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Introduction

1.1 Heat Transfer Processes Containing Periodic Oscillations

1.1.1 Oscillation Internal Structure of Convective Heat Transfer Processes

Real stationary processes of heat transfer, as a rule, can be considered stationary only on the average. Actually (except for the purely laminar cases), flows are always subjected to various periodic, quasiperiodic and other casual oscillations of velocities, pressure, temperatures, momentum and energy fluxes, vapor content and interphase boundaries about their average values. Such oscillations can be smooth and periodic (wave flow of a liquid film or vapor, a flow of a fluctuating coolant over a body), sharp and periodic (hydrodynamics and heat transfer at slug flow of a two-phase media in a vertical pipe; nucleate and film boiling process), on can have complex stochastic character (turbulent flows). Oscillations of parameters have in some cases spatial nature, in others they are temporal, and generally one can say that the oscillations have mixed spatiotemporal character.

The theoretical base for studying instantly oscillating and at the same time stationary on the average heat transfer processes are the unsteady differential equations of momentum and energy transfer, which in case of two-phase systems can be notated for each of the phases separately and be supplemented by the conditions of the physical interface on the boundaries between phases (conditions of conjugation). An exhaustive solution of the problem could be a comprehensive analysis with the purpose of a full description of any particular fluid flow and heat transfer pattern with all its detailed characteristics, including various fields of oscillations of its parameters.

However, at the time being such an approach can not be realized in practice. The problem of modeling turbulent flows [1] can serve as a vivid example.

As a rule at its theoretical analysis, Reynolds-averaged Navier–Stokes equations are considered, which describe time-averaged quantities of fluctuating parameters, or in other words turbulent fluxes of the momentum and energy. To provide a closed description of the process, these correlations by means of various semiempirical hypotheses are interrelated with time-averaged fields of velocities and enthalpies. Such schematization results in the statement of a stationary problem with spatially variable coefficients of viscosity and thermal conductivity. Therefore, as boundary conditions here, it is possible to set only respective stationary conditions on the heat transfer surface of such a type as, for example, "constant temperature," "constant heat flux."

It is necessary to specially note, that the replacement of the full "instant" model description with the time-averaged one inevitably results in a loss of information on the oscillations of fluid flow and heat transfer parameters (velocities, temperatures, heat fluxes, pressure, friction) on a boundary surface. Thus the theoretical basis for an analysis of the interrelation between the temperature oscillations in the flowing ambient medium and in the body is omitted from the consideration. And generally saying, the problem of an account for possible influence of thermophysical and geometrical parameters of a body on the heat transfer at such a approach becomes physically senseless. For this reason, such a "laminarized" form of the turbulent flow description is basically not capable of predicting and explaining the wall effects on the heat transfer characteristics, even if these effects are observed in practice. The problem becomes especially complicated at imposing external oscillations on the periodic turbulent structure that takes place, in particular, in flows over aircraft and spacecraft. Unresolved problems of closing the Navier–Stokes equations in combination with difficulties of numerical modeling make a problem of detailed prediction of a temperature field in the flowing fluid very complicated. In some cases, differences between the predicted and measured local *heat transfer coefficient* (HTC) exceeds 100%.

In this connection the direction in the simulation of turbulent flows based on the use of the primary transient equations [2] represents significant interest. The present book represents results of numerical modeling of the turbulent flows in channels subjected to external fields of oscillations (due to vortical generators etc.). It is shown that in this case an essentially anisotropic and three-dimensional flow pattern emerges strongly different from that described by the early theories of turbulence [1]. In the near-wall zone, secondary flows in the form of rotating "vortical streaks" are induced that interact with the main flow. As a result, oscillations of the thermal boundary layer thickness set on, leading to periodic enhancement or deterioration of heat transfer. Strong anisotropy of the fluid flow pattern results in the necessity of a radical revision of the existing theoretical methods of modeling the turbulent flows. So, for example, the turbulent Prandtl number being in early theories of turbulence [1] a constant of the order of unity (or, at the best, an indefinite scalar quantity), becomes a tensor.

It is necessary to emphasize that all the mentioned difficulties are related to the nonconjugated problem when the role of a wall is reduced only to maintenance of a *boundary condition* (BC) on the surface between the flowing fluid and the solid wall.

1.1.2 Problem of Correct Averaging the Heat Transfer Coefficients

The basic applied task of the book is the investigation into the effects of a body (its thermophysical properties, linear dimensions and geometrical configuration) on the traditional HTC, measured in experiments and used in engineering calculations. Processes of heat transfer are considered stationary on average and fluctuating instantly. A new method of investigation of the conjugate problem "fluid flow–body" is presented. The method is based on a replacement of the complex mechanism of oscillations of parameters in the flowing coolant by a simplified model employing a varying "true heat transfer coefficient" specified on a heat transfer surface.

The essence of the developed method can be explained rather simply. Let us assume that we have perfect devices measuring the instant local values of temperature and heat fluxes at any point of the fluid and heated solid body. Then the hypothetical experiment will allow finding the fields of temperatures and heat fluxes and their oscillations in space and in time, as well as their average values and all other characteristics. In particular, it is possible to present the values of temperatures (exactly saying temperature heads or loads, i.e., the temperatures counted from a preset reference level) and heat fluxes on a heat transfer surface in the following form:

$$\vartheta = \langle \vartheta \rangle + \hat{\vartheta}, \tag{1.1}$$

$$q = \langle q \rangle + \hat{q}, \tag{1.2}$$

i.e., to write them as the sum of the averaged values and their temporal oscillations. For the general case of spatiotemporal oscillations of characteristics of the process, the operation of averaging is understood here as a determination of an average with respect to time τ and along the heat transferring surface (with respect to the coordinate Z). The *true heat transfer coefficient* (THTC) is determined on the basis of (1.1) and (1.2) according to Newton's law of heat transfer [3, 4]:

$$h = \frac{q}{\vartheta}. \tag{1.3}$$

This parameter can always be presented as a sum of an averaged part and a fluctuating additive:

$$h = \langle h \rangle + \hat{h}. \tag{1.4}$$

It follows from here that the correct averaging of the HTC is as follows

$$\langle h \rangle = \left\langle \frac{q}{\vartheta} \right\rangle. \tag{1.5}$$

Therefore we shall call parameter $\langle h \rangle$ an *averaged true heat transfer coefficient* (ATHTC). The problem consists in the fact that the parameter $\langle h \rangle$ cannot be directly used for applied calculations, since it contains initially the unknown information of oscillations $\hat{\vartheta}, \hat{q}$. This fact becomes evident if (1.5) is rewritten with the help of (1.1) and (1.2):

$$\langle h \rangle = \left\langle \frac{\langle q \rangle + \hat{q}}{\langle \vartheta \rangle + \hat{\vartheta}} \right\rangle. \tag{1.6}$$

The purpose of the heat transfer experiment is the measurement of averaged values of an averaged temperature $\langle \vartheta \rangle$ and a heat flux $\langle q \rangle$ on the surfaces of a body and determination of the traditional HTC

$$h_{\mathrm{m}} = \frac{\langle q \rangle}{\langle \vartheta \rangle}. \tag{1.7}$$

The parameter h_{m} is fundamental for carrying out engineering calculations, designing heat transfer equipment, composing thermal balances, etc. However it is necessary to point out that transition from the initial Newton's law of heat transfer (1.3) to the restricted (1.7) results in the loss of the information of the oscillations of the temperature $\hat{\vartheta}$ and the heat fluxes $\hat{q}$ on the wall.

