8 System Control of Complex Objects

This chapter describes the system problems of coordinated control of serviceability and the safety of complex hierarchical systems (CHSs) in real functioning conditions characterized by uncertainty and risks. On both the structural-functional analysis level and the control level there is a need to find a rational solution under conditions of conceptual uncertainty regarding the object's shape, structure, and properties. In addition, it is necessary to take into account other factors [18, 83, 162, 175, 194, 198, 199, 201, 202, 207]. In particular, in practice there is no any a priori information that guarantees a reliable evaluation of constructional and technological abilities and predefined requirements to CHS main properties, indicators, and conditions of usage. The conditions of CHS usage are determined by various factors depending on the CHS's purpose. However, nowadays for most CHS types, the risks connected with accidents and catastrophes have the utmost importance [33, 180].

Specificity investigation and the development of the main principles for solving the system control problems under conditions of uncertainty and risks will be described in this chapter.

8.1 System Control Problem Analysis and Classification

In human practice a lot of control problems arise. In these problems the evaluation and optimization of the quality of an object's control is performed using a single criterion. Such problems include a family of the following simple problems: the maximal operating speed problem, the problem of the highest signal reproduction quality, the finite state optimization problem of the vertically launched rocket, the problem of fuel consumption minimization in order to reach the required trajectory by a rocket, the problem of the energy consumption minimization to perform a given task, etc. All these control problems can be, in general, stated in the following way.

Substantial problem definition. Acceptable state set and acceptable control set of an object are ***defined***. Initial conditions and boundary conditions are ***defined***. It is ***necessary*** to find an acceptable control by which the extreme value of the specified control quality indicator is achieved and the controlled object is transferred from an initial state to a finite one, remaining in the admissible state domain.

Let us note some features and properties of the considered problems. They differ not only in complexity but also in practical significance. The class of maximal operating speed determination problems appeared at the same time as the simplest mechanical regulators were introduced. For example, Watt's regulator, introduced in the 1840s. This circumstance made the name of a control discipline a regulation theory.

The period 1940–1970 witnessed a qualitative change in the problems. This was caused by the rapid technological expansion of aeronautical engineering and also by the creation and usage of space-system engineering.

The principal difference between these problems and traditional regulation problems is as follows. In traditional problem definition the quality of control of slowly changing processes was investigated. In this way, the quality of autopilot compensation of a random disturbing process in real-life flight conditions was investigated. In this problem the flight time is much longer than the time of transition of a random disturbing compensation. Yet, the flight time of a rocket with a working engine is an example of a transient process, because the continuous changes in rocket mass take place. The control problem is complicated by many other factors. In particular, the high price of fuel and high consumption rate of the useful load determines the necessity of minimizing fuel consumption while selecting the flight trajectory. All these factors resulted in the definition of qualitatively new problems that are fundamentally different from traditional regulation problems. Such differences are as follows: the presence of a set of conflicting objectives of control optimization; incompleteness, uncertainty, and fuzziness of initial information about controlled object service and usage in various conditions; and the necessity of control being optimally coordinated throughout the object's entire life cycle.

The necessity and features of applied system problems of control of a complex modern technique in different stages of its life cycle are determined by many factors. It is necessary to emphasize factors of uncertainty and risk in the CHS functioning process. These factors are the root cause of many accidents and catastrophes attended by the loss of lives and running into billions of dollars in damage [33, 180]. It follows that it is practically necessary to find new principles and approaches to the control of modern CHSs, which have a variety of purposes. Technologically and ecologically dangerous technical systems belong to such CHs. In finding a solution to this problem, in addition to risk factors it is necessary to take into account that functioning effectiveness and the possibility of an accident or a catastrophe directly depend on control structure properties of a given system. Thus, in working out a solution it should be taken into account that the controllability, adaptability, stability, coordination capability, survivability, and effectiveness of modern systems with their varying purposes are defined in many respects by the properties, possibilities, and effectiveness of a control.

Among other factors a continuous increase in requirements for technical systems is significant. This stimulates wide application of high science-intensive technologies at all stages of a system life cycle. It follows that it is practically necessary to perfect processes of forming, substantiating, and making rational decisions in strategic planning and control, in designing and manufacturing innovative and other promising technical products, and also in the allocation of resources and money for their development and manufacture. However, the versatile interconnection and multifarious interrelations among strategic decisions at different stages of life cycles of complex systems and other promising technical products have essentially complicated the problem of forming and making a decision at different levels of hierarchical control systems, particularly in evaluating practical necessity, technological possibility, and economical availability.

As a result there appeared multiple classes of nonformalizable or hardly formalizable control problems. For their solution, it is necessary to provide system unity of control principles, approaches, and criteria.

Thus the classical Winner's definition of control obtained two principally different interpretations. ***The first interpretation*** defines the control as a general category that includes all forms and kinds of changes in properties and behavior of controlled systems. In such an interpretation the control includes such processes as the transformation of the structure, directions, goals, and criteria of specified sorts of human activity. It also includes forming and making a decision about the evaluation, planning, and realization of promising new directions in human activity.

The second interpretation defines the control as specialized purposeful changes in operations and properties of systems within predetermined limits. The first interpretation is typical for control in industrial, economical, social, and other scopes of activity. The second one is typical for more specialized kinds of activity, for example, for medium and small-scale business of any kind, including firms that are developing tools of small-scale automation, automatic regulators, and dedicated controllers. The class of dedicated controllers includes airplane and car onboard computers and other similar specialized computers; each of them manages the specified technological process in the current situation. For example, a car computer can provide rational fuel consumption in different driving conditions. It is evident that the second interpretation does not fully suit general purposes, problems, and principles of a system control. Thus, later on it is expedient to consider problems of a system control according to the first interpretation, because it is more common and based on a broader understanding of the essence, principles, and problems of system control. In accordance with this approach the following formulation of the term "control" is offered:

A control is any purposeful action directed toward the achievement of specified useful changes or transformations in a controlled complex system.

Actions in different kinds of human activity may differ in many properties and indicators, in particular in principles and manner of influence on controlled object abilities. Because of the broad meaning of the term "control" in the first interpretation it is necessary to introduce the following definitions:

Behavior control (C1). This control provides the transition of an object from one state to another on limited set of possible states. C1 is realized during process of simple interactions between a controlled object and an environment, in a form of specified reaction of an object on an external action.

Property control (C2). This control provides changes in the forms and methods of forming a reaction to an external action in accordance with a current situation. C2 provides adaptation of an object to the current situation by means of changing a reaction to an external action in accordance with an external action peculiarity.

Structure control (C3). This control provides changes in the composition, structure, and interconnections of an object in accordance with an external action. C3 allows an object to adapt to a current situation by means of structural changes of an object in accordance with newly formed object goals.

Evolution control (C4). This control provides purposeful changes in the goals, properties, structure, and activity forms of an object in accordance with changes in conditions of interaction with the environment. C4 is the main kind of control during planning of the compound production complex development, complex space system development (i.e., "Mir" space station, GPS satellite navigation system, etc.), complex weapons, and defense technology (i.e., aircraft-carriers, submarines, etc.).

Purpose control (C5). This control provides changes in the objectives and problems of an object's operation in current conditions. C5 is the kind of control that is active in situations where previously predicted conditions of an object's operation are changing so fundamentally that no one of the aforementioned kinds of control allows the object to adapt to new conditions.

Note that the practical significance and complexity of control problems increase while switching from class C1 to class C5. This is conditioned by many reasons and factors, among which is, firstly, an increasing scale of human activity and of complexity. Such an increase is conditioned by the application of new technologies and essential enhancement of the possibilities and scope of use of various modern systems.

The main control purposes are enhancement of the effectiveness of a controlled object and ensuring of its adaptation to a higher level of uncertainty and complexity of external actions. Indeed, C1 class provides optimal control only within boundaries of a previously defined set of actions under a priori known external action restrictions. If the current conditions in which a real object is used differ essentially from accepted, possible conditions of external actions, then the control may be so ineffective that the object will be quite uncontrollable.

The consequences of such a state may be extremely serious. In particular, similar conditions accompanied the Chernobyl nuclear power plant disaster in Ukraine in 1986. At the same time the control of class C2 or higher allows one to react to unforeseen situations more adequately and promptly. This is realized by changing the properties (C2) or the structure (C3) of an object and during the process of fast evolving changes in a normal mode or unexpected external factors (wind velocity, temperature, etc.).

Other principles are realized in the implementation of the C4 or C5 class of control because their purposes and problems are qualitatively different from those of C1, C2, and C3. In addition, classes C4 and C5 are oriented toward a sufficiently long time period. In fact, fundamental changes in the direction of an object's evolution under C4 or changes in purpose under C5 require a large volume of previous examinations and researches, and transforming them into real products and technologies. Thus, the C4 and C5 class controls are implemented in the process of slowly changing strategic situations. The global experience with the development of various complex systems proves it. In particular, there is the experience with creating, using, and developing space systems of different purpose (GPS, satellite communications, aerography, earth remote sensing, etc.) The classical example of prolonged development and purposeful expansion of the abilities of a complex system is the Mir space station, which was used and developed in the USSR for more than 15 years.

It is significant to note one more necessary feature of the considered control classes: while switching from class C1 to class C5 the control problem complexity is fundamentally increasing. The main factor in complexity increase is the transition from very limited formalized control problem set of class C1 to a wide nomenclature of problems of class C5, by which an avalanchelike increase in the quantity of nonformalizable or hardly formalizable problems in the total volume of control problems occurs.

It is significant to take into account one more necessary factor in the development of system analysis methodology. Its essence is that at the same time as system analysis methodology development the qualitative change of the notion of "control" takes place. From a narrow interpretation of this notion, which follows from the goals and objectives of control problems of the C1 class, there occurs a gradual transition to a more broad, system understanding of control problems of the C5 class. However, these classes remain essentially within the bounds of a typical narrow understanding of a control and, therefore, do not include a number of practical necessary aspects of behavior of complex systems of varying purposes. In particular, there is the barest necessity of creation of a unified methodology of a system investigation and optimization of a control of modern complex systems in real conditions of uncertainty and risks. Therefore, assume it is expedient not to limit the classification by classes described above and to supplement it with a system control class.

System control (C6). This control provides purposeful changes in the functioning goals and/or other properties of an object under investigation for the sake of system-coordinated guarantee of a required level of operational capability, safety, and efficiency in real conditions of conceptual uncertainty and multifactor risks.

Taking into account that the substantial definition of the notion of "system control" is missing, we introduce the following definition.

The system control is an unlimited sequence of procedures of forming, substantiating, selecting, and realizing system interconnected and functionally interdependent decisions and actions that are coordinated under the goals, objectives, time constraints, resources, and expected results for the sake of ensuring the required level of operational capability, safety, and efficiency of the controlled object and/or achievement of specified changes in the controlled object in the presence of multifactor risks and uncertainty of external influences.

The notion of ***"unlimited sequence"*** means the sequence of procedures that are executed as needed in accordance with the current circumstances, conditions, and goals of the controlled object's operation. Thus the quantity of procedures is not limited a priori.

Now we consider the properties and features of complex object control problems in detail.

First, let us consider the C1 problems class by the example of a solution to the problem of dynamic object control. The main purpose of such an approach is to reveal the properties and features of the formalization and solution to control problems of this class. This will allow us not only to discover similarities and differences between approaches (accepted in control theory and in system analysis) to a control problem solution, but also to analyze the possibilities, merits, and demerits of typical system analysis apparatus. To this end it is necessary to analyze the specificity and conditions of problem formalization. Thus, let us consider the features of a formalized definition of an optimal control problem [99].

Formalized problem definition. Let the dynamic object movement be described by a system of differential equations represented in a vectorial form:

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = \bar{f}\,(\bar{x}, \bar{u}, t)\ , \tag{8.1}$$

where $\bar{x}$, $\bar{u}$, and $\bar{f}$ are vectors with components x_i, u_i, and f_i, respectively; $\bar{x}(t)$ is an object-state vector (phase coordinate vector) that defines an object's state at moment t; $\bar{u}(t)$ is the control vector. Vectors $\bar{x}(t)$ and $\bar{u}(t)$ may change in some admissible domain:

$$\bar{x}\,(t) \in G_x\ , \tag{8.2}$$

$$\bar{u}\,(t) \in G_u\ , \tag{8.3}$$

or briefly

$$(\bar{x}, \bar{u}) \in G; \quad G = G_x \times G_u\ . \tag{8.4}$$

Condition (8.2) is used to name a state limitation (or phase limitation), and condition (8.3) to name a control limitation. In addition to limitations, the initial and finite states of a controlled object are also defined:

$$\bar{x}\,(t_0) \in E_0, \quad \bar{x}\,(T) \in E_T\,, \tag{8.5}$$

where t_0 and T are respectively the time of an initial state and the time of a finite state of a controlled object and E_0 and E_T are a set of admissible values of, respectively, an initial state and a finite state that are assumed to be defined.

