable (geodesy);
– precisely determine the position of a ground or space radio source if the positions of the antennas are known;
– determine the spectra of the radio emission.

Processing at Each Antenna

The data received by the antennas are processed in the following way before being correlated (many steps of the process, such as amplification, filtering, etc. are omitted in this schematic description).

1. They are down converted to a baseband signal by mixing them with a local oscillator. The accuracy and stability of this local oscillator must be consistent with the phase shifts to be measured.
   Suppose we have an input signal

$$x(t) = X\exp\left[j(2\pi\nu t + \phi\right)$$

and a local oscillator

$$x_{\mathrm{LO}} = X_{\mathrm{LO}} \exp\left[j(2\pi\nu_{\mathrm{LO}}t + \phi_{\mathrm{LO}})\right] .$$

Mixing these two signals uses a non-linear operator, which produces output components at various frequencies, the sum and difference of the multiples of the frequencies ν and ν_{LO}. From these components, it is easy to select, with a filter, the one whose frequency is $\nu_{\mathrm{IF}} = \nu - \nu_{\mathrm{OL}}$ (this frequency is called the intermediate frequency) and whose phase is $\phi - \phi_{\mathrm{LO}}$ (these relations apply in the case where $\nu_{\mathrm{LO}} < \nu$). The baseband is centered on this frequency $\nu - \nu_{\mathrm{LO}}$. The phase fluctuations of the IF signal are consequently the sum of that of the signal and of the local oscillator,

$$x_{\mathrm{IF}} = KXX_{\mathrm{LO}} \exp j([2\pi(\nu - \nu_{\mathrm{LO}})t + \phi - \phi_{\mathrm{LO}}] . \tag{9.61}$$

This is not a problem if all the signals of the interferometer are down converted using the same local oscillator, since it is the phase difference between them that is the pertinent information. On the contrary, in the case of a VLBI, the signals from different antennas are down converted using a different local oscillator, located in the same station as the antenna; the phase of each local oscillator must consequently be very precisely defined.

2. The resulting signal is sampled and recorded in a digital media, along with a precise timestamp.
3. The recorded data are then sent to the correlator to be further processed.

Delay Compensation

Due to the delay between the two antennas whose signal are to be correlated and to the finite linewidth of the line being studied, the fringe visibility is decreased (see Sect. 9.1.1). This can be compensated, since it is possible to shift the two recorded data to optimize the value of their cross-correlation.

Digital Correlator

The correlator is the masterpiece of VLBI signal processing. Extensive descriptions can be found, for instance, in [27, 107].

In the case of a digital processing, the cross-correlation of the discrete-time process function of $f(n)$ and $g(n)$ is easily computed,

$$\gamma(n) = f \star g(n) = \sum_{p=-\infty}^{\infty} f^*(p)g(n+p) . \tag{9.62}$$

A schematic block diagram of a cross-correlator is shown in Fig. 9.3. It uses memory to implement delays of a multiple of the sampling time T_{s}, multipliers and accumulators.

In fact, the summation does not extend from $-\infty$ to $+\infty$ and the output of the device is an estimator of the cross-correlation.

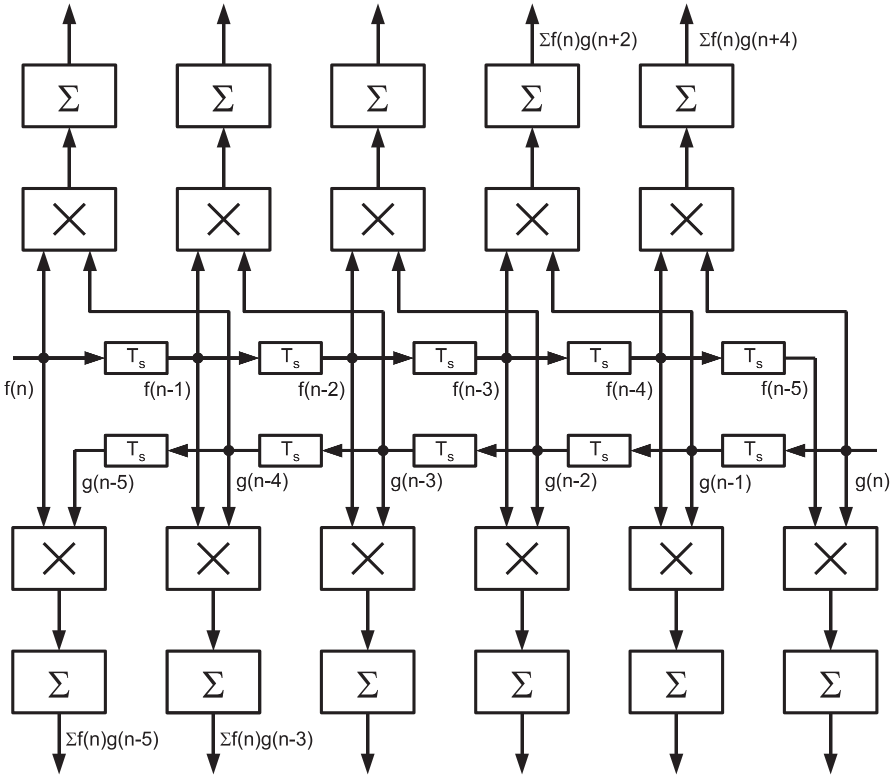

Fig. 9.3. Schematic block diagram of a cross-correlator

9.2 Applications of VLBI

It was shown in Sect. 9.1 that VLBI may have various applications in astronomy (position, spectra and imaging of astronomical objects) and in geodesy (the relative position of the antennas, absolute position relative to reference astronomical objects, rotation of the Earth).

9.2.1 Astronomy

VLBI was developed first as a radio-astronomical tool and remains a powerful and high resolution tool for observing radio sources. It allows submilliarcsecond imaging [73, 60] and detection [118] of extragalactic objects.

9.2.2 Geodesy

In this kind of application, the astronomical sources are known and used as references to determine some parameters of the Earth.

Rotation of the Earth

Very distant quasars provide an inertial reference frame that is much more accurate than the fundamental catalog of fix stars FK5 [126].

The antennas of a VLBI array are then in a situation that may be compared to that of a differential GPS experiment; they receive the signal emitted by the same source. Nevertheless, in the case of VLBI, the astronomic sources appear as a point-source with no motion. There is consequently no need to construct a model for their motion.

Since the radio telescopes are fixed on the rotating Earth, VLBI measures the orientation of the Earth in the inertial reference frame defined by these quasars as a function of time, monitoring the Earth rotation and orientation. It is consequently possible to measure all the components of the Earth's rotation:

- the position of the Earth's spin axis in space,
- the position of the Earth's spin axis relative to the Earth crust, and
- the velocity of the rotation, which allows one to connect the two time scales UT and UTC (see Sect. 7.1).

This information allow one to perform orbit controls of satellites, including GPS satellites (see, for instance [110, 95]).

Monitoring of Plate Potions

This application of VLBI, joined to the GPS technique, is well known. These space geodetic techniques allow the direct measurements of plate motions. Motions of a few cm per year are clearly visible (see, for instance [49, 7, 47]). The results of these measurements are used in Earthquake research.

Precise Localization on the Earth

The precise measurement of the position of the VLBI and GPS stations allow one to maintain the realization of the International Terrestrial Reference System (see, for instance [59, 88]).

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