Thus, it is logical to assume that the influence of the material and the wall thickness of the body taking part in the heat transfer process on HTC h_{m} uncovered in experiments is caused by noninvariance of the value of h_{m} with respect to the Newton's law of heat transfer. For this reason we shall refer further to the parameter h_{m} as to an *experimental heat transfer coefficient* (EHTC).

Thus, we have two alternative procedures of averaging the HTC: true (1.5) and experimental (1.7). The physical reason of the distinction between $\langle h \rangle$ and the h_{m} can be clarified with the help of the following considerations:

- Local values $\langle \vartheta \rangle$ and $\langle q \rangle$ on a surface where heat transfer takes place are formed as a result of the thermal contact of the flowing fluid and the body.
- Under conditions of oscillations of the characteristics of the coolant, temperature oscillations will penetrate inside the body.
- Owing to the conjugate nature of the heat transfer in the considered system, both fluctuating $\hat{\vartheta}, \hat{q}$, and averaged $\langle \vartheta \rangle, \langle q \rangle$ parameters on the heat transfer surface depend on the thermophysical and geometrical characteristics of the body.
- The ATHTC $\langle h \rangle$ directly follows from Newton's law of heat transfer (1.3) (which is valid also for the unsteady processes) and consequently it is determined by hydrodynamic conditions in the fluid flowing over the body.

– The EHTC h_m by definition does not contain the information on oscillations $\hat{\vartheta}, \hat{q}$, and consequently it is in the general case a function of parameters of the interface between fluid and solid wall.
– Aprioristic denying of dependence of the EHTC on material properties and wall thickness is wrong, though under certain conditions quantitative effects of this influence might be insignificant.

From the formal point of view, the aforementioned differences between the true (1.5) and experimental (1.7) laws of averaging of the actual HTC is reduced to a rearrangement of the procedures of division and averaging. This situation is illustrated evidently in Fig. 1.1.

Using the concepts introduced above, the essence of a suggested method can be explained rather simply. We shall assume that for the case under investigation the HTC h is known: $h = h\,(Z, \tau)$, where Z and τ are the coordinate along a surface where heat transfer takes place and the time, respectively. According to the internal structure of the considered processes this parameter should have periodic, quasiperiodic, or generally fluctuating nature, varying

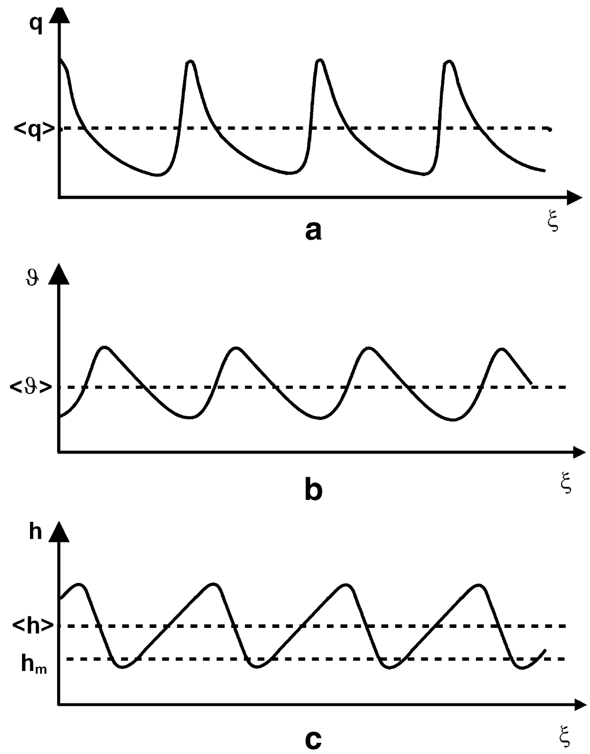

Fig. 1.1. True and experimental laws of the averaging of the heat transfer coefficient: (**a**) heat flux density on the heat transfer surface, (**b**) temperature difference "wall–ambience," and (**c**) heat transfer coefficient

about its average value $\langle h \rangle$: $h = \langle h \rangle + \hat{h}(Z, \tau)$. This information is basically sufficient for the definition of actual driving temperature difference $\vartheta(Z, \tau)$ heat fluxes $q(Z, \tau)$ in a massive of a heat transferring body, and, hence, on the heat transfer surface. Thus, the calculation is reduced to a solution of a boundary value problem of the unsteady heat conduction equation [5]

$$\frac{\partial \vartheta}{\partial \tau} = \alpha \left(\frac{\partial^2 \vartheta}{\partial X^2} + \frac{\partial^2 \vartheta}{\partial Z^2} \right) + \frac{q_V}{c\rho}, \tag{1.8}$$

with the boundary condition (BC) of the third kind on the heat transfer surface

$$-k \frac{\partial \vartheta}{\partial X} = h\vartheta \tag{1.9}$$

and suitable BC on the external surfaces of the body.

It is essential for our analysis that up to the same extent in which the information about the function $h = h(Z, \tau)$ is trustworthy, the computed parameters $\vartheta(Z, \tau)$ and $q(Z, \tau)$ are determined also authentically. The basis for such a statement is the fundamental theorem of uniqueness of the solution of a boundary value problem for the heat conduction equation [5]. In other words, the temperature field ϑ and heat fluxes q found in the calculation should appear identical to the actual parameters, which could be in principle measured in a hypothetical experiment. Further based on the known distributions ϑ and q, it is possible to determine corresponding average values $\langle \vartheta \rangle$ and $\langle q \rangle$, and finally (from (1.7)) the parameter h_{m}, which appears to be a function of the parameters of conjugation. It follows from the basic distinction of procedures of averaging of (1.5) and (1.7) that an experimental value of the actual HTC is not equal to its averaged true value:

$$h_{\mathrm{m}} \neq \langle h \rangle \,, \tag{1.10}$$

The analytical method schematically stated above, in which "from the hydrodynamic reasons" the following relation is stated

$$h(z, \tau) = \langle h \rangle + \hat{h}(Z, \tau) \,, \tag{1.11}$$

and further from the solution of the heat conduction equation in a body the parameter h_{m} is determined, outlines the basic essence of the approach developed in the present book. Different aspects of this method are discussed below in more detail.