As an example let us represent (8.1) in an explicit form that describes a spaceship moving while achieving an orbit:

$$\begin{aligned} &\dot{x}_1 = x_3, \quad \dot{x}_2 = x_4, \quad \dot{x}_3 = \frac{1}{m}\,(\varphi_1 + u_1 \cos u_2)\,, \\ &\dot{x}_4 = \frac{1}{m}\,(\varphi_2 + u_1 \sin u_2)\,, \quad \dot{m} = -F\,(u_1)\,. \end{aligned} \tag{8.6}$$

Here x_1 and x_2 are the current position coordinates of a spaceship, x_3 and x_4 are the current velocity values, m is the total carrier rocket and spaceship mass while orbitting. u_1 is the thrust value, u_2 is the angle between the thrust direction and the x_1-axis, $F\,(u_1)$ is the instant fuel consumption and thus characterizes the total mass change. Gravity influence, atmospheric resistance, and other hampering factors are taken into account in φ_1 and φ_2 variables, where φ_1 and φ_2 are the x_1-axis and x_2-axis projections of the aforementioned forces. Set G is a Cartesian product of sets G_x and G_u. Set G_x is a region of space around the Earth, within the limits of which the spaceship trajectory should pass.

In particular, trajectory limitations are evident when the pilot spacecraft starts. For example, the trajectory should not pass through the Earth's radiation belts and should not cross the ground surface. Set G_u is defined taking into account that the spacecraft flight control should be realized by changing the rocket thrust vector direction and value. The control vector is defined by thrust u_1 and angle u_2. Thus, G_u is a set of allowed values u_1 and u_2 defined by the construction, technological, and other constraints including an allowed human acceleration value.

The initial and the finite state should be defined in the form (8.5) in accordance with the spacecraft launch purpose. For the problem (8.6) solution it is necessary to define corresponding conditions for each equation. Therefore, there should be defined an initial spacecraft position, starting speed, and mass values. In this case it is possible to consider the initial state in the form:

$$\begin{aligned} &x_1\,(t_0) = x_{01}, \quad x_3\,(t_0) = x_{03}\,, \\ &x_2\,(t_0) = x_{02}, \quad x_4\,(t_0) = x_{04}, \quad m\,(t_0) = m_0\,. \end{aligned} \tag{8.7}$$

Here x_{01}, x_{02}, x_{03}, x_{04} and m_0 are specified fixed values and m_0 is the total starting mass of the spacecraft and carrier rocket.

The finite state after achieving an orbit is defined by orbit parameters. When a circular orbit is achieved with the specified radius R, the finite state of the spaceship will be defined with the following conditions:

$$\begin{aligned} x_1^2(T) + x_2^2(T) &= R^2\,, \\ x_3(T)\,x_1(T) + x_4(T)\,x_2(T) &= 0\,, \\ x_3^2(T) + x_4^2(T) &= V_R^2\,, \end{aligned} \tag{8.8}$$

where V_R is a specified velocity of the spacecraft in orbit. The first condition (8.8) means that the point with coordinates (x_1, x_2) at the moment $t = T$ is located on the circle of the specified radius R. The second condition means vectors $\bar{r}$ and $\bar{V}$, with components (x_1, x_2) and (x_3, x_4), respectively, are orthogonal, i.e., the spacecraft velocity vector $\bar{V}$ at the moment $t = T$ is directed at a tangent to a circle of the specified radius and is orthogonal to vector $\bar{r}$. The third condition shows us that the velocity of the spacecraft in orbit should be equal to a specified value. Thus, condition (8.8) guarantees that a free movement of the spacecraft in a circular orbit (with radius R and velocity V_R) will occur if the engine shuts down at the moment $t = T$.

Now, it is necessary to formalize a criterion of control efficiency. It is evident that the process by which a spacecraft achieves a specified orbit, i.e., the process of transition of a spacecraft from an initial state (8.7) to a finite state (8.8) may be realized in many different ways, where each of them is characterized by a specific control program. We can propose various criteria of program efficiency evaluation. As described earlier, one of the most necessary practical problems is the fuel-consumption-minimization problem. The mathematical representation of the given problem can be found on the basis of the last equation in (8.6). Since the value $F(u_1)$ defines the instant fuel consumption, then the total fuel consumption as the spacecraft approaches its target orbit will be defined by the integral

$$I(\bar{u}) = \int_{t_0}^{T} F(u_1)\,\mathrm{d}t\,. \tag{8.9}$$

It follows that the optimal program $\bar{u}^*$ of a spacecraft approaching it target orbit corresponds to the condition

$$I(\bar{u}^*) = \min_{U \in G_U} I(\bar{u})\,, \quad \bar{u}^* = \operatorname{Arg}\min_{u \in G} I(\bar{u})\,. \tag{8.10}$$

For short, condition (8.10) will be used in the form

$$I(\bar{u}) \to \min \tag{8.11}$$

or, if the criterion is maximized, in the form

$$I(\bar{u}) \to \max\,. \tag{8.12}$$

In the given problem control optimization is achieved via the selection of a control vector function. In a more common definition the optimization may get complicated. In particular, it may be provided also by selecting a control vector function and a state vector function, i.e., at the expense of the $\bar{u}(t)$ and the $\bar{x}(t)$ selection under conditions (8.11) or (8.12).

The given conditions of formalization of the optimal control problem allow one to represent the mathematical definition of the mentioned problem in the following, more common, form.

Mathematical optimal control problem definition. It is required to determine such vector functions $\bar{x}^*(t) \in R^n$ and $\bar{u}^*(t) \in R^m$ while $t \in [t_0, T]$ that provide the minimization of the functional

$$I = I(\bar{x}, \bar{u}) \tag{8.13}$$

in the form of the condition

$$I(\bar{x}, \bar{u}) \to \min \quad \text{or} \quad I(\bar{x}, \bar{u}) \to \max \tag{8.14}$$

under differential constraints

$$\dot{x} = \bar{f}(\bar{x}, \bar{u}, t) , \tag{8.15}$$

conditions

$$(\bar{x}, \bar{u}) \in G; \quad t \in [t_0, T] , \tag{8.16}$$

and edge conditions

$$(\bar{x}, t_0) \in E_0, \quad (\bar{x}, T) \in E_T , \tag{8.17}$$

where G is some specified region of space $R^n \times R^m$, and E_0 and E_T are regions defined in the space $R^n \times R^1$.

Conditions (8.13)–(8.17) define different types of optimal control problems. These problems can be divided into three groups in accordance with methods whose use allows one to set the specified problem properties and features.

For ***group 1*** the different forms of functional (8.13) presets are typical. It contains the following problem types:

1.1. Lagrange
1.2. Mayer
1.3. Boltz
1.4. Operating speed

Group 2 contains different forms of conditions (8.16). It contains the following problem types:

2.1. Control restrictions
2.2. State restrictions
2.3. Combined control and state restrictions
2.4. Integral restrictions (isoperimetric problem)

Group 3 contains different forms of the presets of edge conditions (8.17). It contains the following problem types:

3.1. Fixed trajectory ends
3.2. Free trajectory end
3.3. Traveling ends

The solution methods for the optimal control problems are described in the book by Moisejev [99].

8.2 System Control Problems of Complex Objects' Operational Capability and Safety

New approaches to the creation of modern engineering define qualitatively new requirements for technological and ecological safety. Such a demand is conditioned not only by the fact that the loss from partial or full destruction of a piece of machinery or a construction project may be ten times more than its production cost but also by the fact that catastrophes may have international or global scales of influence on inhabitants and the environment. Furthermore, several catastrophes with losses in the millions and billions of dollars have taken place in the last decades of the 20th century in almost all industrially developed countries. This proves that the present principles and tools of complex object safety control do not meet modern requirements [33, 180].

Thus, there is a practical necessity to qualitatively change the principles and the structure of operational-capability controls and the safety of modern engineering systems in real conditions of multifactor risk influence. First of all, the control of complex objects should be systemwide which means that there should be system coordination of operability control and safety control not merely by the corresponding goals, tasks, resources, and expected results but also, importantly, by the immediacy and effectiveness of interaction in real conditions of abnormal situations. Such coordination should provide immediate and effective interaction between the mentioned control systems. On the one hand, there should be provided effectiveness of the safety system for timely detection of abnormal situations, evaluation of risk degree and level, and the definition of an permissible risk margin during the process of forming recommendations about immediate actions given to the decision maker. On the other hand, the system of operational capability control after receiving a signal about abnormal situations should, in an effective and operative manner, make a complex object ready for an emergency transition to an offline state and should make it possible to effect this transition within the limits of permissible risk. The main objectives of the system control of complex objects follow this principle.

Let us show the mathematical formulation of this problem with a priori set variation intervals of main indicators of the system in the normal mode and predefined permissible boundes of the influence of external factors.

Mathematical formulation of complex object system control problem. **It is known** that system functioning is characterized by the following sequence of complex system states: $E_1, E_2, \ldots, E_k, \ldots$. Every state E_k is characterized by specified indicators of system function processes (Y_k, X_k, U_k) and specified indicators of external environmental influence and risk factors Ξ_k:

$$E_k = \{(Y_k \in Y) \wedge (X_k \in X) \wedge (U_k \in U) \wedge (\Xi_k \in \Xi)\} , \tag{8.18}$$

where the meaning of indicators at the moment $T_k \in T^{\pm}$ is defined by the following relations:

$$\begin{aligned}
&Y_k = \hat{Y}\left[T_k\right], \quad X_k = \hat{X}\left[T_k\right], \quad U_k = \hat{U}\left[T_k\right], \quad \Xi_k = \hat{\Xi}\left[T_k\right],\\
&T_k = \{t_k | t_k > t_{k-1}\}, \; T_k \in T^{\pm}, \; T_0 = \left\{t | t_0 = t^-\right\}, \; T^{\pm} = \left\{t | t^- \le t \le t^+\right\},\\
&Y = \left(Y_i | i = \overline{1, m}\right), \quad X = \left(X_j | j = \overline{1, n}\right),\\
&U = \left(U_q | q = \overline{1, Q}\right), \quad \Xi = \left(\Xi_p | p = \overline{1, P}\right).
\end{aligned} \tag{8.19}$$

Here Y is a set of external parameters Y_i that includes technical, economic, and other indicators of system-function quality; X is a set of internal parameters X_j that includes constructional, technological, and other indicators; U is a set of control parameters U_q; Ξ is a set of external environmental influence parameters and parameters of risk factor influence Ξ_p; $\hat{Y}\left[T_k\right], \hat{X}\left[T_k\right], \hat{U}\left[T_k\right]$, and $\hat{\Xi}\left[T_k\right]$ are sets of meanings of appropriate parameters at the moment T_k; and $T^{\pm}$ is a specified or predicted complex object functioning period.

It is required to find such meanings of indicators of the degree of risk η_i and level of risk W_i and also a margin of permissible risk T_{ar} that provide the possibility of transition from the mode $\overset{\cup +}{R}_{tr}$ during the time period $\overset{\cup \pm}{T}_{tr}$ in the normal mode before the critical moment T_{cr} of an abnormal mode becomes an emergency situation or a catastrophe. Here, the mode $\overset{\cup +}{R}_{tr}$ is a controlled functioning mode conditioned by the control influence U_{tr} of a safety control system. During the time period $\overset{\cup \pm}{T}_{tr}$ this mode leads to the reduction of the abnormal mode R_{os} to the normal mode R_{sd}. The mode $\overset{\cup +}{R}_{tr}$ is characterized by the following functional:

$$\overset{\cup +}{R}_{tr}\colon R_{os} \overset{U_{tr}}{\rightarrow} R_{sd} , \tag{8.20}$$

which defines the process of the reduction of the abnormal mode R_{os} to the normal mode R_{sd} under the influence of the control system U_{tr}.

The main system property is an operational capability characterized by given quality indicators defined by the set Y.

We will consider the ***notion "system safety"***. Here, this means an ability to timely prevent a consecutive transfer from a normal mode to an accident or a catastrophe on the basis of timely detection of essential risk factors and elimination of the possibility of their conversion into catastrophic risk factors. Safety is characterized by the following indicators: degree of risk η_i and risk level W_i; the reserve T_{ar} of permissible risk of an abnormal mode; the reserve T_{as} of permissible risk of an accident; and the reserve T_{ds} of permissible risk of a catastrophe.

The quantitative values of safety indicators are defined on the basis of the general problem of multifactor risk analysis, whose mathematical definition is described in this chapter.

Strategy for solving the problem of system control of complex objects. First, note the principal differences between the given problem and typical control problems. The main difference is that the initial information about a complex object contains only a small part of information about its state, properties, functioning processes, and operational capability characteristics. This information represents only the state and work characteristics of such objects in normal mode. Undoubtedly, this information is enough for decision making during the complex object control only on the condition that the normal mode continue for a long time. However, in real objects in view of existing technical diagnosis systems, oriented toward failure and malfunction detection, it is impossible to ensure that a malfunction or a failure will not appear within the next 5–10 min. It is a priori unknown how much time it will take to repair a malfunction. It may take from a few minutes up to several hours or even days and months. And, consequently, the possible damage is a priori unknown, and thus the safety control system is, essentially, a recorder of information about facts and damage.

A fundamentally different approach can be realized on the basis of the system control of complex objects. The essence of such control is systemically coordinated evaluation and adjustment of the operational capability and safety during the functioning process of an object.

