

Introduction

Typically, models of neural networks are divided into two categories in terms of signal transmission manner: feed-forward neural networks and recurrent neural networks. They are built up using different frameworks, which give rise to different fields of applications.

1.1 Backgrounds

1.1.1 Feed-forward Neural Networks

Feed-forward neural network (FNN), also referred to as multilayer perceptrons (MLPs), has drawn great interests over the last two decades for its distinction as a universal function approximator (Funahashi, 1989; Scalero and Tepedelenlioglu, 1992; Ergezinger and Thomsen, 1995; Yu *et al.*, 2002). As an important intelligent computation method, FNN has been applied to a wide range of applications, including curve fitting, pattern classification and nonlinear system identification and so on (Vemuri, 1995).

FNN features a supervised training with a highly popular algorithm known as the error back-propagation algorithm. In the standard back-propagation (SBP) algorithm, the learning of a FNN is composed of two passes: in the forward pass, the input signal propagates through the network in a forward direction, on a layer-by-layer basis with the weights fixed; in the backward pass, the error signal is propagated in a backward manner. The weights are adjusted based on an error-correction rule. Although it has been successfully used in many real world applications, SBP suffers from two infamous shortcomings, i.e., slow learning speed and sensitivity to parameters. Many iterations are required to train small networks, even for a simple problem. The sensitivity to learning parameters, initial states and perturbations was analyzed in (Yeung and Sun, 2002). Behind such drawbacks the learning rate plays a key role in affecting the learning performance and it has to be chosen carefully. If the learning rate is large, the network may exhibit chaotic

behavior so learning might not succeed, while a very small learning rate will result in slow convergence, which is also not desirable. The chaotic phenomena was studied from a dynamical system point of view (Bertels *et al.*, 2001) which reported that when the learning rate falls in some unsuitable range, it may result in chaotic behaviors in the network learning, and for non-chaotic learning rates the network converges faster than for chaotic ones.

Since the shortcomings of the SBP algorithm limit the practical use of FNN, a significant amount of research has been carried out to improve the training performance and to better select the training parameters. A modified back-propagation algorithm was derived by minimizing the mean-squared error with respect to the inputs summation, instead of minimizing with respect to weights like SBP, but its convergence heavily depended on the magnitude of the initial weights. An accelerated learning algorithm OLL (Ergezinger and Thomsen, 1995) was presented based on a linearization of the nonlinear processing nodes and optimizing cost functions layer by layer. Slow learning was attributed to the effect of unlearning and a localizing learning algorithm was developed to reduce unlearning (Weaver and Polycarpou, 2001). Bearing in mind that the derivative of the activation has a large value when the outputs of the neurons in the active region, a method to determine optimal initial weights was put forward in (Yam and Chow, 2001). This method was able to prevent the network from getting stuck in the early stage of training, thus increasing the training speed.

Existing approaches have improved the learning performance in terms of the reduction of iteration numbers, however, none of them dealt with dynamical adaption of the learning rate for different parameters and training phases, which certainly contributes to the sensitivity of such algorithms. An optimal learning rate for a given two layers' neural network was derived in the work of (Wang *et al.*, 2001), but a two-layer neural network has very limited generalization ability. Finding a suitable learning rate is a very experimental technique, since for the multilayer FNN with squashing sigmoid functions, it is difficult to deduce an optimal learning rate and even impossible to pre-determine the value of such a parameter for different problems and different initial parameters. Indeed, the optimal learning rate keeps changing along with the training iterations. Finding a dynamical optimal learning algorithm being able to reduce the sensitivity and improve learning motivate developing a new and efficient learning algorithm for multilayer FNN.

1.1.2 Recurrent Networks with Saturating Transfer Functions

Unlike feed-forward neural networks, recurrent neural networks (RNN) are described by a system of differential equations that define the exact evolution of the model dynamics as a function of time. The system is characterized by a large number of coupling constants represented by the strengths of individual junctions, and it is believed that the computational power is the result of the collective dynamics of the system. Two prominent computation models

with saturating transfer functions, the Hopfield network and cellular neural network, have stimulated a great deal of research efforts over the past two decades because of their great potential of applications in associative memory, optimization and intelligent computation (Hopfield, 1984; Hopfield and Tank, 1985; Tank and Hopfield, 1986; Bouzerdoum and Pattison, 1993; Maa and Shanblatt, 1992; Zak *et al.*, 1995b; Tan *et al.*, 2004; Yi *et al.*, 2004).

As a nonlinear dynamical system, intrinsically, the stability is of primary interest in the analysis and applications of recurrent networks, where the Lyapunov stability theory is a fundamental tool and widely used for analyzing nonlinear systems (Grossberg, 1988; Vidyasagar, 1992; Yi *et al.*, 1999; Qiao *et al.*, 2003). Based on the Lyapunov method, the conditions of global exponential stability of a continuous-time RNN were established and applied to bound-constrained nonlinear differentiable optimization problems (Liang and Wang, 2000). A discrete-time recurrent network solving strictly convex quadratic optimization problems with bound constraints was analyzed and stability conditions were presented (Pérez-Ilzarbe, 1998). Compared with its continuous-time counterpart, the discrete-time model has its advantages in digital implementation. However, there is lack of more general stability conditions for the discrete-time network in the previous work (Pérez-Ilzarbe, 1998), which deserves further investigation.