1.2 Physical Examples

For the practical realization of this method it is necessary for each investigated process to specify the parameter $h(Z, \tau)$ (i.e., THTC) periodically varying with respect to its average value. A difficulty thus consists in the fact

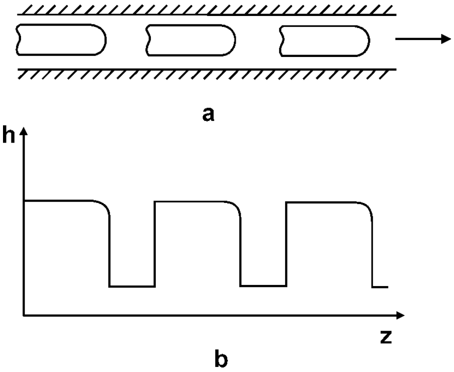

Fig. 1.2. Slug flow of a two-phase fluid: (**a**) schematic of the process, (**b**) variation of the THTC with the longitudinal coordinate

that, generally speaking, a valid function outlining the change of the THTC (with all its details) is unknown for any real periodic process. Therefore, the specification of this parameter is possible only approximately. This freedom in choice of the THTC inevitably makes results of the analysis dependent on the accepted approximations and assumptions. Thus the approximate nature of the developed method consists namely in this aspect. From the mathematical point of view, all constructions, solutions, estimations, and conclusions are obtained quite strictly and precisely. Physical features of some characteristic processes of heat transfer with periodic oscillations are discussed below.

Slug flow of a two-phase medium. A schematic image of this type of flow frequently met in engineering applications is given in Fig. 1.2. Oscillations of the heat transfer intensity in each section of the channel are caused here by the periodic passage of a large steam bubble and a liquid volume. Instant picture of the HTC variation over the height of a pipe is shown in the same figure. The thickness of the liquid film δ_{f} formed on a wall during passage of a steam bubble, can be determined using known recommendations documented in [6, 7]. The THTC is practically equal to thermal conductivity of a liquid layer $k_{\mathrm{f}}/\delta_{\mathrm{f}}$, where k_{f} is the heat conductivity of the liquid phase. During the passage of the liquid, the heat transfer intensity is determined by the relations for heat transfer to a turbulent flow. Thus the character of the variation of the THTC with respect to time and to the vertical coordinate can be considered periodic step function. The curve of $\delta_{\mathrm{f}}\,(Z, \tau)$ here will move upward with speed of movement of the steam bubbles along the wall of a pipe. For the considered case, it is essential that the function $h\,(Z, \tau)$ is determined by fluid flow peculiarities in the two-phase medium and consequently does not depend on the thermophysical properties and thickness of the wall.

Flow over a body in the vicinity of the stagnation point. The schematization of this type of flow is shown in Fig. 1.3. It is easy to show that at presence of the periodic oscillations of the velocity of a fluid about its average value, the heat

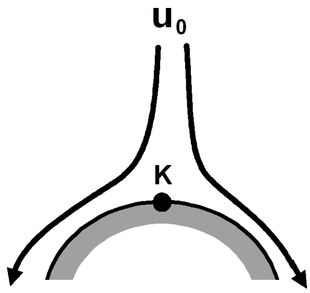

Fig. 1.3. Flow over a body in the vicinity of a stagnation point

transfer intensity will be also periodic in time. In other words, if the period of change in the fluid velocity is essentially larger than the time needed for the individual particles of a liquid to pass by zone where heat transfer is studied (in the vicinity of the frontal stagnation point K), the instant behavior of heat transfer can be considered quasistationary, with the function $h\,(\tau)$ being equal to the stationary dependence $h\,[u_0\,(\tau)]$.

In the considered case, the time variation of the heat transfer intensity follows from the hydrodynamic conditions of flow, and THTC remains actually constant for various materials of the surface.

Flow in a laminar boundary layer. Let us consider stationary flow in a laminar boundary layer on which periodic velocity oscillations are imposed. From the same reasons, as in the example of the fluid flow over a body in the vicinity of a stagnation point considered above, the process of heat transfer here can be considered quasistationary: $h\,(\tau) = h\,[u_0\,(\tau)]$. For a case where the amplitude of the velocity oscillations is comparable to the velocity's average value, it is necessary to expect backward influence of the imposed oscillations on the average level of heat transfer. As known [4], a stationary HTC h_0 in a laminar boundary layer depends on the velocity as

$$h_0 = C\sqrt{u}. \tag{1.12}$$

Here $C = 0.332\rho_{\mathrm{f}}c_{\mathrm{f}}/Pr^{2/3}\sqrt{\nu_{\mathrm{f}}/X}$, X is the distance from the initial stagnation point of a plate. Imposing of harmonic velocity oscillations on the stationary flow $u \to \langle u\rangle\,[1 + b\cos{(2\pi\tau/\tau_0)}]$ results in corresponding oscillations of the THTC $h_0 \to h_0(1 + \tilde{h})$, so that (1.12) takes the following form:

$$h_0(1 + \tilde{h}) = C\sqrt{\langle u\rangle\,[1 + b\cos{(2\pi\tau/\tau_0)}]}. \tag{1.13}$$

Averaging (1.13) over the period of oscillations τ_0 gives:

$$h = Cf\,(b)\,h_0. \tag{1.14}$$

Here $f\,(b)$ is a rather complex function of the oscillations amplitude, which weakly decreases with increasing b: $b = 0, f\,(b) = 1; b = 1, f\,(b) \approx 0.9$. Subtracting (1.14) of (1.13) term-by-term, one can find the fluctuating component of the THTC. In the case of negligibly small amplitude $b \to 0$, these oscillations will look like as a cosine function:

$$h_0 = C\,(b/2)\cos\left(2\pi\tau/\tau_0\right). \tag{1.15}$$

In a limiting case of the maximal amplitude $b = 1$, it can be deduced from (1.13):

$$h_0 = C\left[\pi/2\left|\cos\left(\pi\tau/\tau_0\right)\right| - 1\right]. \tag{1.16}$$

As it is obvious from (1.16), at transition from $b \to 0$ to $b = 1$ oscillations of the heat transfer intensity are strongly deformed: the period decreases twice, and the form sharpens and is pointed from top to bottom. On the other hand, the average heat transfer level changes thus only by $\cong 10\%$: at maximal amplitude ($b = 1$) the ATHTC equals to $h \approx 0.9h_0$. Thus, the strong change in the amplitude of oscillations leads only to minor change of the average heat transfer level.