The general strategy of such an approach is shown in Fig. 8.1. The "?" symbol means that there is incomplete, fuzzy information about the object functioning state at the moment $T_k \in T^{\pm}$. This information is not enough for decision making. This implies the significant property of such an approach. This property means that the situation analysis and decision making are provided not only in typical conditions of exact recognition of a normal or an abnormal system mode, but also in conditions where there is only fuzzy, incomplete information about a situation. It is significant that this approach, in conditions of fuzzy information about a situation, allows one, if necessary, to make a timely decision on emergency stop of the system operation. In the following control strategy in blocks 1–3 there are realized procedures of com-

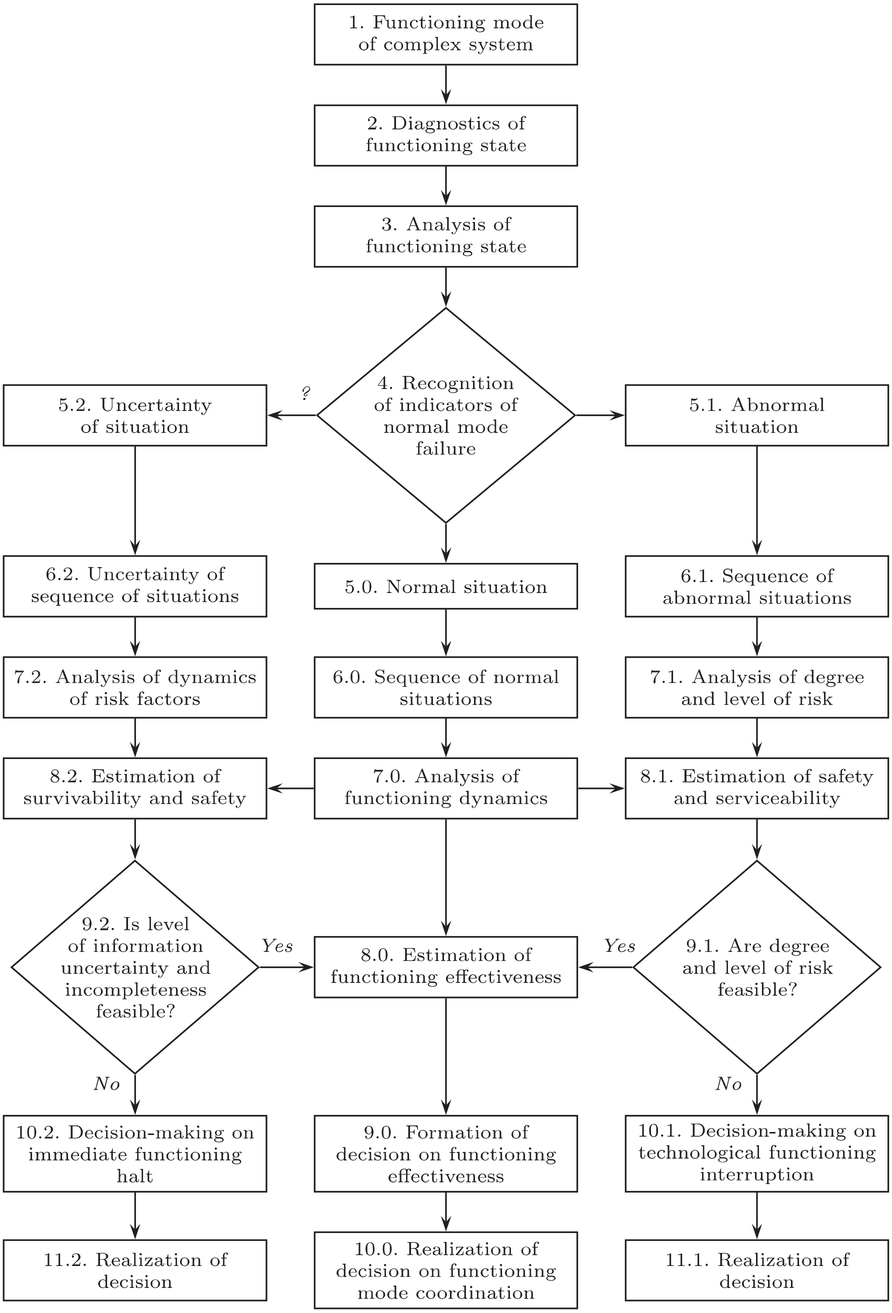

Fig. 8.1. System control strategy of a complex objects' capability and safety

plex object functioning diagnostics and analysis. In block 4, on the basis of the results of the execution of the procedures of blocks 2 and 3, the recognition of a normal functioning mode occurs. During this process, three possible variants of a complex object state are analyzed: the normal functioning mode remains (transition of a control to block 5.0); signs of a normal mode violation appears that make it possible to reveal that at time $T_k \in T^{\pm}$ the situation is abnormal (transition of a control to block 5.1); or at time $T_k \in T^{\pm}$ the situation becomes undefined (transition of a control to block 5.2).

In the first variant the system operation in a normal mode, and quality control is executed (blocks 6.0–10.0).

In the second variant on the basis of the sequence of abnormal situations the following actions are realized. The risk degree and level of an abnormal-situation sequence is analyzed, and the safety and operational capability of complex objects are evaluated (blocks 5.1–9.1), and a decision is made regarding the scheduled stop of the complex object functioning (transition of a control to blocks 10.1 and 11.1) or a decision is made regarding the continuation of the complex object functioning if the values of risk degree and level are acceptable (transition of a control to blocks 8.0–10.0).

In the third variant, an evaluation of survivability and safety of the system in conditions of uncertain information about abnormal situations is made. For this, the following sequence of operations is realized. An analysis is made of risk factors sequence of abnormal situations, on the basis of which the complex object survivability and safety are evaluated (blocks 5.2–9.2). If a certain uncertainty level and incompleteness level are acceptable, then the decision about the continuing functioning of an object is made (transition of a control to blocks 8.0–10.0). Otherwise, the decision on emergency stop of an object functioning is made (transition of a control to blocks 10.2 and 11.2).

In the following figures we will consider in detail the developed algorithm for system control of complex objects operational capability and safety in conditions of multifactor risks. Let us firstly consider the structure scheme of an algorithm of safety control in abnormal situations (Fig. 8.2), which details procedures of blocks 5.1–9.1 (Fig. 8.1). In this scheme blocks 2.0 and 3.0 realize diagnostic procedures and evaluation of abnormal situations during the process of transition of a complex system normal functioning mode into a sequence of abnormal situations. Using the results of these procedures a database and a scenario of a sequence of abnormal situations is formed (blocks 4.0 and 6.0). The obtained information is used for making a decision about the following actions (block 7). In this block the possibility of a transition of a complex object from abnormal situations to a normal mode is determined.

Three variants are realized: the transition is possible (variant 1); the transition is impossible (variant 2); there is not enough information about abnormal situations to make a decision (variant 3). If the transition to a normal mode is possible, then a procedure for evaluating an object's survivability

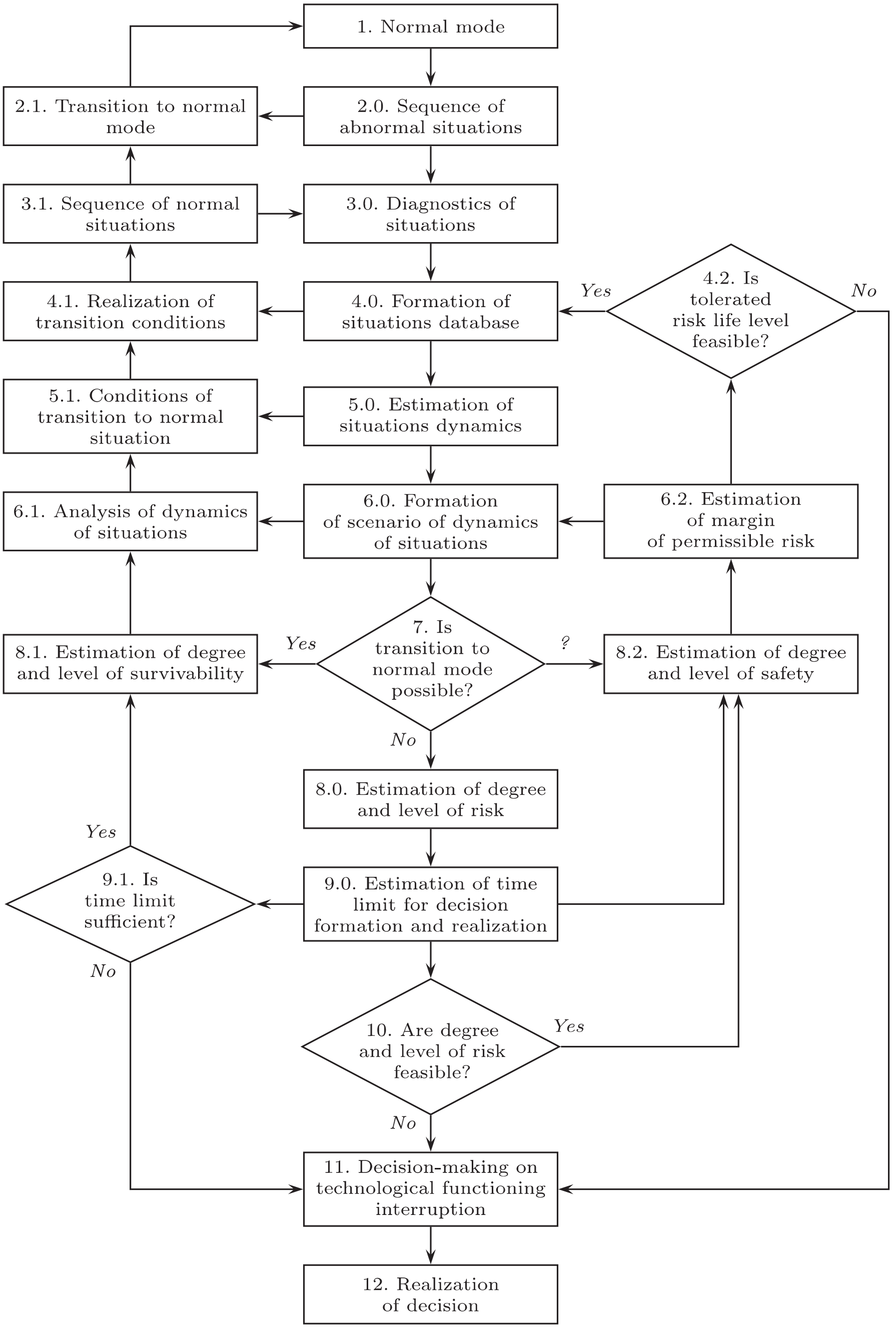

Fig. 8.2. Structure scheme of an algorithm of a complex object safety control in abnormal situations

degree and level is executed (block 8.1). If the transition to a normal mode is impossible, then a procedure for evaluating the risk degree and level of an abnormal situation sequence is executed (block 8.0). If there is not enough information about abnormal situations for making a decision about the possibility or impossibility of a transition to a normal mode, then the procedure for evaluating the safety degree and level of an object is executed (block 8.2).

The following actions of a control system in conditions of abnormal situations are oriented toward the elimination of the possibility of an accident or a catastrophe. According to variant 1, measures are taken for the transition of abnormal situations to a normal mode on the basis of step-by-step execution of procedures that are defined in blocks 9.1 to 2.1. According to variant 2 measures are taken to evaluate the risk degree and level and the time limit for forming and realizing a decision on scheduled stop of the object operation. This evaluation is made following the step-by-step execution of procedures described in blocks 9.0 and 10–12. According to variant 3 measures are taken to evaluate the safety degree and level, margin of permissible risk, time limit on forming and realizing a decision about scheduled stop of the object operation. This evaluation is made following the step-by-step execution of procedures described in blocks 9.0 and 8.2–4.2. All variants are oriented toward accident prevention before the moment T_{cr}.

Then let us consider the structure of a knowledge base and a database (Fig. 8.3). This structure is the detailed content of block 4.0 of a safety-control algorithm (Fig. 8.2) and blocks 7.0–9.0, 7.1–9.1, and 7.2–9.2 of an operational-capability and safety-control algorithm (Fig. 8.1).

Let us note the most important properties and features of this structure. It provides the forming, accumulation, and usage of three types of information in the system control of operational capability and safety. These three types are a priori knowledge, current diagnostic information, and information about complex-object control results. A priori knowledge is formed by system 1, which consists of a casual knowledge base, an empirical knowledge base, an experimental knowledge base, and an expert knowledge base. This knowledge is used in blocks 4.0, 5.0, 6.0, 6.1–3.1, and 8.0–8.2 of the safety-control algorithm (Fig. 8.2) and is especially demanded when an abnormal situation first emerges because it provides the possibility of comparing processes under way with known analogs and prototypes.