Solving NP-hard optimization problems, especially the traveling salesman problem (TSP) using recurrent networks has become an active topic since the seminal work of (Hopfield and Tank, 1985) showed that the Hopfield network could give near optimal solutions for the TSP. In the Hopfield network, the combinatorial optimization problem is converted into a continuous optimization problem that minimizes an energy function calculated by a weighted sum of constraints and an objective function. The method, nevertheless, faces a number of disadvantages. Firstly, the nature of the energy function causes infeasible solutions to occur most of the time. Secondly, several penalty parameters need to be fixed before running the network, while it is nontrivial to optimally set these parameters. Besides, low computational efficiency, especially for large scale problems, is also a restriction.

It has been a continuing research effort to improve the performance of the Hopfield network (Aiyer *et al.*, 1990; Abe, 1993; Peng *et al.*, 1993; Papageorgiou *et al.*, 1998; Talaván and Yáñez, 2002a). The authors in (Aiyer *et al.*, 1990) analyzed the dynamic behavior of a Hopfield network based on the eigenvalues of connection matrix and discussed the parameter settings for TSP. By assuming a piecewise linear activation function and by virtue of studying the energy of the vertex at a unit hypercube, a set of convergence and suppression conditions were obtained (Abe, 1993). A local minima escape (LME) algorithm was presented to improve the local minima by combining the network disturbing technique with the Hopfield network's local minima searching property (Peng *et al.*, 1993).

Most recently, a parameter setting rule was presented by analyzing the dynamical stability conditions of the energy function (Talaván and Yáñez,

2002a), which shows promising results compared with previous work, though much effort has to be paid to suppress the invalid solutions and increase convergence speed. To achieve such objectives, incorporating the winner-take-all (WTA) learning mechanism (Cheng *et al.*, 1996; Yi *et al.*, 2000) is one of the more promising approaches.

1.1.3 Recurrent Networks with Nonsaturating Transfer Functions

In recent years, the linear threshold (LT) network which underlies the behavior of visual cortical neurons has attracted extensive interests of scientists as the growing literature illustrates (Hartline and Ratliff, 1958; von der Malsburg, 1973; Douglas *et al.*, 1995; Ben-Yishai *et al.*, 1995; Salinas and Abbott, 1996; Adorjan *et al.*, 1999; Bauer *et al.*, 1999; Hahnloser, 1998; Hahnloser *et al.*, 2000; Wersing *et al.*, 2001a; Yi *et al.*, 2003). Differing from the Hopfield type network, the LT network possesses nonsaturating transfer functions of neurons, which is believed to be more biologically plausible and has more profound implications in the neurodynamics. For example, the network may exhibit multistability and chaotic phenomena, which will probably give birth to new discoveries and insights in associative memory and sensory information processing (Xie *et al.*, 2002).

The LT network has been observed to exhibit one important property, i.e., multistability, which allows the networks to possess multiple steady states coexisting under certain synaptic weights and external inputs. The multistability endows the LT networks with distinguished application potentials in decision, digital selection and analogue amplification (Hahnloser *et al.*, 2000). It was proved that local inhibition is sufficient to achieve nondivergence of LT networks (Wersing *et al.*, 2001b). Most recently, several aspects of LT dynamics were studied and the conditions were established for boundedness, global attractivity and complete convergence (Yi *et al.*, 2003). Nearly all the previous research efforts were devoted to stability analysis, thus the cyclic dynamics has yet been elucidated in a systematic manner. In the work of (Hahnloser, 1998), periodic oscillations were observed in a multistable WTA network when slowing down the global inhibition. He reported that the epileptic network switches endlessly between stable and unstable partitions and eventually the state trajectory approaches a limit cycle (periodic oscillation) which was shown by computer simulations. It was suggested that the appearance of periodic orbits in linear threshold networks was related to the existence of complex conjugate eigenvalues with positive real parts. However, there was lack of theoretical proof about the existence of limit cycles. It also remains unclear what factors will affect the amplitude of the oscillations.