Wave flow of a liquid film. At film condensation of a vapor on a vertical surface and also at evaporation of liquid films flowing down, one can observe a wave flow of the film already at small values of the film Reynolds numbers [6,7]. Under these conditions, the wavelength essentially exceeds the film thickness, and the phase speed of its propagation is of the same order as the average velocity of the liquid in the film. As the Reynolds numbers increase, the character of flow changes: a thin film of a liquid of approximately constant thickness is formed on the surface, on which discrete volumes of a liquid periodically roll down. At a wave mode of the film flow, the THTC is rather precisely described by the dependence $h\,(Z,\tau) = k_\mathrm{f}/\delta_\mathrm{f}\,(Z,\tau)$ specified for the first time by Kapitsa in his pioneer works [8,9]. It follows from this dependence that at a harmonic film structure the THTC is characterized by an inverse harmonic function (Fig. 1.4). At a flow with a "rolling down" liquid, a description of the THTC can be constructed similarly to the case of the slug flow of a two-phase medium considered above, i.e., also independently of the

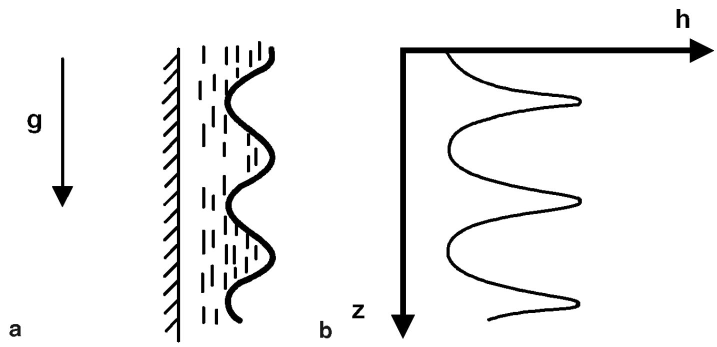

Fig. 1.4. Wave flow of a liquid film: (**a**) schematic of the process (**b**) variation of the THTC with the longitudinal coordinate (g is the gravitational acceleration)

thermal influence of a solid body. At a wave mode of condensation of vapor of liquid metals (sodium, potassium), nonequilibrium molecular-kinetic effects in the vapor phase play a significant role, due to the process of capturing (condensation) of the molecules of vapor. Therefore for a calculation of the heat transfer for vapor condensation (as well as for liquid film evaporation) of a liquid metal, these effects should be taken into account together with the thermal resistance of the liquid film itself determined by the formula of Kapitsa.

Near-wall turbulent flows. The structure of the hydrodynamic oscillations in the turbulent flows are very complex and include a wide spectrum of oscillations with various scales and amplitudes. Along with the so-called stochastic noise, typical for casual processes in a flow, there exist also large-scale periodic oscillations caused by periodic entrainment of accelerated portions of a fluid from the core of the flow into the near-wall region. The average time intervals between these periodic entrainments, and also characteristics of oscillations of the wall friction have been determined in a number of experimental investigations (see, e.g., [10, 11]). On the basis of the Reynolds analogy, it is possible to expect that the wall heat flux will undergo also similar oscillations. It is essential for our analysis that oscillations of parameters are connected with the movement of large turbulent vortical streaks and are consequently caused by the hydrodynamics of the flow. It is again obvious in the examined case that the THTC is independent of the material of a solid body.

1.3 Numerical Modeling of Conjugate Convective–Conductive Heat Transfer

The needs of modern engineering applications (in particular, aerospace engineering) dictate extremely strict requirements for termally loaded surfaces and of critical conditions of the flow aerodynamics. In order to meet these requirements, it is necessary to have an effective tool for the solution of various problems of conjugate convective–conductive heat transfer. Numerical modeling of the velocity field in a fluid flow as well as conjugated temperature fields in a solid body and in the fluid was carried out in [12]. For the calculation of temperature fields at any spatial location and at any moment of time, a Finite-Element method was used. Compact representation of the conjugated fields of temperatures as a uniform symmetric matrix has allowed the author of the work [12] to carry out an effective calculation of a solid body and a fluid for different geometries, thermophysical properties and conditions of heat transfer. Thus, the necessary information on the distribution of temperatures along the heat transfer surface for a number of applied problems (a supersonic flow over an aircraft, flow in compact heat exchangers of a complex configuration, a three-dimensional flow around turbine airfoils etc.) can be obtained. The problem of the thermal interface "fluid flow–body" was schematically represented in [12] as "an aerodynamic triangle." This triangle shows that in any case an interaction between two components takes place, while the third component remains passive. Possible pair interactions are listed below:

– Ambient medium (fluid) and a body cooperate by means of friction and convection. The fluid determines the quantitative, but not the qualitative character of interaction.
– Interaction of a fluid and a body is determined by combination of their thermophysical properties (for example, viscosity and density), and also the nature of a fluid (liquid, gas, or a two-phase stream) independently of a solid body bordering with the fluid.
– A fluid and a body interact through temperature fields and "catalytic effects" independently of the flow regime (laminar or turbulent, incompressible or compressible, etc.).

Ideally, an analysis on the basis of the aerodynamic triangle is called to give an exhaustive description of any conjugate problem. However, as it is pointed in [12], in practice in a real numerical experiment only separate parts (or "legs") of the triangle are used. In other words, by modeling of the particular conjugate problem one should distinguish the main characteristic feature (turbulence, unsteadiness, chemical reactions, etc.). Depending on this, respective simplifications of the mathematical description will be further carried out: linearization of separate terms, replacement of the numerical solution of the system of equations by iterative procedure etc. Thus, the initially global structure of a numerical method results in practice in the necessity of particularly relevant approximations, estimations, neglecting of terms, etc. An application of the specified approximations within the framework of an apparently strict and self-sufficient numerical method is explained in [12] by the primary approximate nature of the used discrete numerical methods, and also by the necessity of minimization the computational time. These inherent features of numerical methods persist until now, despite of the rapid development of these methods over the last decades. Ideologically rather similar to [12] numerical research of the conjugate problem fluid flow–body has been carried out in [13].