Real-time diagnostic information is formed by system 2 during the process of system object functioning both in a normal mode and in conditions of abnormal situations. Diagnostic information displays the state of technological processes, technological mechanisms and constructions, technical control means, processes of complex object operational capability, and safety control. Diagnostic results are used in blocks 4.0, 5.0, 6.0, 6.1–3.1, and 8.0–8.2 of the safety control algorithm (Fig. 8.2) and are especially demanded in abnormal situations, providing not only the possibility of analysis of processes that are under way, but also margins of permissible risk, and a time limit on forming

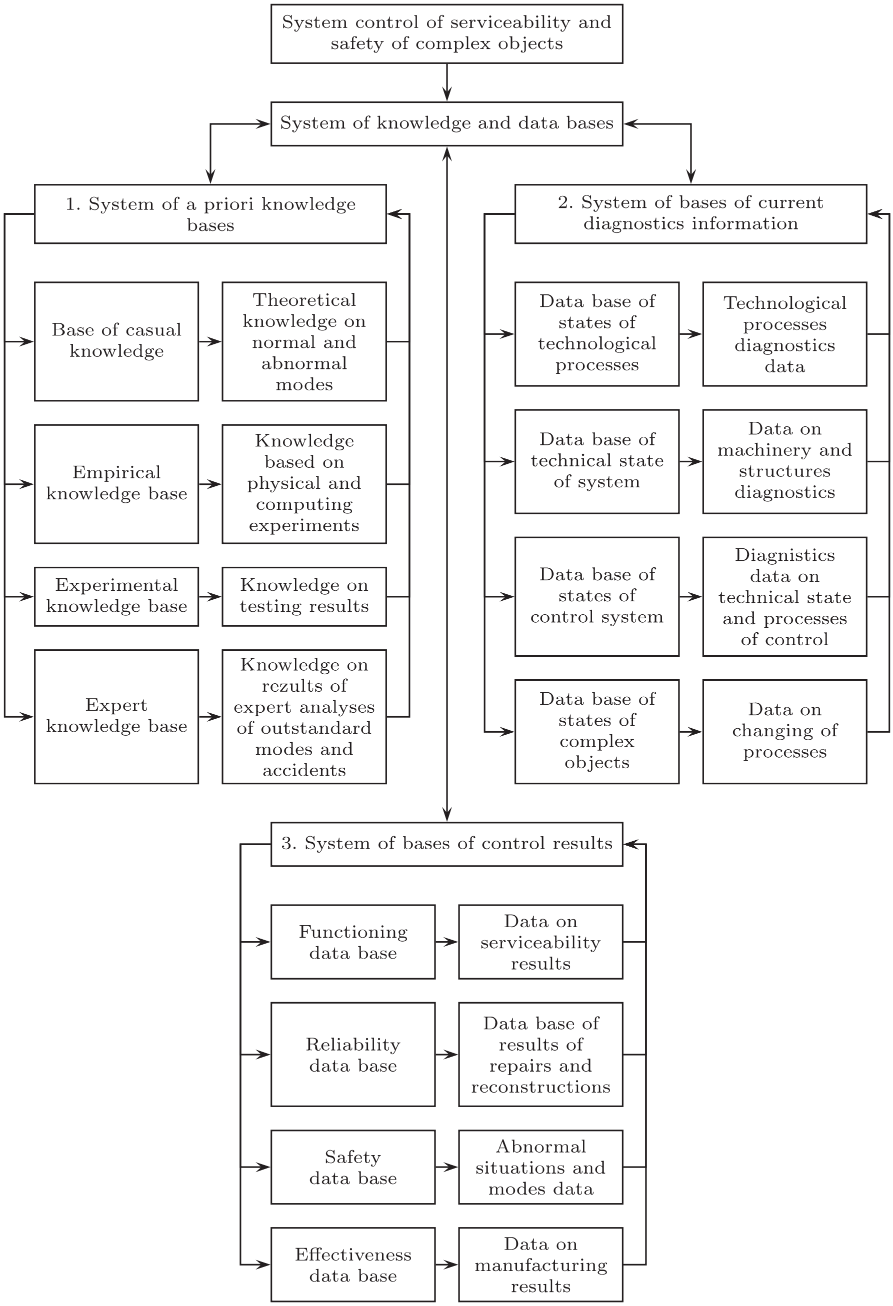

Fig. 8.3. Structure of a knowledge base and a database for the system control of the operational capability and safety of a complex object

and realizing a decision about schedulled stop of an object operation. A measurement discreteness may essentially vary during diagnosis depending on changing of abnormal situation. The purpose of such variation is, firstly, to reduce the time of schedulled stop of the object functioning, and, secondly, to create conditions to timely realize this procedure before the moment T_{cr}.

System 3 forms information about the results of object control. Information represents the results of an object's operational capability, of reconstructions and repairs, of actions in conditions of abnormal situations, and of manufacture. This information also represents the effectiveness of different functioning processes, the specificity of different planned preventive and emergency repair tasks, and also other features of a specific object.

Information obtained from systems 1–3 is used in a system-control algorithm of an object's operational capability and safety (Fig. 8.1) and is the basis for making a decision in the correction of object functioning processes in normal mode, and for making a decision of safety control for the purpose of accident and catastrophe prevention.

However, we will notice that the presence of the aforementioned information is a necessary, but insufficient, condition for accident and catastrophe prevention in abnormal situations. Practical experience demonstrates that abnormal situations during some period of time may become accidents and catastrophes if adequate measures are not taken. Thus, it is necessary to provide such control system characteristics that will guarantee a timely transition of an abnormal situation into a normal one before the moment T_{cr}, or a timely schedulled stop of an object functioning. For this it is necessary to provide on the basis of the aforementioned information a timely solution of a sequence of risk analysis tasks during control of the complex objects operational capability and safety. The list of these tasks is shown in a scheme of interrelations between risk analysis tasks during the complex object's control process (Fig. 8.4).

In this scheme the main attention is focused on specific safety tasks and objectives. The main objective is to prevent possible accidents and catastrophic situations before the moment T_{cr}. The main tasks are reliable evaluation and online forecasting of the dynamic of main safety indicators in different risk situations for the purpose of forming, making, and realizing a decision in a timely manner.

To achieve the stated objectives two task groups are solved at the same time: ***analysis of degree and level of risk of different abnormal situations and analysis of margin of permissible risk analysis***. The initial data are the results of a solution of tasks of detecting signs of abnormal situations and the results of risk situation recognition (blocks 1 and 2). Then there follow tasks of a risk category evaluation and a risk change evaluation (blocks 3.1 and 3.2), an evaluation of risk degree and level and a margin of permissible risk evaluation (blocks 5.1 and 5.2; blocks 8.1 and 8.2), ranking of problems by degree of importance and level of complexity (blocks 10.1

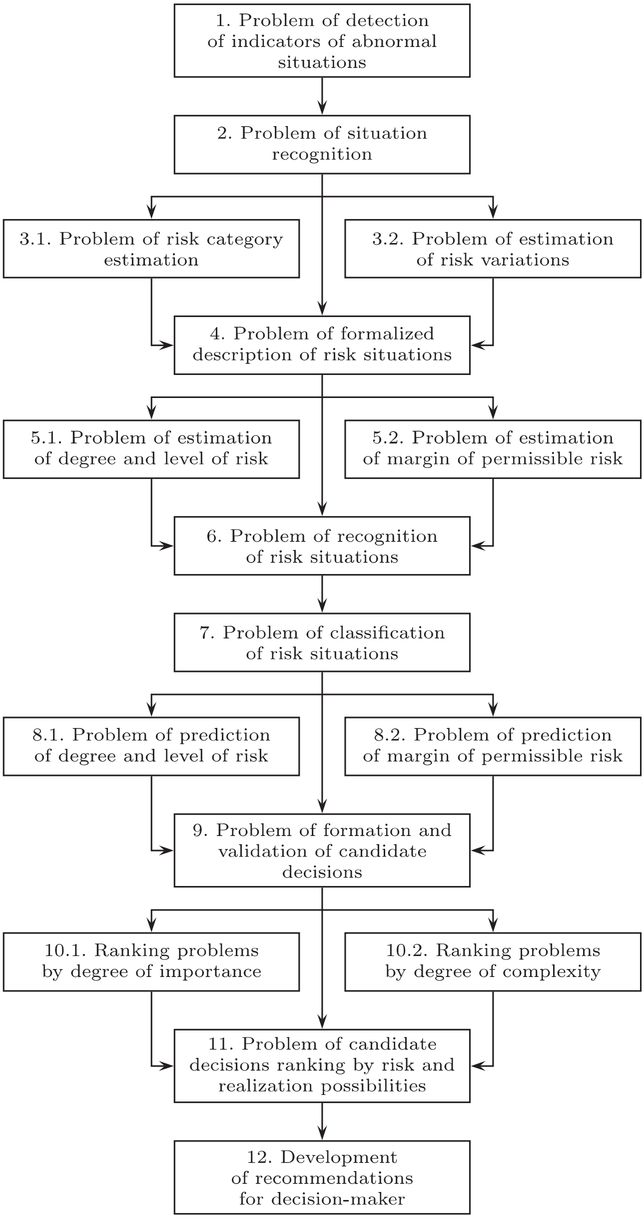

Fig. 8.4. Interrelations between risk analysis tasks during complex object's control process

and 10.2). In conclusion the following tasks are solved: the task of ranking of solution variants by risk and possibility of realization and the task of developing recommendations for the decision maker (blocks 11 and 12). The other tasks (blocks 4, 6, 7, and 9) are used by way of "supporting points", in which the results of the aforementioned task solutions are synthesized and generalized.

8.3 System Control of Complex-Object Structure and Properties

Let us proceed to examine the properties and features of other classes of control problems. First of all, it is necessary to discuss factors that determine the practical necessity of creating and using more complex classes of object control problems.

The problems considered earlier related to operational capability and safety control include many problems of complex object control in conditions of normal functioning mode. But at the same time the given class of control problems has limited capabilities. These limitations follow from the mathematical definition of an optimal control problem, where the number of practically significant factors are not taken into account, while these factors are related to real conditions of modern complex system functioning. In particular, real interactions between an object and its environment are not fully taken into account. For example, such important features as incompleteness, uncertainty, and inaccuracy of initial information, information lag, control lag, and the non-Markovian process of an object state change.

These disadvantages are taken into account in classes of control problems of higher levels, particularly in problems connected with the necessity of complex object properties and structure control. First of all, let us consider some techniques for eliminating incompleteness, uncertainty, and inaccuracy of initial information in control systems. In real conditions of designing the system of complex object control the insufficient level of decision maker awareness may be caused by different factors. The most typical situations are as follows.

Situation 1. During the control system design it may be found that properties and indicators of external influences on a controlled object are partially or completely unknown, and thus many indicators of a control system are virtually unknown. Such situations were typical in various spacecraft. Particularly, in Moon-buggy research information about an environment was virtually missing. There was only one property known: there is no atmosphere on the Moon. But there was no important information about the Moon's ground mechanical, physical, chemical, and other properties and indicators. In such conditions the demands made on a control system are principally different with the demands accepted in developing class C1 control systems.

In particular, the control system should meet the principally new requirement that is missing in class C1 control systems. The essence of this requirement is as follows: During a functioning process the control system should provide missing information about an environment and, using it, realize a decision about appropriate changes in a controlled object's parameters, properties, and structure. Therefore, it is required that the control system, during the functioning process, be able to perform control functions of class C1 (the behavior control), class C2 (the properties control), and class C3 (the structure control). During this the procedures for making changes to properties and structure may be single and multiple.

In the case of realization of a single procedure, the control system works in the following mode: missing information is provided, correction of properties and structure is carried out. Such a procedure is executed on conditions that environmental characteristics are almost constant while a controlled object is in use. This variant of a procedure is typical, particularly in cross-country vehicle usage in harsh climatic conditions (swamp, sand, firm soil, viscous soil, etc.). An example would be use of a cross-country vehicle in different seasons in transpolar terrain.

Situation 2. There is sufficient initial information about environmental properties and a controlled object for the creation of a control system. However, while a controlled object is in operation, environmental or object properties may change dramatically as a result of the influence of various factors. For example, in an aircraft during flight its mass changes because of fuel consumption, and thus the center of mass changes its position. At the same time as flying altitude changes, air density changes as does aerodynamics. The mentioned object property changes and nonstationary turbulent atmosphere processes may cause the control system, designed on the basis of initial information, will not provide required qualitative indicators of an object in operation during changes to controlled object properties and the environment. In such conditions there appears the necessity of providing continuous change in the properties and structure of the object that will be adequate to changes in external influences. Such a control system should ensure the presence of the required qualitative indicators of objects in operation at any moment of a current situation.

Situation 3. This situation is a generalization of two previous ones and is characterized by insufficient awareness about external conditions and factors at the stage of development of a controlled object, and also by the possibility of unpredictable, nonstationary changes in external conditions while an object is in operation.

The general property of the situations considered earlier is the necessity of adapting to new conditions while the situations are changing. Such accommodation is realized in different ways: by changing only the properties, by changing only the structure, or by changing both the structure and properties of a controlled object. The ability of a nonstationary system to adapt to environmental changes or object characteristic is called ***adaptation***. System that possess the ability to adapt to conditions change are called ***adaptive***.

Here, by nonstationary system we mean an entire object that consists of a controlled object and a control system that are structurally interconnected and functionally interacting for the sake of specified goals.

Thus, the above brief analysis shows that in real conditions of incompleteness, inaccuracy, and contradictoriness of initial information the effective functioning of modern complex systems is possible only if those systems are able to adapt to changes as they occur. Adaptation is possible only given a necessary level of informedness about external environmental properties and controlled object properties while in operation. It follows that adaptation and optimization problems are narrowly interconnected with problems of adaptive optimal information processing while a current situation is undergoing changes. Such problems are outside the scope of approaches of control theory class C1. The given above stimulated the evolution of different tools and methods of adaptive control and promoted the birth of methods of control classes C2 and C3.

At the same time, the continuous increase in the volume and level of demands on modern manufacturing makes for fundamentally new, essentially more complex theoretical and practical control problems. Only the first steps have been made toward the theoretical investigation of more actual control problems of classes C4 and C5. However, practical problems of class C4, particularly problems of complex multilayer, multipurpose system evolution, have been solved in many respects intuitively and empirically for the last few decades.