Studying recurrent dynamics is also of crucial concern in the realm of modeling the visual cortex, since recurrent neural dynamics is a basic computational substrate for cortical processing. Physiological and psychophysical data suggest that the visual cortex implements preattentive computations

such as contour enhancement, texture segmentation and figure-ground segregation (Kapadia *et al.*, 1995; Gallant *et al.*, 1995; Knierim and van Essen, 1992). Various models have addressed particular components of the cortical computation (Grossberg and Mingolla, 1985; Zucker *et al.*, 1989; Yen and Finkel, 1998). A fully functional and dynamically well-behaved model has been proposed to achieve the designed cortical computations (Li and Dayan, 1999; Li, 2001). The LEGION model uses the mechanism of oscillation to perform figure-ground segmentation (Wang and Terman, 1995; Wang and Terman, 1997; Wang, 1999; Chen and Wang, 2002). The CLM model, formulated by the LT network, realizes an energy-based approach to feature binding and texture segmentation and has been successfully applied to segmentation of real-world images (Ontrup and Ritter, 1998; Wersing *et al.*, 1997; Wersing and Ritter, 1999). Dynamic binding in a neural network is of great interest for the vision research, a variety of models have been addressed using different binding approaches, such as temporal coding and spatial coding (Hummel and Biederman, 1992; Feldman and Ballard, 1982; Williamson, 1996). Understanding the complex, recurrent and nonlinear dynamics underlying the computation is essential to explore its power as well as for computational design.

These facts have provided substantial motivations for the extensive investigations of neural networks, both in dynamics analysis and applications.

1.2 Scopes

One focus of this book lies in the improvement of training algorithms for feed-forward neural networks by analyzing the mean-squared error function from the perspective of dynamical stability. The dynamical learning method is able to adaptively and optimally set the value of learning rate, hence the elimination of sensitivity of FNN networks with a fixed learning rate can be expected, as well as the reduction of convergence iterations and time.

Another emphasis is on the neurodynamics. The dynamics of the recurrent networks with saturating and nonsaturating transfer functions are analyzed extensively. New theoretical results on the nondivergence, stability and cyclic dynamics are established, which facilitate the applications of the recurrent networks in optimizations and sensory information segmentation. As an important application of the attractor networks, the analog associative memory of the LT network is also investigated. It shows that the LT network can successfully retrieve gray level images.

A special focus is on developing a competitive network incorporating winner-take-all mechanism. The competitive network deals with the constraints in optimization problems in an elegant way, so it has attractive advantages both in suppressing invalid solutions and in increasing convergence speed. The latter is a great concern when solving large scale problems. Probabilistic optimization methods, such as simulated annealing and local minima

escape, are also applicable to the competitive network, which can further improve the solution quality.

The significance of this book falls into two basic grounds. Above all, the book will serve the purpose of exploring the computational models of neural networks, and promoting our understanding of the functions of biological neural systems such as computation, perception and memory. Secondly, the theories and methods in this book can provide meaningful techniques for developing real-world applications.

1.3 Organization

The first chapter motivates the issue of dynamics analysis as a crucial step to understand the collective computation property of neural systems and describes the scope and contributions of the book.

The second chapter describes the typical learning algorithm of feedforward networks and several prominent modified algorithms among existing approaches. Chapter 3 presents a new dynamical optimal training algorithm for feed-forward neural networks. The new training method aims to avoid the serious drawback of the standard feed-forward neural network's training algorithm, i.e., sensitivity to initial parameters and different problems.

Chapter 4 introduces the fundamentals of mathematical analysis for linear and nonlinear systems, which underlie the analysis of neuro-dynamics.

Chapter 5 is devoted to various computational models based on recurrent neural networks and winner-take-all networks. Some useful applications, such as linear and nonlinear programming, extracting eigenvalues, feature binding and segmentation are introduced. In Chapter 6, a class of discrete-time recurrent networks is discussed and is applied to the typical nonlinear optimization problems. The global exponential stability condition is established which ensures the network globally convergent to the unique optimum.

Chapters 7 and 8 are focused on the neural networks applied to combinatorial optimization problems, where the issue of parameter settings of Hopfield networks, and new competitive model is presented respectively. Subsequently, the competitive model is extended as an algorithm for image segmentation in Chapter 9. In Chapter 10, the model is proposed to solve the multi traveling salesman problems. Chapter 11 is focused on studying the local minima problem of the competitive network and an improvement strategy is provided. In Chapter 12, a new algorithm for finding the shortest path based on the pulsed coupled networks is proposed.

The next consecutive Chapters (13-15) are devoted to a prominent biologically motivated model, i.e., the recurrent network with linear threshold (LT) neurons. In Chapter 13 qualitative analysis is given regarding the geometrical properties of equilibria and the global attractivity. Chapter 14 analyzes one of important dynamic behaviors of the LT networks, periodic oscillation. Conditions for the existence of periodic orbits are established. Chapter 15 presents

new conditions which ensure roundedness and stability for nonsymmetric and symmetric LT networks. As an important application, the analog associative memory is exploited in terms of storing gray images. The stability results are used to design such an associative memory network.

Chapters 16 and 17 are more focused on approaches of studying the dynamical properties of recurrent neural networks: delayed networks with time varying inputs and background neural networks with uniform firing rate and background input.

Neural Networks: Computational Models and
Applications

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2007, XXII, 300 p. 103 illus., Hardcover

ISBN: 978-3-540-69225-6