As a conclusion, one can note that by modeling of the conjugate systems fluid flow–body in [12,13] important and interesting results have been obtained allowing, in particular, to analyze temperature fields in different interacting media. However, the authors of [12, 13] have not dealt with the problem of averaging of the actual HTC at presence of periodic oscillations in the flow (as well as they have not addressed the whole range of issues associated with this problem and discussed in the present book). As we believe, the reason for this lies not in the computational (mathematical) aspects of the problem, but in the issues that have fundamental (physical) character. On the one hand, the use of the rapidly developing modern computer codes indeed allow solving effectively two- and three-dimensional unsteady transport equations for the conjugated media. On the other hand, as far as it is known to the author, any comprehensive technique has not been created so far that could allow displaying real oscillations of thermohydraulic parameters as respective terms in the transport equations.

Meanwhile there is an urgent need for the everyday engineering and thermophysical practice in creation of a justified tool for a reliable prediction of the thermal energy transfer at the presence of periodic oscillations of thermohydraulic parameters in the flow. So, for example, the account for the dependence of the heat transfer intensity at nucleate boiling of a liquid on the thermophysical properties of a body till now is carried out on the basis of empirical recommendations of [14]. The listed reasons testify in favor of the benefit of the approximate method of the analysis of the periodic connected heat transfer developed in the present book.

1.4 Mechanism of Hydrodynamic Oscillations in a Medium Flowing Over a Body

1.4.1 Van Driest Model

Let us consider the known model of Van Driest [4] describing the law of attenuation of the velocity oscillations in the near-wall region of a turbulent flow. The model is based on the classical exact solution of the Navier–Stokes equations (second problem of Stokes [15]). Consideration is given to an unsteady multilayer flow caused by harmonic oscillations (with the frequency ω) of an infinite solid surface around its own plane. By the virtue of the no-slip BC on the surface, oscillation of the wall results in the fact that the fluid on the solid surface of interface ($y = 0$) possesses some velocity varying under the law:

$$y = 0 : u\,(0, \tau) = u_0 \cos\,(\omega\tau)\,. \tag{1.17}$$

The system of the Navier–Stokes equations is reduced to one equation for the longitudinal velocity, with its convective terms being identically equal to zero:

$$\frac{\partial u}{\partial \tau} = \nu_{\mathrm{f}} \frac{\partial^2 u}{\partial y^2}. \tag{1.18}$$

Solution of (1.18) with the BC (1.17) results in:

$$u\,(y, \tau) = u_0 \exp\left(-y\sqrt{\frac{\omega}{2\nu_{\mathrm{f}}}}\right) \cos\left(\omega\tau - y\sqrt{\frac{\omega}{2\nu_{\mathrm{f}}}}\right). \tag{1.19}$$

According to (1.19), the fluid performs oscillations with amplitude decreasing away of the wall

$$u = u_0 \exp\left(-y\sqrt{\frac{\omega}{2\nu_{\mathrm{f}}}}\right). \tag{1.20}$$

Oscillations of the fluid layer, which is located at the distance y counted from the wall, have a phase shift $y\sqrt{\omega/(2\nu_{\mathrm{f}})}$ in comparison to the oscillations at a wall. The phase shift is directed opposite to movement of the wall. As the surface $y = 0$ is actually at rest, a flow corresponding to synchronous oscillations of the whole infinite volume of a fluid with the velocity $u_0 \cos(\omega\tau)$ is imposed on the obtained flow. This means that in order to provide the required character of the velocity oscillations, an indefinitely extended source of momentum is entered into the right-hand side of (1.18) without any substantial justifications. Extension of the Van Driest scheme for the problem of attenuation of the temperature oscillations results in the necessity of introduction of the similar nonphysical source terms in the energy equation for the fluid. At last, an attempt to state the conjugate problem based on the similar approach results in the physically absurd introduction of virtual thermal sources both in the fluid, and in the body.

As far as it is known to the author of the present book, the mentioned obvious incorrectness of the widely known Van Driest model has not been commented anyhow in the literature. It once again confirms the conclusion that a correct statement of the problem of conjugation of temperature fields in the environment and in a body in view of a real behavior of oscillations (as well as the derivation of its solution) encounters serious difficulties. In this connection, correct approximate models of thermohydraulic processes with periodic intensity gain more importance. A simple model describing interrelation of laws of friction and heat transfer in the turbulent near-wall flow is stated below.

1.4.2 Periodic Model of the Reynolds Analogy

As it is known, for flow in a turbulent boundary layer for $Pr = 1$ a similarity of the longitudinal velocity and temperature fields takes place, from which the classical Reynolds analogy [3, 4, 15] follows

$$St = C_{\mathrm{f}}/2. \tag{1.21}$$

Here

$$St = \frac{q}{\rho_{\mathrm{f}} c_{\mathrm{f}} u_0 \vartheta_0} \quad \text{and} \quad C_{\mathrm{f}}/2 = \frac{\varGamma}{\rho_{\mathrm{f}} u_0^2}, \tag{1.22}$$

are the Stanton number and friction coefficient, respectively; q is the heat flux density; $\varGamma$ is the shear stress. At $Pr \neq 1$, the similarity of the velocity and temperature distributions holds for a turbulent core of the flow, however it is broken in the near-wall region. This case, which is described within the framework of different schemes of the so-called extended Reynolds analogy, results in the use of different correction factors in the right-hand side of (1.21). These corrections are determined, as a rule, with the help of rather labor-consuming procedures (introduction of the radial velocity distributions and friction coefficients, calculation of the Lyon's integrals, etc.). Known correlation for the extended Reynolds analogy [3] looks like

$$St = \frac{C_{\mathrm{f}}/2}{1 + 11.7\sqrt{C_{\mathrm{f}}/2}\left(Pr^{2/3} - 1\right)}. \tag{1.23}$$

Let us show that expressions like (1.23) can be derived from a simple flow model describing the interaction between a wall and a flow periodically entrained from the core of the accelerating cold fluid flow. A physical basis of this model is the phenomenon of the above mentioned "bursting" described in [10,11]. These works mentioned for the first time the existence in near-wall regions of flow of specific coherent structures in form of pair vortices extended in the direction of flow and periodically pushed out into the turbulent core of the fluid. Let us accept that after collisions with a wall the homogeneous volume of a fluid with parameters u_0, ϑ_0 continues moving downstream, leaving on the wall its trace in the form of a laminar boundary layer (Fig. 1.5). Velocity and temperature difference on external boundary of the near-wall layer will be equal to $u_\delta, \vartheta_\delta$, respectively. Let us write down the known laws of friction and heat transfer for a laminar boundary layer [3]

$$\Gamma_\delta\left(Z\right) = A\left(Z\right)\rho_{\mathrm{f}} u_\delta^2, q_\delta\left(Z\right) = \frac{A\left(Z\right)}{Pr^{2/3}}\rho_{\mathrm{f}} c_{\mathrm{f}} u_\delta \vartheta_\delta. \tag{1.24}$$