Class C5 control problems are the next step in the development of class C4 problems. This complication lies in the fact that, during the controlled object evolution its purpose is changed, and thus object goals are changed. In such conditions the problem becomes more difficult than developing the same object. In fact, the developer of a new object may choose the form, structure, functions, and elements of each hierarchical level and the object as a whole. In the case of purpose control, the situation in which the problem is solved is fundamentally different. The object already exists, already has established technologies, an established system of production supply and sale, and also an existing collective of factory and office workers with specific knowledge of practice in a specific environment and a specific vocational training. The main object property, its purpose, must be changed, but at the same time one must keep as much as possible of the existing manufacturing infrastructure and maintain its maximal effective usage in the new scope of practical activity. It is evident that providing an effective and timely solution to organizational, technological, economic, scientific and technical, social, and other problems that arise in such situations is possible only in the presence of their system coordination by objectives, tasks, terms, resources, and expected results and in the presence of multilayer control.

To understand all the complexity of a solution of system control problems of the C3, C4, and C5 classes we will briefly consider a class C3 problem.

This problem belongs to a group of simplest, but has necessary practical significance in real conditions of usage of technologically and ecologically dangerous objects. It is solved at the stage of creation and testing of control systems of complex hierarchical objects.

The formulation of a problem of a rational control of a hierarchical system in conditions of multifactor risks is based on the results of structural-functional analysis, considered in Chap. 6. We will cite a substantial formulation and mathematical definition of a considered problem.

Substantial problem formulation. For a complex hierarchical system there ***are defined*** structures of all levels and project decisions about functional elements (FEs) of all hierarchical levels in accordance with preliminary specifications. There are known requirements for main properties and indicators of control of system functioning processes in fuzzy and incompletely specified conditions.

It is required to find the form and structure of a control system, to develop project decisions about FEs of all hierarchical levels based on conditions of achievement of a required system control quality in normal, abnormal, and critical situations.

Mathematical problem definition. For a complex multilayer system there is a ***specified*** structural interconnection between FEs of different hierarchical levels in the following form:

$$f_q : \; V_Q \to \bigcup_{q=1}^{Q} V_q; \quad f_{qp} : V_q \to \bigcup_{p=1}^{P_0} V_{qp} \,, \tag{8.21}$$

$$f_{qp}^{-1} : \; \bigcup_{p=1}^{P_0} V_{qp} \to V_q; \quad f_q^{-1} : \bigcup_{q=1}^{Q} V_q \to V_Q \,, \tag{8.22}$$

where V_Q is a set of FEs of an object as a whole, V_q is a set of FEs of the qth hierarchical level, and V_{qp} is the pth FE of the qth hierarchical level. The functional interconnection of indicators of an entire object's functioning quality with FE parameters is defined in the form of a function with monotonous variables including:

$$\begin{aligned}&\bar{K}_Q \\ &= \tilde{F}_Q\left(\bar{X}_Q, \tilde{F}_{Q-1}\left(\bar{X}_{Q-1}, \tilde{F}_{Q-2}\left(\ldots \tilde{F}_l\left(\bar{X}_l, \tilde{F}_{l-1}\left(\ldots \tilde{F}_1\left(\bar{X}_1 \ldots\right)\right) \ldots\right)\right)\right)\right)\,,\end{aligned} \tag{8.23}$$

where $\bar{K}_Q$ is a quality indicator vector, $\bar{X}_Q$ is a vector of FE parameters of a higher level of a hierarchical structure (level of an object as a whole), $\bar{X}_q$ is a vector of FE parameters of the qth hierarchical level, $q = 1, 2, \ldots, l, \ldots, Q$, $\bar{X}_1$ is a vector of FE parameters of a lower hierarchical level (for example, the module level), $\tilde{F}_Q, \ldots, \tilde{F}_l, \ldots, \tilde{F}_1$ are functions of interconnection between indicators of FE of different hierarchical levels. These functions are specified inaccurately, fuzzily, and incompletely.

It is required to build a sequence of transformations

$$\Phi_q : \ W_Q \to \bigcup_{q=1}^{Q_W} W_q; \quad \Phi_{qp} : W_q \to \bigcup_{p=1}^{P_W} W_{qp} , \tag{8.24}$$

$$\Phi_{qp}^{-1} : \ \bigcup_{p=1}^{P_W} W_{qp} \to W_q; \quad \Phi_q^{-1} : \bigcup_{q=1}^{Q_W} W_q \to W_Q , \tag{8.25}$$

that define the structure of a control system and parameters of its FEs from the condition of achievement of a necessary quality level of an object's functioning at any time $t \in [0; T]$ in predictable risk situations $S_k \in S_0, k = \overline{1, N}$.

In this formulation we will draw attention to the interconnection between reference (8.21) and (8.22), and also (8.24) and (8.25). Naturally, there arises the question as to why it is necessary to have two kinds of interconnection between FEs of a hierarchical structure. The first kind of interconnection is defined by references (8.21) and (8.24), and the second is defined by references (8.22) and (8.25). The practical necessity follows from real conditions and techniques of complex system development. The point is that in preliminary development specifications there are specified requirements for an object as a whole. The customer is not interested in what structure and what FEs will be used to create an object. He is interested in seeing that the object meets the specified requirements. At the same time, the designer should efficiently transform general requirements for an object into requirements for a specific FE of each hierarchical level of a designed system. For this purpose it is necessary to have a description of the interconnection between FEs in the form (8.21). Therefore, references (8.21) are the basis for a decomposition procedure realization.

The necessity of an interconnection between FEs in the form (8.22) follows from the technology of a real complex hierarchical system design. System projection begins from the development of the FEs of the lowest level (modules or components); then higher-level FEs are designed, and then the final object composition is designed. For example, for radio systems designed for different purposes the design procedure is performed in the following sequence: functional module (chip) $\mapsto$ functional block (high-frequency amplifier) $\mapsto$ functional unit (receiver) $\mapsto$ functional system (receiving chain as a system of N receivers, N receiving antennas, and other functional units) $\mapsto$ object as a whole (radio-relay station). Consequently, formula (8.22) is the basis for an aggregation procedure realization in a system analysis.

For the control system, relation (8.24) is similar to the formula (8.21) in purpose, and relation (8.25) to relation (8.22). Note also that formulas (8.21) and (8.22) define a structural interconnection between the FEs of an object, and formula (8.23) defines a functional interconnection between FEs through the quality indicators.

General solution strategy. The fundamental feature of the given problem is that it possess at the same time properties of a structural-functional analysis problem (control system structure selection) and a multifactor-risk system analysis problem (analysis and minimization of risk situations, ensuring the required system quality in specified risk situations). This feature should be taken into account while forming a solution strategy. First of all, we will consider that this problem in tasks related to the selection of a system structure and its FE parameter selection is similar to a structural-functional analysis problem, whose solution techniques were given above. Thus, we will consider the strategy of a control influence optimization in conditions of multifactor risks.

We will assume relation (8.23) as a basis. It defines the interconnection between object quality indicators $\bar{K}_Q$ and FE parameters $\bar{X}_q$, $q = 1, 2, \ldots, l, \ldots, Q$ of all hierarchical levels. The given relation must be transformed to take into account risk factors and control influence. For this we will represent the relation between the quality-indicator vector and the FE parameter vector, the risk-factor vector, and the control-influence vector in the generalized form:

$$\bar{K}_Q = \tilde{F}_Q\left(\bar{\chi}_Q, \tilde{F}_{Q-1}\left(\bar{\chi}_{Q-1}, \tilde{F}_{Q-2}\left(\ldots \tilde{F}_l\left(\bar{\chi}_{l-1}\left(\ldots \tilde{F}_1\left(\bar{\chi}_1\right)\ldots\right)\ldots\right)\ldots\right)\right)\right), \tag{8.26}$$

where $\bar{\chi}_q$ is a tuple for the qth hierarchical level of a complex hierarchical system, which is defined by the relation:

$$\bar{\chi}_q = \left\langle \bar{X}_q, \overline{u}_q, \bar{\rho}_q \right\rangle . \tag{8.27}$$

Here $\bar{X}_q$ is a FE parameter vector of the qth hierarchical level, $\bar{u}_q$ is a control-influence vector, and $\bar{\rho}_q$ is a risk-factor vector.

We will take into account that all variables, defined by relation (8.27), are time functions and are represented at the time moment $t = t_r$ in the form

$$\begin{aligned} \bar{X}_q^r &= \bar{X}_q\left(t_r\right) , \\ \bar{u}_q^r &= \bar{u}_q\left(t_r\right), \quad \bar{\rho}_q^r = \bar{\rho}_q\left(t_r\right) . \end{aligned} \tag{8.28}$$

In the general case the complex system control problem, both as a problem of object control as a whole and as a problem of its qth hierarchical level control, lies in the simultaneous fulfillment of two requirements:

1. To provide a stable meaning of specified quality indicators $\bar{K}_{1q}$ at any fixed time moment $t_r \in [0, T]$.
2. To ensure time changes of certin quality indicators $\bar{K}_{2q}$ by set requirements.

The first requirement in the ideal case means $\bar{K}_{1q} = \text{const}\ \forall t_r \in [0, T]$. In real conditions one is allowed to make designated changes to indicators under the influence of disturbing risk factors on the qth level and a correcting influence in a specified interval. The given requirement is defined by the relation:

$$\bar{K}_{1q}\left(\bar{u}_q^r, \bar{\rho}_q^r, t_r\right) \in \left[\bar{K}_{1q}^+, \bar{K}_{1q}^-\right] \quad \forall t_r \in [0, T], \quad q = \overline{1, Q}, \tag{8.29}$$

where $\bar{K}_{1q}^+$ and $\bar{K}_{1q}^-$ are predefined values. The second requirement in the ideal case means that every component of vector $\bar{K}_{2q}$ is a specified function of time. In real conditions under the influence of disturbing risk factors and a programmed control influence at any time moment $t_r \in [0, T]$ one must meet the following condition:

$$\bar{K}_{2q}\left(\bar{u}_q^r, \bar{\rho}_q^r, t_r\right) \in \left[\bar{K}_{2q}^+(t_r), \bar{K}_{2q}^-(t_r)\right] \quad \forall t_r \in [0, T], \quad q = \overline{1, Q}. \tag{8.30}$$

Here $\bar{K}_{2q}^+(t_r)$ and $\bar{K}_{2q}^-(t_r)$ are, respectively, specified upper and lower boundaries of an allowed variation range of a function $\bar{K}_{2q}(t_r)$ at the moment t_r.

To justify the strategy we will describe in detail the control procedure using a bilevel system as an example. In this case quality indicators $\bar{K}_Q$ are defined by the relation:

$$\bar{K}_Q = \tilde{F}_Q\left(\bar{\chi}_Q, \tilde{F}_1(\bar{\chi}_1)\right). \tag{8.31}$$

Here on the basis of (8.27) for the lower level $q = 1$ and upper level $q = Q$ we have:

$$\bar{\chi}_Q = \left\{\bar{X}_Q, \bar{u}_Q, \bar{\rho}_Q\right\}; \quad \bar{\chi}_1 = \left\{\bar{X}_1, \bar{u}_1, \bar{\rho}_1\right\}. \tag{8.32}$$

Now we specify control $\bar{u}_Q^*$ and $\bar{u}_1^*$ from condition (8.30), assuming quality indicators and parameter behavior in time is known, based on a structure-functional analysis problem solution, in the form:

$$\bar{K}_Q = \bar{K}_Q^*(t), \quad \bar{K}_1 = \bar{K}_1^*(t), \tag{8.33}$$

$$\bar{X}_Q = \bar{X}_Q^*(t), \quad \bar{X}_1 = \bar{X}_1^*(t). \tag{8.34}$$

As an initial solution we will assume a solution that is found without taking into account risk factors. Then, $\overline{\rho}_Q = 0, \overline{\rho}_1 = 0$. In this case from (8.31) and taking into account (8.32) we get:

$$\bar{K}_Q = \tilde{F}_Q\left(\bar{X}_Q, \bar{u}_Q, \tilde{F}_1\left(\bar{X}_1, \bar{u}_1\right)\right). \tag{8.35}$$

We will define a discrete series on the interval $[0, T]$:

$$t_i = t_0 + i\Delta t; \quad \Delta t = \frac{T}{M_0}; \quad i = \overline{0, M}_0; \quad t_0 = 0; \quad t_{M_0} = T. \tag{8.36}$$

Then the problem of a structure control definition is reduced to an equality $\bar{K}_Q = K_Q^*(t)$ ensuring at any time moment t_i [from relations (8.35) and (8.36)]. As a result we get the following system of equations:

$$\tilde{F}_Q\left(\bar{X}_Q(t_i), \bar{u}_Q(t_i), \tilde{F}_1\left(\bar{X}_1(t_i), \bar{u}_1(t_i)\right)\right) = K_Q^*(t_i), \quad i = \overline{0, M_0}. \quad (8.37)$$

Here, components of control vectors $\bar{u}_Q(t_i)$ and $\bar{u}_1(t_i)$ are unknown, but other variables are known, because their corresponding functions are defined in a problem of structural-functional analysis.

The system (8.37) solution defines discrete values of desired control functions at points specified in (8.36). Based on a solution to system (8.37) it is possible to build continuous functions $\bar{u}_Q(t_i)$ and $\bar{u}_1(t_i)$ using approximation methods. In the system of equations (8.37) the number of components of vectors $\bar{X}_Q, \bar{X}_1$ and $\bar{u}_Q, \bar{u}_1$ is specified. So, the number of unknown variables N is equal to the total number of components of vectors $\bar{u}_Q$ and $\bar{u}_1$. The number of equations M is specified by a quantity of discrete values in (8.34) and is equal to $M = M_0 + 1$. Because the quantity of discrete values is selected during the problem solution, it is necessary to take into account the following features:

1. It is expedient to choose the number of equations $M \geq N$, i.e., equal or greater than a number of unknown variables.
2. With increasing M the approximation error for functions $\bar{u}_Q(t)$ and $\bar{u}_1(t)$, based on discrete values $\bar{u}_Q(t_i)$ and $\bar{u}_1(t_i)$, $i = \overline{0, M_0}$ will decrease. But system (8.37)'s computational complexity will increase.

The system (8.37) solution selection method depends on the problem dimension, the ratio between M and N, and features of functions in (8.37). Further, based on (8.26) it is necessary to take into account risk factors $\bar{\rho}_Q(t)$ and $\bar{\rho}_1(t)$ in function (8.35) and to reduce the task of control-function improvement to one of well-known forms of a problem of disclosing situation uncertainty or to a problem of minimization of degree and level of multiple factor risks.

Determination of the controlling functions in cases where the total number of levels $Q > 2$, may be reduced to a sequence of tasks for a bilevel system or directly reduced to a system like (8.37), but with the number of variables equal to the total number of components of all control vector functions $\bar{u}_q(t)$, $q = \overline{1, Q}$.

Thus the strategy of complex hierarchical system control for which the structural-functional analysis problem is solved may be reduced to the following sequence of tasks:

1. Solution of a system of nonlinear equations (like (8.37)).
2. Approximation of the desired functions based on their discrete values of previous problem solution results.

3. Taking into account risk factors based on methods of disclosing situational uncertainty or reduction to a multifactor risk analysis problem.
4. Development of design solutions of control FE.

In practice it is not always possible to obtain an acceptable solution to a system control problem as a result of a single iteration. Derived control functions may require FE structure corrections and function corrections for some hierarchical levels of an object or its control system. This will require the next iteration execution in an object's structural-functional analysis problem and, consequently, the solution correction in control problems of an object under investigation.

Solutions of a structural-functional analysis problem and a control problem allow one to proceed to the next problem, which lies in a technical-economical analysis of the functionality and operational capability of a complex object.

8.4 Technical-Economical Analysis of Complex Object System Control

The technical-economical analysis is the final problem in a sequence of system problems of researching the properties, structure, serviceability, and control of complex hierarchical systems under conditions of multifactor risks.

In previous sections we considered problems whose solution defined the shape and the structure of an object and its control system, as well as design solutions regarding FEs of all hierarchical levels. The considered problem should give an answer to the following question: how rationally the previous problems have been solved by comparing the gained effect and expenditures of resources for its achievement. The practical importance of the problem is obvious—only a comparison of the cost and efficiency of solutions makes it possible to estimate such major characteristics of the developed systems as the technical and economic level of complex object production or levels of product efficiency and competitiveness.

The problem statement of technical and economic analysis of system control in conditions of multifactor risks is determined by the general purpose of system research of the given class of complex objects in the process of their design, production, operation, and control in real conditions of multifactor risks.

Contensive statement of problem. Structure and design solutions regarding the FEs of all levels of a complex hierarchical system ***are assigned***, properties and quality indicators of an object ***are defined***, the structure and design solutions of a multilevel control system of an object are defined, and the design solutions and qualitative indicators of functional elements of a control system ***are known***.

It is required to determine the technical and economic efficiency of the system control of an object under real conditions of multifactor risks.

The mathematical statement of the problem follows from the general functional interconnection of indicators of an object quality as a whole and parameters of FEs of all hierarchical levels, risk factors and control actions of a control system, as well as interconnections of the cost of component parts, custom-designed modules, and structures of FEs of all hierarchical levels with technical indicators of the design solutions.

Mathematical statement of problem. For complex multilevel hierarchical systems the interconnections between the quality indicators and an object's FE parametrs and the technical means of a control system **are given** in the form:

$$\bar{K}_Q = \tilde{F}_Q\left(\bar{\chi}_Q, \tilde{F}_{Q-1}\left(\bar{\chi}_{Q-1}, \tilde{F}_{Q-2}\left(\ldots \tilde{F}_l\left(\bar{\chi}_{l-1}\left(\ldots \tilde{F}_1\left(\bar{\chi}_1\right)\ldots\right)\ldots\right)\ldots\right)\right)\right), \tag{8.38}$$

where $\bar{\chi}_q$ is a tuple for the qth hierarchical level $(q = 1, 2, \ldots, l, \ldots, Q)$ of the complex system, which is defined by the relationship:

$$\bar{\chi}_q = \left\langle \bar{X}_q, \bar{u}_q, \bar{\rho}_q \right\rangle .$$

Here $\bar{X}_q$ is a vector of parameters of FEs of the qth hierarchical level, $\bar{u}_q$ is a vector of controls, and $\bar{\rho}_q$ is a vector of risk factors.

It is required to establish the indicators that will allow one to determine in a generalized form the effectiveness of the object's operation; to determine their dependence on FE parameters; to find the interconnection of the total cost of the designed product and the technical, structural, and technological parameters of FEs of each qth hierarchical level of an object itself and its control system; to determine the technical and economic efficiency of an object's operation as a whole in conditions of multifactor risks.

General strategy of problem solution. The general strategy of estimation of technical and economic efficiency of system control of complex objects in conditions of multifactor risks is based on the following basic regulations:

– Techniques and methods of value analysis.
– Techniques and methods of control of complex-object functionality.
– Techniques and methods of analysis and minimization of risks.
– Techniques and methods of system control of safety and serviceability.

We assume that there is no need to deal with the principles, techniques, and methods of value analysis since they are considered in detail in special subdisciplines of economics (microeconomics, economic productive efficiency, economic design calculations, etc.). Let us consider only the general idea of obtaining parameters and characteristics that are to be determined according to the mathematical statement of the problem. First of all, the interconnections between control vector $\bar{u}_q$ and the control system's FE parameters are defined in the form:

$$
\begin{aligned}
\bar{Y}_Q &= \bar{f}_{u_Q}\left(\bar{u}_Q, \bar{Y}_{Q-1}\right), \\
\bar{Y}_{Q-1} &= \bar{f}_{u_{Q-1}}\left(\bar{u}_{Q-1}, \bar{Y}_{Q-2}\right), \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
\bar{Y}_l &= \bar{f}_{u_l}\left(\bar{u}_l, \bar{Y}_{l-1}\right), \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
\bar{Y}_2 &= \bar{f}_{u_2}\left(\bar{u}_2, \bar{Y}_1\right), \\
\bar{Y}_1 &= \bar{f}_{u_1}\left(\bar{u}_1\right),
\end{aligned}
\tag{8.39}
$$

where $\bar{Y}_q(q = 1, 2, \ldots, Q)$ is a vector of parameters of FEs of the qth hierarchical level of a control system and $\bar{u}_q$ is a vector function of control influences at the qth hierarchical level.

Then, an interconnection is established between the cost of development, production, and operation of functional elements of the qth hierarchical level and the parameters of an object itself and its control system:

$$
C_c = \tilde{f}_c\left(\bar{X}_Q, \tilde{f}_{cQ}\left(\bar{X}_{Q-1}, \tilde{f}_{cQ-1}\left(\ldots \tilde{f}_{cl}\left(\bar{X}_l, \ldots\left(\tilde{f}_{c1}, \left(\bar{X}_1\right)\ldots\right)\ldots\right)\ldots\right)\right)\right),
\tag{8.40}
$$

$$
C_u = \tilde{f}_u\left(\bar{Y}_Q, \tilde{f}_{uQ}\left(\bar{Y}_{Q-1}, \tilde{f}_{uQ-1}\left(\ldots \tilde{f}_{ul}\left(\bar{Y}_l, \ldots\left(\tilde{f}_{ul}, \left(\bar{Y}_1\right)\ldots\right)\ldots\right)\ldots\right)\right)\right).
\tag{8.41}
$$

Here relationship (8.40) establishes the interconnection between the cost C_c and parameters of FEs of all hierarchical levels of a controlled object, and relationship (8.41) establishes a similar interconnection for a control system.

In the general case the total cost of an object C_Σ as a cost function of FEs of system C_c and control system C_U is defined as:

$$
C_\Sigma = f_\Sigma\left(C_c, C_U\right).
\tag{8.42}
$$

The effectiveness of the object's operation as a generalized estimate of its serviceability results for a definite period is defined through some generalized indicators, which in quality control are traditionally referred to as an integral object quality indicator. It is expressed through quality indicators in the form:

$$
K_\Sigma = F_\Sigma\left(\bar{K}_Q\right).
\tag{8.43}
$$

Then the technical and economic efficiency defined as a ratio of the generalized effectiveness of the object's operation to the total cost for the period of the life cycle of an object may be represented in the following form:

$$
E_\Sigma = \frac{K_\Sigma}{C_\Sigma}.
\tag{8.44}
$$

It is necessary to pay attention to one practically important feature of the determination of total cost. The expenditures at the stage of development and production of an object begin to be covered only at the operation stage. Therefore, financial, material, and other resources spent at the stages of development and production of an object for some time equal to the duration of these stages are "frozen" and do not make any profit. This results in an increase in the total cost from the moment of the onset of operation. This circumstance is taken into account by a certain coefficient that depends on the duration of margin "freezing". For example, such accounting can be made on the basis of the following approximate relationship:

$$C_t = \frac{C_0}{(1-E)^t},$$

where C_0 is the cost value at the beginning of an object elaboration ($t = 0$), C_t is the cost value over t years, E is a normative coefficient, frequently assumed to be $E = 0,1,\ldots,0,15$.

The strategy of technical and economic efficiency estimation can be formulated briefly as follows:

1. The total cost of a product is determined taking into account the expenditures for the development, production, and operation of a technical system along with a control system and technical diagnostics system.
2. The total effectiveness of a complex object's operation is determined on the basis of an integral indicator, which is expressed through indicators of serviceability and quality of an object's operation, taking into account the losses on liquidation of abnormal situations, failures, and malfunctions.
3. Technical and economic efficiency is determined as a ratio of effectiveness to costs.
4. The degree of conformity of technical and economic efficiency to the requirements given in the requirements specification is verified.
5. The degree and level of risk are determined on the basis of the general principles of the system analysis of risk.
6. The degree to which the requirements are met with regard to technical and economic efficiency is estimated. If the requirements are not met, a new iteration of estimating and choosing functional elements and solving a structural-functional analysis begins.

When the required indicators of effectiveness and the technical-economic efficiency of a complex object's operation based on the data of its functioning during an a priori set period are obtained, one can assume that the general problem of a complex hierarchical system analysis is solved.

It should be noted that the above considered approach of the estimation of effectiveness and technical-economic efficiency of a complex object's operation is the simplest one and does not take into account a number of practically important factors. A more general and sufficiently detailed representation of the system estimation of the technical and economic efficiency of complex objects is given by the example of the serviceability of modern information-telecommunications systems [165, 166].

General strategy of technical-economical analysis. Technical-economical analysis is a major component of system analysis of functioning effectiveness of various complex objects.

It is obvious that the practical necessity of estimating the influence of various factors and realizing the basic principles of ensuring technical and economic efficiency requires a certain strategy of optimization of the properties and capabilities of complex objects. It is possible to give the two most common formulations of the optimization problem. The first formulation consists in the development of a strategy for choosing a rational compromise of contradictory requirements in the interests of attaining the highest level of technical and economic efficiency with the assigned restrictions on effectiveness indicators. The second formulation of the optimization problem requires provision of the maximal effectiveness with the assigned restrictions of technical and economic efficiency.

In spite of a certain distinction of the content and accepted goal functions, these problems have one practically important general property. Its essence lies in the fact that only the realization of the systemness of technical and economic efficiency can ensure a high credibility of the obtained estimation. At first sight, reasoning about the systemness of efficiency analysis seems to be absurd. It seems very simple—increase the numerator and reduce the denominator in formula (8.44) and the required level of credibility and estimation effectiveness will be guaranteed. Reducing the denominator means that it is necessary to reduce the expenses at each stage of a product's life cycle. However, experience has shown that so straightforward an approach may lead to absurd results. To understand this, one should pay attention to one feature of the accepted indicator of formula (8.44). The value E_Σ remains constant if the numerator and denominator in (8.44) are increased or reduced proportionally and simultaneously.

Also, it is necessary to pay attention to one more important feature of indicator (8.44)—the effectiveness value is a nonlinear and a priori unknown function of costs. And, first of all, as the experience shows, the maximum payback of costs is achieved for the earliest stages of a life cycle and, mainly, when the appearance, principles of operation, and other initial preconditions of a product design are chosen. From here it follows that it is necessary to provide a multipronged system analysis of accounting of factors that determine various aspects of the technical and economic efficiency of a product.

We list only those areas where it is required to realize the systemness of technical-economical analysis.