Here $A\left(Z\right) = 0.332/\sqrt{Re_Z}$, $Re_Z = u_\delta Z/\nu_{\mathrm{f}}$ is the local Reynolds number. In accordance with the phenomenon of "bursting," after a certain time period τ_0 there should be a replacement of the fluid volume drifting over a wall by the new volume invading into the near-wall layer from the turbulent core flow. During this time period, individual particles of the fluid in the laminar wake of the previous fluid volume reach a certain coordinate $Z_0 = u_\delta \tau_0$. The subsequent emission of the decelerated heated fluid from the near-wall region and its replacement with a new portion of the accelerated cold fluid will lead

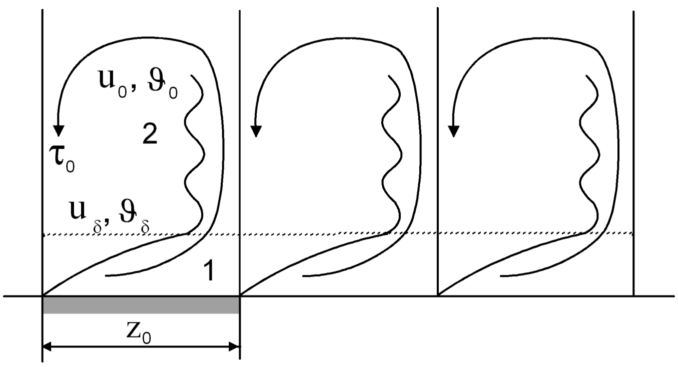

Fig. 1.5. Schematic of the near-wall turbulent flow: 1 – laminar boundary layer, 2 – turbulent core of the flow

to a renewal of a laminar boundary layer on the wall and a repetition of all the subsequent actions. On the external boundary of the near-wall layer, there will be momentum and heat exchange with the fluid invading from the turbulent core. This process can be approximately described with the one-dimensional transient equations for the differences of velocities $u_0 - u_\delta$ and temperatures $\vartheta_0 - \vartheta_\delta$ on border of semi-infinite bodies

$$\mathit{\Gamma}_0\left(\tau\right) = \mu_\mathrm{f}\frac{u_0 - u_\delta}{\sqrt{\pi\nu_\mathrm{f}\tau}}, \quad q_0\left(\tau\right) = k_\mathrm{f}\frac{\vartheta_0 - \vartheta_\delta}{\sqrt{\pi\alpha_\mathrm{f}\tau}}. \tag{1.25}$$

According to the described model, spatial (in near-wall regions) and temporal (in the core of the flow) periodic flow pattern exists. Natural conditions of the interface between these regions will be the equality of the respective time-averaged (with respect to spatial Z_0 and time τ_0 scales) momentum and heat fluxes

$$\langle \mathit{\Gamma}_\delta\left(Z\right)\rangle = \langle \mathit{\Gamma}_0\left(\tau\right)\rangle = \mathit{\Gamma}, \quad \langle q_\delta\left(Z\right)\rangle = \langle q_0\left(\tau\right)\rangle = q. \tag{1.26}$$

Then from (1.22) to (1.26) it is possible to obtain a correlation for the extended Reynolds analogy:

$$St = \frac{C_\mathrm{f}/2}{\sqrt{Pr}\left[1 + \sqrt{(C_\mathrm{f}/2)/\langle A\rangle}\left(Pr^{1/6} - 1\right)\right]}, \tag{1.27}$$

where $\langle A\rangle = 0.664/Re_{Z_0}$, $Re_{Z_0} = u_\delta Z_0/\nu_\mathrm{f}$. For the expression (1.27) to pass to (1.23) in the limiting case of $Pr \to \infty$, it is necessary to put: $\langle A\rangle = 1/11.7^2$. It is interesting to note, that at values of $Pr \geq 1$ correlation (1.27) reduces to the relation

$$St \approx \frac{C_\mathrm{f}/2}{\sqrt{Pr}}, \tag{1.28}$$

which agrees well with the solution given in [16]. The resulting simple model evidently illustrates the physical expediency of taking into account of internal fluctuating structures in real heat transfer processes.

1.4.3 Model of Periodical Contacts

A simple evident model of the conjugate problem fluid flow–body is a scheme of periodic collisions with a surface of a solid body (conductive supply of heat into the system) of the volumes of fluid constantly replacing each other (convective removal of heat) – Fig. 1.6. Since a constant heat flux is supplied from the depth of a solid body, the distribution of the average temperature in the body should look like linear functions. On this linear distribution, temperature oscillations with increasing amplitude (as approaching to the surface) will be imposed. In doing so, the "conductive condition of periodicity" should be fulfilled: temperature distribution in the solid body at time $\tau = \tau_0$ should exactly repeat the respective distribution at time $\tau = 0$. The temperature

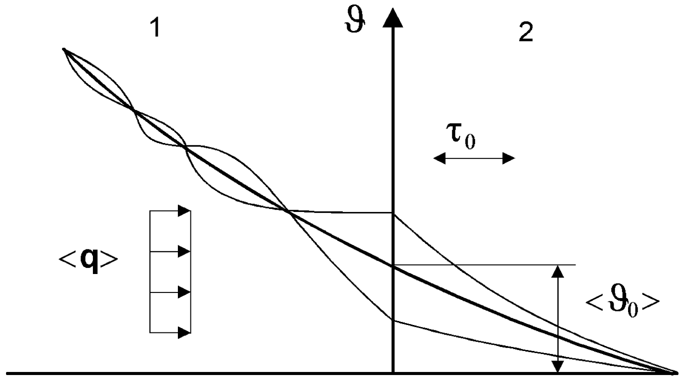

Fig. 1.6. Schematic of the periodical contacts of two media: 1 – body, 2 – ambient fluid

of a surface of the next cold fluid volume will always grow in time (stepwise at the initial moment of time, and then as a monotonic function during the entire period of interaction). The "convective condition of periodicity" will be expressed in the replacement of a heated volume after the end of the interaction of a wall with a new cold volume. The mathematical description of the problem includes the unsteady one-dimensional equations of heat conduction for the solid body and the volume of fluid completed with the conditions of conjugation at the interface (equality of temperatures and heat fluxes). The described model of periodic contacts contains a unique dimensionless parameter, which is the ratio of thermal potentials of the contacting media $K = \sqrt{(kc\rho)/(k_\mathrm{f}c_\mathrm{f}\rho_\mathrm{f})}$. Nevertheless, apparent simplicity of the problem is deceptive. Its solution with the help of the Green's function [5] results in obtaining a complex integro-differential equation. Let us consider the heat conduction equation for a volume of fluid for the limiting cases allowing an analytical solution:

(a) $\vartheta = \text{const}$. The limiting case for $\vartheta = \text{const}$ will be reached for $K \to \infty$. In this case, oscillations of temperatures and averaged temperature gradient in a body will be negligibly small. The known solution [5] for a case $\vartheta = \text{const}$ gives: $q = k_\mathrm{f}\vartheta/\sqrt{\pi\alpha_\mathrm{f}\tau}$. It follows from here that the heat flux averaged over the period of contact t_0 will be equal to $\langle q\rangle = 2k_\mathrm{f}\vartheta/\sqrt{\pi\alpha_\mathrm{f}\tau_0}$. Under these conditions, the EHTC and ATHTC will be equal to each other: $h_\mathrm{m} = \langle h\rangle = 2k_\mathrm{f}/\sqrt{\pi\alpha_\mathrm{f}\tau_0}$. One should note that in the general case final values of the complex K under conditions of conjugation of a flowing fluid and body temperature oscillations will penetrate into the body, and the isothermal wall condition thus will be broken.

(b) $q = \text{const}$. In the other limiting case $K \to 0$, temperature oscillations in a body will reach their maximum. It follows from the Fourier law that at $k \to 0$ an infinitely large average temperature gradient corresponds to a final average heat flux in a body. This means from a physical point of view an unlimited increase in the heat flux rate, in relation to which any finite oscillations will be considered negligibly small. This corresponds to a limiting case $q = \text{const}$. The known solution [5] for this case gives the law of monotonic increase of temperature in time: $\vartheta = 2q\sqrt{\alpha_\mathrm{f}\tau}/(\sqrt{\pi}k_\mathrm{f})$. It means that in the limiting case $K \to 0$ the surface temperature at change of the volumes of fluid falls down abruptly to zero value, and then starts to increase monotonically. Let us obtain relations for the following quantities:

- Averaged temperature difference $\langle\vartheta\rangle = 4q\sqrt{\alpha_\mathrm{f}\tau_0}/(3\sqrt{\pi}k_\mathrm{f})$
- ATHTC $\langle h\rangle = \sqrt{\pi}k_\mathrm{f}/\sqrt{\alpha_\mathrm{f}\tau_0}$
- EHTC $h_\mathrm{m} = 3\sqrt{\pi}k_\mathrm{f}/\left(4\sqrt{\alpha_\mathrm{f}\tau_0}\right)$

An analysis of the transition from the case $K \to \infty$ to $K \to 0$ results in the following conclusions:

- Despite the radical reorganization of the temperature field, oscillations in a body, the EHTC and the ATHTC differ from each other insignificantly (no more than by 25%). Even though this fact is unexpected, it agrees with the physically natural (in other words, physically expected) way of the thermal effect of a wall.
- The EHTC not only does not decrease, but also, on the contrary, increases by $\approx$18%. This result is completely unexpected. The reason of this metamorphosis consists, apparently, in reorganization actually of the ATHTC: for the case of $q = \text{const}$, it appears to be $\pi/2$ times higher, than for the case of $\vartheta = \text{const}$.
- An uncontroversial conclusion follows from the above-mentioned limiting estimations that there is practically no effect of the thermal conjugation within the framework of the model of periodic contacts. More precisely: this effect is so weak that it is not visible on the background of the changes in the character of oscillations of the THTC. This discouraging circumstance can induce quite a critical analysis of applications of the model of periodic contacts available in the literature.

For example, in [17] a method of calculation of the influence of a solid body's material on the growth rate of a steam bubble over a heated surface is proposed for the nucleate boiling regime in a liquid. The method is based on the model of a one-time thermal contact. It is implicitly supposed in this method that, after the termination of the interaction, both volumes (liquid and solid) are replaced with new ones. As a matter of fact, this means the replacement of a periodic problem considered above by a problem of one-time thermal contact of two media with homogeneous initial distributions of temperatures. As it is known [5], a solution of latter can be written in the following simple from

$$\frac{\vartheta}{\vartheta_0} = \frac{K}{1+K}, \tag{1.29}$$

where ϑ_0 is a difference of temperatures between the isothermal bodies before their contact, ϑ is the temperature difference for a fluid volume after the contact. One should point out that, at the given problem statement, the condition $\vartheta = \text{const}$ holds for the entire period of contact. Thus, in the model of one-time thermal contact "conductive condition of periodicity" is absent, with a completely new pair "fluid – solid body" being used for a description of each new contact. The confusion and misunderstanding arising as a result of this in determining of the average temperatures and heat fluxes on a heat transfer surface makes this model incorrect. Apparently, (1.29) has laid a foundation of the correlation from [14], providing introduction of an empirical correction factor such as $\sqrt{K}$ in the formula for a stationary HTC at nucleate boiling.

At the same time, noticeable influence of the complex $\sqrt{kc\rho}$ (coefficient of the thermal activity of a wall) on the measured HTC at nucleate boiling of a liquid is an experimentally established fact. So, it was found in experiments [18] that replacing the heater's material from copper to stainless steel results in a decrease in the heat transfer intensity at boiling cryogenic liquids by an order of magnitude: ≈12 times at boiling of nitrogen and ≈40 times at boiling of helium. Therefore, there is an open question in front of the theory of nucleate boiling to search for the correct models describing thermal influence of a wall on the average intensity of heat transfer.

1.5 Hydrodynamic HTC

As it was mentioned above, an exact specification of all parameters of the THTC is possible only in view of the exact knowledge of all fields of velocities and temperatures for each particular process with allowance for temperature conjugation between the flowing fluid and the body. Such situation can take place as a result of either (a) a global solution of the system of the unsteady differential equations for the substance transfer in the contacting media or (b) a global experiment, which has been carried out with the help of an ideal instrumentation measuring fields of temperatures and heat fluxes in the coolant and in the wall. Acquisition of the full information for the real unsteady (stochastic) process is believed to be unreal, owing to well-known difficulties in mathematical solution and measuring techniques. For the overwhelming majority of applications, however, so detailed information on fluctuating fields of actual parameters is redundantly detailed and superfluous. Therefore, use of the THTC "specified from the outside" cardinally simplifies this situation: an initial conjugate problem for a system "coolant – wall" is replaced by a boundary value problem for the heat conduction equation in the wall. Thus there is an opportunity to obtain analytical solutions for a series of interesting and actual cases in the applications for the EHTC. It is especially significant in that sense that the structure of real processes, as a rule, is defined by simultaneous influence of many factors. Therefore, direct numerical solutions of a particular problem will inevitable reflect only some special cases of the general multiparameter problem. For the determination of the EHTC, we shall attribute a characteristic (typical) function $h(Z, \tau)$, i.e., a "hydrodynamically determined THTC," to each considered process. As shown above, for a series of processes (such as slug flow of a two-phase medium, wave flow of liquid films, a pulsing jet flow over a body, near-wall turbulent flows), a correct definition of the THTC "from hydrodynamic reasons" is physically quite justifiable. An important specificity of the considered processes consists, thus, in an opportunity of a solution of the heat conduction equation for a wall with "an externally specified" (independent of the thermal influence of a wall) BC of the third kind.