1. Systemness of costs.

1.1. Systemness of costs of an object's structure.
1.2. Systemness of costs of an object's life cycle.
1.3. Systemness of analysis of reasons and factors that increase costs.
1.4. Systemness of analysis of reasons and factors that decrease costs.
1.5. Systemness of estimation of risks during life cycles of complex objects.

2. Systemness of effectiveness analysis.

2.1. Systemness of effectiveness of decisions concerning the object's structure.

2.2. Systemness of effectiveness of decisions concerning the object's life cycle.
2.3. Systemness of conditions and factors of increases in efficiency.
2.4. Systemness of conditions and factors of decreases in efficiency.
2.5. Systemness of conditions and factors of reducing multifactor risks.

3. Systemness of analysis of application conditions.
3.1. Systemness of technical and economic efficiency.
3.2. Systemness of production and control efficiency.
3.3. Systemness of the analysis of possible market demand and supply.
3.4. Systemness of the analysis of competition.
3.5. Systemness of estimation of product development.

8.5 Example of Solving a Problem of System Control of Serviceability and Safety of a Complex Engineering Object

We will consider a problem of serviceability and safety control of a complex engineering object in the process of its transition from a normal to abnormal mode by the example of an aircraft engine [127].

Contensive statement of problem. We believe it expedient to consider a safety control problem as a system problem involving the detection of risk factors whose influence may result in abnormal situations. At the same time it is taken into consideration that safety and functioning control is implemented under conditions of incompleteness and uncertainty of dynamics of the transition of a normal to abnormal mode. The normal mode is not permanent and changes considerably at different stages of a system operation cycle. These conditions are met by the operation of an aircraft engine, the modes of which during takeoff, cruising, and landing differ fundamentally. The influences most critical and sensitive to risk factors are the takeoff and landing modes. These modes are in their physical essence transitional between two stationary modes: the first is an off mode, when airplane stands still; the second is a cruising flight mode, when the airplane flies at a specified altitude and the engine works in a permanent mode during the flight period.

The normal functioning mode consists of a number of basic stages. Crucial are the following stages: the basic stage is a stationary cruising flight mode, the transitional stages are non-stationary takeoff and landing modes. The engine parameters vary synchronously during a transition from one mode to another and in transitional modes. Among these parameters are traction, fuel mass, oil pressure, air supply, etc. These parameters vary in different modes as time functions depending on the purposeful actions of a control system and uncontrolled influence of external factors of climatic, geographical, geological, technological, and other processes [13, 14, 102, 191].

General properties and peculiarities of an object under investigation. The main properties, indicators, and the characteristics of a normal functioning mode of an object are known. The mode is determined by the following basic stages and properties:

- Stage 1 $\rightarrow [t_0^-, t_1]$ is the period of engine start-up and warm-up.
- Stage 2 $\rightarrow [t_1, t_2]$ is the preparation for takeoff, starting up and testing the engine transition from a normal power mode to an afterburning mode.
- Stage 3 $\rightarrow [t_2, t_3]$ is takeoff execution in the engine afterburning mode, which is characterized by the engine power increasing by 1.2–1.3 times in comparison with the standard power.
- Stage 4 $\rightarrow [t_3, t_4]$ is a transition process from the afterburning to cruising mode, characterized by a reduction in the engine power level to 0.75–0.85 of its standard value.
- The main stage 5 $\rightarrow [t_4, t_5]$ is the cruising mode; the main functioning indicators are 0.75–0.85 of their standard values.
- Stages 6–8 are similar to takeoff stages by general indicators, so we represent them in a simplified version. Stage 6 is a transition from the cruising mode and the landing mode. Stage 7 is the landing mode, stage 8 is an engine-shutdown mode, which ends with the transition to ***the mode of waiting and preparation*** to the next start-up. Typical functioning modes are presented in Fig. 8.5.

Characteristics and peculiarities of modes are known and determined by the following properties and accepted tolerances.

1. Each stage is characterized by its specific duration, the initial and final values of each indicator y_i, determined respectively at the initial and final moment of the time period. Variations y_i during the period are determined by a corresponding model. The period duration and initial and final values are a priori assigned.

2. All indicators y_i are synchronous and cophased.

These properties consist in a simultaneous increase or decrease of indicators without a time delay under the influence of various factors.

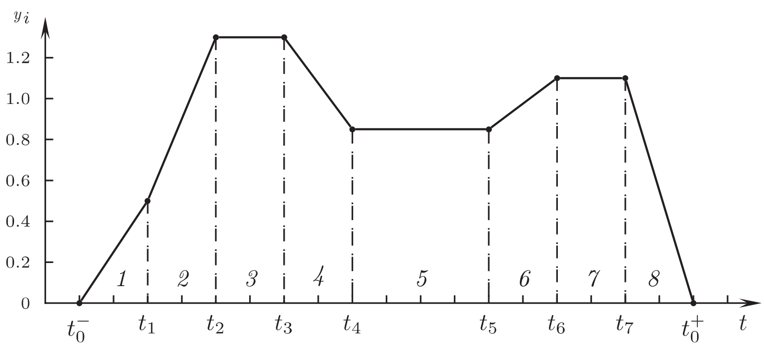

Fig. 8.5. Typical modes of an aircraft engine functioning

3. Control action $U = \left(U_j | j = \overline{1, m}\right)$ is inertialess, i.e., without a time delay between a control action and the object's reaction to it.

4. Risk factors $\bar{\rho} = \left(\rho_q | q = \overline{1, n_q}\right)$ change the value of influence on the engine in time, and with an increase in time of influence the degree and level of risk increase.

5. Control action can slow down risk-factor influence or stop their negative influence on the controlled object under the condition that the speed of control exceeds the growth rate of the level of risk-factor influence.

6. Termination of negative influence of risk factors can be ensured only under the condition that a decision is developed, made, and realized before the critical moment T_{cr} comes, when risk-factor influence results in irreversible consequences of an accident or a catastrophe.

The recognition of abnormal situations will be considered **on the basis of the given above properties and features** as a problem of recognition of risk factors whose influence may result in an abnormal situation. We shall take into account the main differences between normal and abnormal modes, which are enumerated in Chap. 7.

Mathematical statement of problem. It is known that an object's normal functioning mode is characterized by a set:

$$Y = \left\{ y_i | y_i^- \le y_i \le y_i^+ ,\ i \in N_0 ,\ N_0 = \overline{1; n_0} \right\} . \tag{8.45}$$

Values of the object's functioning indicators $y_i \in Y$ are limited by the control system action $\bar{U} = \left(U_j , j = \overline{1, J_0}\right)$ and random influences of risk factors $\bar{\rho} = \left(\rho_q | q = \overline{1, n_q}\right)$. The dependence of indicators on the control system actions and influences of random risk factors is defined by the relationship:

$$y_i = f_i \left(\bar{U}, \bar{\rho}\right) , \tag{8.46}$$

where $f_i(\bar{U}$ and $\bar{\rho})$ are unknown functions to be determined by discrete data of the diagnostics system.

The parameters y_i are measured in the known discrete time moments $t_k = t_0 + k\Delta t$ with known values U_j and unknown values ρ_q. The measurement results are known in the form of the finite sample of observations during the given period $[t_0, t_{k_0}]$ and the finite sample of observations during the period $[t_{k_1}, t_{k_0}]$, where $t_0 < t_{k_1} < t_{k_0}$. The samples are represented in the form of the following arrays:

$$B_0 = \left\{ \left\langle \bar{y}\left(t_k\right), \bar{U}\left(t_k\right) \right\rangle | t_k = t_0 + k\Delta t;\ k = \overline{0; k_0} \right\} , \tag{8.47}$$

$$B_{k_1} = \left\{ \left\langle \bar{y}\left(t_k\right), \bar{U}\left(t_k\right) \right\rangle | t_k = t_0 + \hat{k}\Delta t;\ \hat{k} = \bar{k}_1; k_0 \right\} , \tag{8.48}$$

where $\bar{y} = \left(y_i | i = \overline{1, n_0}\right)$.

There is a set $\hat{Y}$ of dynamically synchronous indicators:

$$\hat{Y} = \left\{ \hat{y}_{i_1} | \left\langle U_{j_1} \uparrow,\ j_1 \in \hat{M}_1 \right\rangle \Rightarrow \left\langle \hat{y}_{i_1} \uparrow,\ i_1 \in \hat{N}_1 \right\rangle \right\} ,$$
$$\hat{y} \subseteq Y,\ M_1 = \overline{1; J_0};\ j \le J_0;\ \hat{N}_1 \subseteq N_0;\ \hat{N}_1 = \overline{1; n_1};\ n_1 \le n_0 . \tag{8.49}$$

The values of all control vector components at the moment t_0 are known:

$$\bar{U}_0 = \left\{U_j^0 | U_j^0 = U_j\left(t_0\right)\right\}, \quad j = \overline{1, J_0}\,, \tag{8.50}$$

and the required values of normal mode indicators, without the influence of uncontrolled risk factors, are also known:

$$\bar{y}_0 = \left\{y_j^0 | y_j^0 = \hat{f}\left(\bar{U}_0\right);\; y_i^0 \in Y;\; i = \overline{1; n_0}\right\}. \tag{8.51}$$

For the specified period of operation $\overset{\cup}{T} = [t_0; t]$ permissible deviations of parameters y_i under the influence of uncontrolled factors are defined in the form of the following restrictions:

$$y_i^- \le \overset{\cup-}{y}_i < \overset{\cup+}{y}_i \le y_i^+\,.$$

If this condition is not fulfilled simultaneously for several synchronous indicators $\hat{y}_{i_1} \in \hat{Y}$ and $\hat{y}_{i_2} \in \hat{Y}$, the situation is abnormal. Formally, the condition for an abnormal situation will be described by the following relationship:

$$\begin{aligned} &\left\{\left[\left(\hat{y}_{i_1} \in \hat{Y}\right)\right] \wedge \left[\left(\hat{y}_{i_1} < y_{i_1}^-\right) \vee \left(y_{i_1}^+ < \hat{y}_{i_1}\right)\right]\right\} \wedge \\ \wedge &\left\{\left[\left(\hat{y}_{i_2} \in \hat{Y}\right)\right] \wedge \left[\left(\hat{y}_{i_2} < y_{i_2}^-\right) \vee \left(y_{i_2}^+ < \hat{y}_{i_2}\right)\right]\right\}. \end{aligned} \tag{8.52}$$

It is required:

- To ensure the recognition of an abnormal situation on the basis of samples B_0 and B_{k_1}.
- To define the degree and level of risk at each moment t_k.
- To find the moment and rate of possible transition from a normal mode to an abnormal one.

Initial values of indicators

1. The quantity of indicators $\hat{y}_{i_1} \in \hat{Y}$; $\hat{y}_{i_2} \in \hat{Y}$; $\langle i_1; i_2 \rangle \in \hat{N}_1$; $N_1 = \left\{i | i = \overline{1, n_1}\right\}$ of an object is limited by $n_0 = 2$; $i_1 = 1$; $i_2 = 2$. The indicators are dynamically synchronous and time dependent $\hat{y}_{i_1} = y_1(t)$, $\hat{y}_{i_2} = y_2(t)$ and correspond to condition (8.49). The maximal changes take place during the transition modes of stages 1 and 8, where conditions (8.52) are most probable. The limits of variation for these stages are assigned:

$$y_1[t_0] = 0; \quad y_1[t_k] = 0.45; \quad y_2[t_0] = 0; \quad y_2[t_k] = 0.35\,.$$

Henceforth the partial values of indicators are normalized by their general values during the flight.

2. The mode variation is performed by a control system whose indicators $U_j \in \bar{U}$; $\bar{U} = \left(U_j,\; j = \overline{1, J_0}\right)$ are time functions $U_j = U_j(t)$. Conditions (8.50) and (8.51) are fulfilled at the moment t_0. It is assigned $J_0 = 3$, and

the functions $\langle U_1(t), U_2(t), U_3(t)\rangle \in \bar{U}$ are restricted and represented in the form:

$$\begin{aligned}
&0 \le U_1(t) \le 1; \quad && U_1(t) = d_{10} + d_{11}t + d_{12}t^2;\\
&0 \le U_2(t) \le 0.7; && U_2(t) = d_{20} + d_{21}t;\\
&0 \le U_3(t) \le 0.5; && U_3(t) = d_{30} + d_{31}t + d_{32}t^2. \qquad (8.53)
\end{aligned}$$

3. In the system at the definite discrete time moments $t_k = t_0 + k\Delta t$ parameters $y_i = y_i[t_k]$ are measured with known values $U_j = U_j[t_k]$ and unknown values ρ_q. Measurement results are represented in the forms (8.47) and (8.48).

It is known that the measuring interval constitutes in minutes t the period $t \in [0; 0,3]$. Within this interval its discrete analog is formed, which fulfills the following conditions: $k = \overline{1;11}$; for $k = 1$, $t = t_0 = 0$; for $k = k_0 = 11$, $t = t_{k_0} = 0.3$. In the process of technical diagnostics a sequence of discrete intervals during the flight is formed, which includes intervals of all stages from 1 to 8.