A considerably more complex case of thermal interface is represented by the process of nucleate boiling. As it is known [19], heat transfer intensity at boiling is determined by such factors as velocities and the periods of growth of steam bubbles, density of the bubble-producing cites, a temperature head at the beginning of boiling (superheating) etc. These characteristics generally depend on thermophysical properties and thickness of a heat transferring wall, and in some cases (for example, at nucleate boiling of liquids) effects of this influence can be rather significant. Hence, the THTC describing the process of nucleate boiling also should depend on parameters of conjugation.

It is necessary to emphasize that the method developed in the present work and based on the use of the THTC does not depend on the type of functions $h(Z, \tau)$ and is universal in this sense. However, from the point of view of a practical use of this method, a method of specification of the THTC is important. As shown above, information on the hydrodynamic structure of the flow is sufficient for this purpose in some cases. In this case, a replacement of one heat transferring wall by another (made of a different material, having different thickness, heat input conditions), with a two-layer plate or a body of different geometry, etc. does not result in a change of the behavior of the THTC. Then, having solved the heat conduction equation for various bodies with a BC of the third kind, it is possible to obtain a certain "set" of values of the EHTC. Distinction of these EHTC-functions among themselves will also express qualitative and quantitative effects of thermal influence of a body on the averaged heat transfer intensity. For the case of nucleate boiling, a change of the conjugation parameters should result also in a change of the actual THTC. A remedy here can consist, apparently, in a development of initial theoretical models for the THTC, taking into account initial influence on them of the conjugation parameters. Then our method can allow introducing correctly additional amendments to such parameters taking onto account the effect of conjugation. One should also note, that a physical class of the heat transfer processes with the periodic intensity including "hydrodynamically determined" THTC is rather wide and covers, apparently, overwhelming majority of the engineering applications. This circumstance is a powerful argument in favor of the actuality of the present research.

1.6 Previous Investigations of Heat Transfer Processes with Periodic Intensity

Experimental and numerical investigations of heat transfer at laminar flow in a pipe under conditions of periodic oscillations of pressure were carried out in [20, 21]. Similar studies applicable to a flow of gas in regenerators under conditions of an intermittently reversed mass flow-rate have been carried out in [22, 23]. These works based their analysis on a nonconjugate problem statement, i.e., used an initially set wall temperature (fixed value). One should notice that this fact is quite justifiable for the conditions of those particular experiments. It is clear that for the use of air as the coolant, treatment of a physical problem in a thermally conjugate statement is practically unnecessary. Thermophysical properties are many times less than those of metals, and consequently gases cannot basically render any appreciable influence on the temperature field of in a body. On the other hand, the interesting experimental and theoretical information on local HTC periodically changing in time obtained in [20–23] makes a valuable database for a computation of parameters under conditions of the hydrodynamically determined HTC. An indirect confirmation of the presence of the thermal influence of a solid body was obtained in [24]. An experimental research of temperature oscillations in a wall for turbulent flow of water in a channel performed in this work has shown that these oscillations appear for a case of the wall made of stainless steel and are practically completely absent for the case of copper wall. The class of conjugate stationary problems of heat transfer in a laminar boundary layer has been analytically investigated in a series of works by Dorfman [25–27]. An important achievement of the specified works is the substantiation of generalization of the self-similar variables proposed by Blasius [15] and their further use for the case of thermal conjugation. Later analytical solutions of the stationary conjugate problems have been obtained at flow of liquids in channels using a similar approach [28, 29]. The authors of [30, 31] have numerically investigated a stationary conjugate problem for a flow in a channel with discrete sources of heating. It represents an important step on studying of spatioperiodic type of the thermal conjugation. However, in the specified works there is no generalization given concerning the results of the investigated thermal influence.

1.7 Analytical Methods

As known, the majority of problems of hydrodynamics and heat transfer are described by partial differential equations. So, Navier–Stokes and energy equations represent quasilinear partial differential equations, which solution in most cases can only be obtained with the help of numerical methods. This can lead to a "natural" conclusion about an absolute priority of numerical solutions in the specified area of research. However, analytical solutions of the fluid flow and heat transfer problems play a significant role even in the current computer age. They possess the following decisive advantages in comparison with numerical methods:

– The value of the analytical approach consists in an opportunity of the closed qualitative description of the process, revealing of the full list of dimensionless characteristic parameters and their hierarchical classification based on the criteria of their importance.
– Analytical solutions possess a necessary generality, so that a variation of boundary and inlet conditions allows carrying out parametrical investigations.
– In order to validate numerical solutions of the full differential equations, it is necessary to have basic (often rather simple) analytical solutions of the equations for some obviously simplified cases (after an estimation and omission of negligible terms).
– In a global aspect, an analytical solution can be used for a direct validation of the correctness in the statement of numerical investigations applicable to a particular problem.

Analytical investigations of a wide spectrum of fluid flow and heat transfer problems have been carried out in the book of Weigand [32]. Parabolic, elliptic, and hyperbolic partial differential equations of second order were considered. Solutions of a wide class of problems with the help of the classical method of separation of variables are also presented in the book. Classical and modern methods of the analytical solution of the hydrodynamics and heat transfer problems are considered for flow of a fluid in a channel for various conditions: stationary and unsteady (including periodically fluctuating) flow, flow over a thermal initial length, flow in an axially rotating pipe. A limiting case of large eigenvalues of the solution is considered, as well as asymptotic solutions for small Peclet numbers. The class of nonlinear differential equations, opportunities of their linearization, application of self-similar variables have been also thoroughly investigated. The value of the book of Weigand [32] in the sense of the method proposed in the present work consists in the availability of a representative database for determination of "hydrodynamically determined HTC," i.e., in the formation of a theoretical basis for calculating the EHTC. The present book overviewes and generalizes from a single viewpoint results published by the author in works [33–68].

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