4. On the basis of discrete intervals the values for functions $\langle U_1(t), U_2(t), U_3(t)\rangle \in \bar{U}$ are formed, which are defined by relationship (8.53). The values of functions are defined by the conditions

$$\begin{aligned}
&U_1(t_0) = 0; \quad && U_1(0.3) = 1; \quad && U_1(0.15) = 0.7;\\
&U_3(t_0) = 0; && U_3(0.3) = 0.5; && U_3(0.2) = 0.4;\\
&U_2(t_0) = 0; && U_2(0.3) = 0.7. && \qquad (8.54)
\end{aligned}$$

5. The following properties and features are considered in the analysis of risk-factor influence on an object:

– Risk factors $\bar{\rho} = \left(\rho_q | q = \overline{1, n_q}\right)$ are independent and vary according to random rules whose distribution is a priori unknown.
– Risk factors can have an impact simultaneously on one, a few, or all indicators y_i.
– If under the influence of risk factors the values of at least two indicators y_i change simultaneously, synchronously, and cophased, and it is detected by the diagnostics system, this means that an object is passing from a normal to abnormal mode.

6. On the basis of the above indicated conditions, properties, and features, the influence of risk factors on the object under investigation is taken into account in the following way. Under the influence of risk factors the values of indicator y_i will be defined by the value $\tilde{y}_i$. At the moment t_k indicator $\tilde{y}_i[t_k]$ is defined by the equation:

$$\tilde{y}_i[t_k] = \frac{1}{J_0}\sum_{j=1}^{J_0} \tilde{b}_{ij} \sum_{r=0}^{R_j} a_{jr} T_r^*(U_j); \; \tilde{b}_{ij} = b_{ij} \cdot F_i(\rho_{qk}). \qquad (8.55)$$

Here, value $\rho_{qk} = \rho_q[t_k]$ defines the value of the qth risk factor at the moment t_k. Indicator b_{ij} characterizes the level of influence of the kth risk factor on the indicator y_i of an object at the moment t_k, implemented by acting on indicator U_j of the object control. The extreme values b_{ij} are also assigned: for $i = 1;\ j = \overline{1;3} \Rightarrow b_{11} = 0.2;\ b_{12} = 0.3;\ b_{13} = 0.5$; for $i = 2;\ j = \overline{1;3} \Rightarrow b_{21} = 0.1;\ b_{22} = 0.25;\ b_{23} = 0.4$. Function $F_i(\rho_{q_k})$ characterizes the level of influence of factor ρ_{q_k} on the ith indicator y_i. It is to meet the condition that in the absence of risk factors the equality $\tilde{y}_i = y_i$ is to be fulfilled (i.e., with $\rho_{q_k} = 0$). Several risk factors $\rho_{n_q} = \left\langle \rho_{qk} | \rho_{qk} = \rho_q[t_k];\ q = \overline{1;n_q} \right\rangle$ may simultaneously and independently affect the object. Function $F_i\left(\rho_{n_q}\right)$ is defined by the relationship:

$$F_i\left(\rho_{n_q}\right) = \prod_{q=1}^{n_q} \left(1 - c_{iq}\rho_{qk}\right),$$
$$\rho_{n_q} = \left\{\rho_{qk} | \rho_{qk} = \rho_q(t_k);\ q = \overline{1;n_q}\right\}. \tag{8.56}$$

Here, the following value is assigned: $0 < c_{iq} \le 1;\ i = \overline{1;2};\ q = \overline{1;n_q}$.

Problem solution.

Let us consider the algorithm of the problem solution, where, using the initial data (8.47) and (8.48) on the basis of (8.53)–(8.56), it is required to execute the following procedures:

1. Normalize each stage duration to the corresponding part of the total duration of engine operation $\left[t_0^-, t_0^+\right]$, typical operation modes of which are represented in Fig. 8.5.

Normalized duration τ_1 and τ_2 of stages 1 and 2 are defined by the relationships:

$$\tau_1 = 1 - \frac{t - t_0^-}{t_1 - t_0^-};\quad t \in \left[t_0^-, t_1\right],\quad \tau_2 = 1 - \frac{t - t_1}{t_2 - t_1};\quad t \in [t_1, t_2].$$

Discrete values τ_1 and τ_2 are calculated in accordance with p. 3 of the initial data. Normalization is performed for all other stages in the same way. For each stage the value τ should vary within the interval [0;1]. Such an approach allows one to raise the accuracy of models due to an increase in the number of points for each interval while compiling the corresponding system of equations.

2. Perform function $y_i(t_k)$ ***indicators and control actions*** $U_j(t_k)$ ***normalization*** on the basis of the relationships:

$$\hat{y}_i(t_k) = \hat{y}_i = \frac{y_i(t_k)}{y_i^+};\quad y_i^+ = \max_{t_k \in \left[t_0^-, t_0^+\right]} y_i(t_k);$$
$$\hat{U}_j(t_k) = \hat{U}_j = \frac{U_j(t_k)}{U_j^+};\quad U_j^+ = \max t_k \in \left[t_0^-, t_0^+\right] U_j.$$

Normalization should be performed corresponding to conditions (8.54).

3. Form a system of models for control action indicators $U_j(t_k)$. A system of models is formed in accordance with (8.53) and (8.54) and consists in determining the corresponding function coefficients $d_{01}, \ldots, d_{32}$. Determination of coefficients for the function $U_1(t) = d_{10} + d_{11}t + d_{12}t^2$ according to (8.53) and (8.54) is reduced to solving the following system of equations:

$$\begin{aligned} d_{10} + d_{11}t_0 + d_{12}t_0^2 &= 0\,, \\ d_{10} + d_{11}0.15 + d_{12}(0.15)^2 &= 0.7\,, \\ d_{10} + d_{11}0.3 + d_{12}(0.3)^2 &= 1.0\,. \end{aligned}$$

Note: In this system of equations there is a practically important peculiarity. The moment $t_0 = t_0^- = 0$ corresponds to an offline state of the engine. Thus, in the first equation, all items should be zero. Then, the last two equations have three unknown variables d_{10}, d_{11}, and d_{12}, which results in an unlimited set of solutions and, thus, is practically unacceptable. The solution to this paradox is possible after a fundamental change in the initial reference point. That is, the reference point should be a value of control action in an established mode, which is typical for the cruising stage. Then, value d_{j_0} should be considered as an indicator of steady operation of the control system, d_{j_1} as the speed of control mode switching, and d_{j_2} as the acceleration of the control mode switching. With this approach the value d_{j_0} is not only a priori determined, but can be specified directly during flight. Values of two other indicators are defined directly from the last two equations. As a result, we obtain a final solution in the form of the functions

$$\begin{aligned} U_2(t) &= d_{20} + d_{21}t + d_{22}t^2\,, \\ U_3(t) &= d_{30} + d_{31}t + d_{32}t^2\,, \end{aligned}$$

for which the conditions $d_{20} = d_{30} = \hat{d}_{05}$ are fulfilled and $\hat{d}_{05}$ is an indicator of steady operation of the control system at stage 5 in the cruising mode.

4. Form a system of models for indicators of an object functioning in a normal mode. The basic models of an object's operation will be formed in a normal mode, taking into account the equality $\tilde{y}_i = y_i$ for all periods. In such conditions the equality $F_i\left(\rho_{n_q}\right) = 1$ is satisfied, since for a normal mode by definition $\rho_{q_k} = 0$; $q = \overline{1, n_q}$ and, therefore, according to (8.56), we get $\rho_{n_q} = 0$. Then, the models of object's operation in compliance with restrictions and recommendations of p. 4 of the initial data are defined by the relationship

$$y_i = \frac{1}{J_0} \sum_{j=1}^{J_0} b_{ij} \sum_{r=0}^{R_j} a_{jr} T_r^*(U_j)\,.$$

Systems of equations for defining unknown variables a_{ir} and b_{ij} should be formed on the basis of the discrete sample (8.47)

$$B_0 = \left\{\left\langle \bar{y}(t_k)\,, \bar{U}(t_k)\right\rangle | t_k = t_0 + k\Delta t;\ k = \overline{0, k_0}\right\}$$

of observations during the period $[t_0, t_{k_0}]$. The sample is defined by a sequence of discrete intervals in compliance with p. 3 of the initial data.

5. Form a system of models for indicators of an object functioning in an abnormal mode. Models of an object functioning in an abnormal mode should be formed for all periods on the basis of relationships (8.55) and (8.56):

$$\tilde{y}_i\left[t_k\right] = \frac{1}{m}\sum_{j=1}^{m}\tilde{b}_{ij}\sum_{r=0}^{R_j} a_{jr}T_r^* \cdot \left(U_j\right); \quad \tilde{b}_{ij} = b_{ij} \cdot F_i\left(\rho_{n_q}\right)$$

using the sample data (8.48)

$$B_{k1} = \left\{\left\langle \bar{y}\left(t_k\right), \bar{U}\left(t_k\right)\right\rangle | t_k = t_0 + \hat{k}\Delta t; \hat{k} = k_1, k_0\right\}$$

for the period $[t_{k_1}, t_{k_0}]$, in compliance with restrictions and recommendations of pp. 4–6 of the initial data. Systems of equations for finding the unknown variables a_{ir} and b_{ij} and the function $F_i\left(\rho_{n_q}\right)$ should be formed on the basis of a discrete sample defined by a sequence of discrete intervals in compliance with the requirements of p. 3.

6. Develop a system of versions of a demo system of control and estimation of abnormal mode risks. On the basis of the models of analysis of normal and abnormal process dynamics proposed above, a demo system of detection and estimation of abnormal situation risks was developed. The models allow one to define the values of the degree and level of risk for each moment t_k.

The above indicated problems for a normal mode of an object's operation are solved by using the simplex method for solving incompatible systems of linear equations. The initial data and results of mode correction with determined values of maximum deviation are given in Fig. 8.6.

The moment of possible transition of the normal mode of an object's operation into abnormal mode is shown in Fig. 8.7.

In conclusion we note that the presented theoretical studies and examples of problem solutions show that it is possibile to define, with sufficient practical accuracy, the moment of possible transition of a normal mode of a complex engineering system into an abnormal mode and, thus, ensure the safety of its operation.

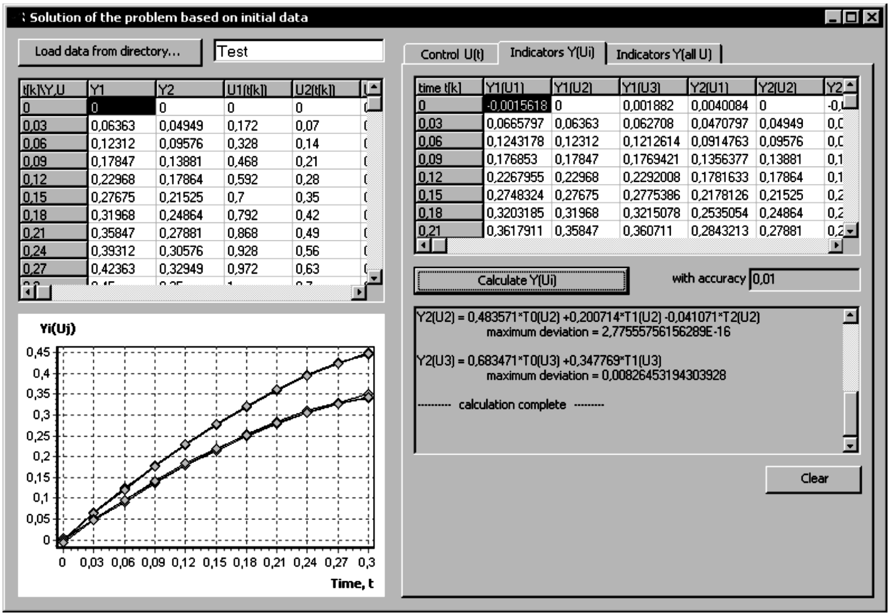

Fig. 8.6. Initial data and results of mode correction

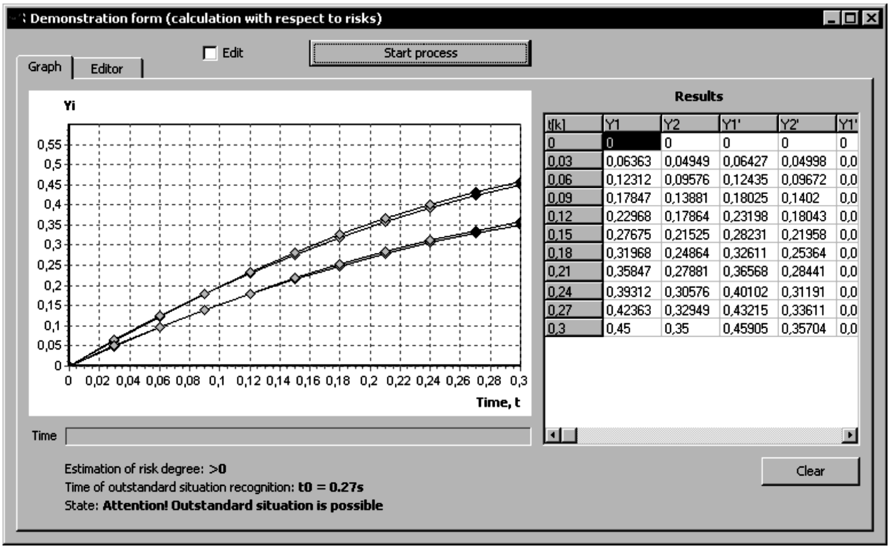

Fig. 8.7. Recognition of abnormal situation

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